I have designed these Rates thus: The Columns of 3, 4 or 5 *per Cent.* for Incomes of Lands, *&c.* that are certain; That Table at 6 *per Cent.* for Houses new or in good Repair, Brick; That of 8 *per Cent.* for the Purchase, *&c.* of Houses that are pretty old; That of 10 *per Cent.* for very old Houses, or not in very good Repair. And they may be used as every one's Discretion directs, according to the foregoing Rules. And as to this Table of Lives, if the Years Purchase be multiply'd by the Annuity, the Product shews the Value of such Estate for such Life. Examples follow.

*The Value of a Life at the several Ages thereof.*

| *Age.* | *Years Purchase.* | *Age.* | *Years Purchase.* | *Age.* | *Years Purchase.* |
|---|---|---|---|---|---|
| 1 | 10.28 | 25 | 12.27 | 50 | 9.21 |
| 5 | 13.4 | 30 | 11.72 | 55 | 8.51 |
| 10 | 13.44 | 35 | 11.12 | 60 | 7.60 |
| 15 | 13.33 | 40 | 10.57 | 65 | 6.54 |
| 20 | 12.78 | 45 | 9.91 | 70 | 5.32 |

*The Use of these Tables of Compound Interest.*

*Prop.* 1.] To find the Amount of any Sum of Money for any Number of Years. This is done by the first Column towards the left hand; so the Amount of *l.*50 in 50 Years Compound Interest allow'd at 5 *per Cent.* is *l.* 573 : 7 : 5 : found by multiplying the Tabular Number against 50 Years by *l.*50 thus.

| | |
|---|---|
| | 11.4674091 |
| | 50 |
| *l.* | 573.3704550 |

*Prop.* 2.] An Office is worth *l.* 500: What may be paid in present Money to enter upon it 15 Years hence? This is done by the 2d Column at 5 *per Cent.* by multiplying the present Worth of *l.* 1 due 15 Years hence by 500, as in the Margin, where the Answer is *l.* 240 : 10 : 2.

| | |
|---|---|
| | .4810171 |
| | 500 |
| *l.* | 240.5085500 |

*Prop.* 3.] An Annuity of 70 *l. per ann.* is forborn 33 Years, what is the Improvement, Compound Interest being

being allow'd the Owner at 5 *per Cent*? By multiplying the Amount of *l.* 1 *per ann.* in 33 Years by the Annuity *l.* 70, you have the Answer *l.* 5604 : 9 : 4 $\frac{1}{2}$.

| | *Years.* | |
|---|---|---|
| *l.* 1 *per ann.* | 33 = | 80.0638328 |
| | | 70 |
| | | *l.* 5604.4682960 |

*Prop.* 4.] There are 13 Years to come in the Lease of an House that is pretty old, but in good Repair, and is *l.* 50 *per ann.* clear; what is a farther Lease for 31 Years worth in present Money, at the Rate of 8 *per Cent*?

I add the 31 Years Reversion to the 13 in *Esse* makes 44.

The present Worth of *l.* 1 due 44 Years hence, is 12.0770735; from which deduct the present Worth for 13 Years, and the Rest is multiply'd by 50 for Answer: *All by the Table of* 8 *per Cent.*

| | *Years.* | |
|---|---|---|
| *l.* 1 *per ann.* | 44 = | *l.* 12.0770735 |
| *l.* 1 . . . . . | 13 = | *l.* 7.9037759 |
| | Rests = | *l.* 4.1732976 |
| | | 50 |
| | Answ. | *l.* 208.66488 |

And if you multiply any of the Numbers in the 4th Columns (according as the Estate is in Land or Houses, as aforesaid) by the Annuity you would know the present Worth of, the Product shews the present Value of such Annuity for any Years required under 62.

*To find the Present Worth of Estates for Lives.*

*Prop.* 5.] To find the Value of an Estate of *l.* 35 *per ann.* in Land for a Life of 45 Years. By the little Table above, as computed by the Learned Dr. *Haley*, that Life is Years Purchase — 9.91
Which being multiply'd by the Annuity 35, the 35
Product is *l.* 346 : 17 : — = the Answer. Prod. *l.* 346.85

*Prop.* 6.] It many times happens, that a Life and so many Years certain to come, are proposed in the same Question; therefore it becomes necessary, as in Computations for two Lives also, *to reduce the Years Purchase of any Life into Years certain to come, or Years in a Lease*: which is done thus.

I find a Life of 50 worth 9.21 Years Purchase. Now if you turn to any of the four Columns in the Tables above, which shew the present Worth of Annuitys according to the several Rates, you will find at 5 *per Cent.* the next to the said 9.21 is 9.393573, against which in the Column under Years is 13 Years to come. At 6 *per Cent.* the next to 9.21 is 9.294984, against which under Years are 14 Years. At 8 *per Cent.* are 9.1216381, against which under Years are

are 17 Years Leaſe. And at 10 *per Cent.* ſtands 9.2372232, (as next to the ſaid 9 21) and againſt that are 27 Years Leaſe.

*Example.* What is the Value of an Houſe of *l.* 96 *per ann.* clear Rent for 23 Years after the Death of a Perſon 60 Years old?

The Age being found = 12 Years to come; therefore by the Table of 8 *per Cent.* Column 4. the Anſwer, as under *Prop.* 4. is found to be as *per* Marg. *l.*395.37504 = *l.* 395 : 7 : 6.

| *Years.* | *Years Purch.* | *Col.*4. 8 *per C.* | *Years to come.* |
|---|---|---|---|
| 60 = | 7.60 —— | or 7.536078 | or = 12 |

*The preſ. Worth of* l.r per ann. —

| | *Years.* | *l.* |
|---|---|---|
| for | 35 = | 11.654568 |
| for | 12 = | 7.536078 |
| of *l.*1 *Reverſion* = | | 4.118490 |
| | | 96 |
| Anſw. = | | 395.37504 |

*Prop.* 7.] *In Computations of the Value of Eſtates for two Lives*; as ſuppoſe 50 and 55. Firſt, find by the little Table of the Value of Lives the Years Purchaſe that the elder Life is worth; then find in the fourth Column at 5 *per Cent.* (ſuppoſing the Purchaſe to be a clear certain Income) the 8.51 found in the ſaid little Table, or the next to it, which is = 8.3064142; right againſt which, in the Column of Years, is 11 Years: to which add the Difference in the Ages (5) and the Sum is 16 Years; againſt which, in the 4th Column from the Years, is 10.8377695, the Value of *l.* 1 *per ann.* for the ſaid two Lives: which multiply'd by the Annuity, gives the Anſwer or Value thereof for the ſaid two Lives, near enough the Truth. Farther Directions might be given on this head; but I have not room to enlarge on things ſo uncertain, and which at beſt depend on many caſual Circumſtances.

## *A* TABLE *shewing by Inspection the Fines payable for any Number of Years lapsed or expired in a Church or College-Lease of their Lands, to make up such Lease* 21 *Years.*

| *Annual Rent of Lands.* | 1 *Year lapsed.* | | | | 2 *Years laps.* | | | | 3 *Years laps.* | | | | 4 *Years laps.* | | | | 5 *Years laps.* | | | | 6 *Years laps.* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Fines Payable.* | | | | *Fines Payable.* | | | | *Fines Payable.* | | | | *Fines Payable.* | | | | *Fines Payable.* | | | | *Fines Payable.* | | | |
| *l.* | *l.* | *s.* | *d.* | *q* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q* | *l.* | *s.* | *d.* | *q.* |
| 1 | | 2 | 0 | 0 | | 4 | 3 | 0 | | 6 | 9 | 0 | | 9 | 6 | 2 | | 12 | 7 | 3 | | 16 | 1 | 2 |
| 2 | | 4 | 0 | 1 | | 8 | 6 | 0 | | 13 | 6 | 0 | | 19 | 1 | 0 | 1 | 5 | 3 | 3 | 1 | 12 | 3 | 0 |
| 3 | | 6 | 0 | 1 | | 12 | 9 | 0 | 1 | 0 | 3 | 1 | 1 | 8 | 7 | 2 | 1 | 17 | 11 | 2 | 2 | 8 | 4 | 2 |
| 4 | | 8 | 0 | 2 | | 17 | 0 | 0 | 1 | 7 | 0 | 1 | 1 | 18 | 2 | 0 | 2 | 10 | 7 | 2 | 3 | 4 | 6 | 0 |
| 5 | | 10 | 0 | 2 | 1 | 1 | 3 | 1 | 1 | 13 | 9 | 1 | 2 | 7 | 8 | 3 | 3 | 3 | 3 | 1 | 4 | 0 | 7 | 2 |
| 6 | | 12 | 0 | 3 | 1 | 5 | 6 | 1 | 2 | 0 | 6 | 1 | 2 | 17 | 3 | 1 | 3 | 15 | 11 | 1 | 4 | 16 | 9 | 1 |
| 7 | | 14 | 1 | 0 | 1 | 9 | 9 | 1 | 2 | 7 | 3 | 1 | 3 | 6 | 9 | 3 | 4 | 8 | 7 | 0 | 5 | 12 | 10 | 3 |
| 8 | | 16 | 1 | 0 | 1 | 14 | 0 | 1 | 2 | 14 | 0 | 1 | 3 | 16 | 4 | 1 | 5 | 1 | 3 | 0 | 6 | 9 | 0 | 1 |
| 9 | | 18 | 1 | 1 | 1 | 18 | 3 | 1 | 3 | 0 | 9 | 2 | 4 | 5 | 10 | 3 | 5 | 13 | 10 | 3 | 7 | 5 | 1 | 3 |
| 10 | 1 | 0 | 1 | 1 | 2 | 2 | 6 | 1 | 3 | 7 | 6 | 2 | 4 | 15 | 1 | 1 | 6 | 6 | 6 | 3 | 8 | 1 | 3 | 1 |
| 20 | 2 | 0 | 2 | 2 | 4 | 5 | 0 | 3 | 6 | 15 | 1 | 0 | 9 | 10 | 10 | 2 | 12 | 13 | 1 | 2 | 16 | 2 | 6 | 3 |
| 30 | 3 | 0 | 3 | 3 | 6 | 7 | 7 | 0 | 10 | 2 | 7 | 2 | 14 | 6 | 4 | 0 | 18 | 19 | 8 | 1 | 24 | 3 | 10 | 0 |
| 40 | 4 | 0 | 5 | 0 | 8 | 10 | 1 | 1 | 13 | 10 | 2 | 0 | 19 | 1 | 9 | 1 | 25 | 6 | 3 | 0 | 32 | 5 | 1 | 1 |
| 50 | 5 | 0 | 6 | 1 | 10 | 12 | 7 | 3 | 16 | 17 | 8 | 2 | 23 | 17 | 2 | 2 | 31 | 12 | 9 | 3 | 40 | 6 | 4 | 3 |
| 60 | 6 | 0 | 7 | 2 | 12 | 15 | 2 | 0 | 20 | 5 | 3 | 0 | 28 | 12 | 7 | 3 | 37 | 19 | 4 | 2 | 48 | 7 | 8 | 0 |
| 70 | 7 | 0 | 8 | 3 | 14 | 17 | 8 | 1 | 23 | 12 | 9 | 1 | 33 | 8 | 1 | 1 | 44 | 5 | 11 | 1 | 56 | 8 | 11 | 1 |
| 80 | 8 | 0 | 10 | 0 | 17 | 0 | 2 | 3 | 27 | 0 | 3 | 3 | 38 | 3 | 6 | 2 | 50 | 12 | 6 | 0 | 64 | 10 | 2 | 3 |
| 90 | 9 | 0 | 11 | 1 | 19 | 2 | 9 | 0 | 30 | 7 | 10 | 1 | 42 | 18 | 11 | 3 | 56 | 19 | 0 | 3 | 72 | 11 | 6 | 0 |
| 100 | 10 | 1 | 0 | 2 | 21 | 5 | 3 | 2 | 33 | 15 | 4 | 3 | 47 | 14 | 5 | 1 | 63 | 5 | 7 | 2 | 80 | 12 | 9 | 1 |
| 200 | 20 | 2 | 0 | 1 | 42 | 10 | 7 | 0 | 67 | 10 | 9 | 3 | 95 | 8 | 10 | 1 | 126 | 11 | 3 | 0 | 161 | 5 | 7 | 0 |
| 300 | 30 | 3 | 1 | 2 | 63 | 15 | 10 | 2 | 101 | 6 | 2 | 2 | 143 | 3 | 3 | 1 | 189 | 16 | 10 | 2 | 241 | 18 | 4 | 1 |
| 400 | 40 | 4 | 2 | 0 | 85 | 1 | 1 | 2 | 135 | 1 | 7 | 2 | 190 | 17 | 8 | 2 | 253 | 2 | 6 | 0 | 322 | 11 | 1 | 2 |
| 500 | 50 | 5 | 2 | 2 | 106 | 6 | 5 | 0 | 168 | 17 | 0 | 0 | 238 | 12 | 1 | 1 | 316 | 8 | 1 | 1 | 403 | 3 | 11 | 0 |

## *A* TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe*, &c.

| *Annual Rent of Lands.* | 7 *Years lapſ.* *Fines Payable.* | 8 *Years lapſed.* *Fines Payable.* | 9 *Years lapſed.* *Fines. Payable.* | 10 *Years lapſ.* *Fines Payable.* | 11 *Years lapſ.* *Fines Payable.* |
|---|---|---|---|---|---|
| *l.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* |
| 1 | 1 | 1 4 4 0 | 1 9 1 3 | 1 14 6 1 | 2 0 6 0 |
| 2 | 2 | 2 8 7 3 | 2 18 3 1 | 3 9 1 0 | 4 1 0 1 |
| 3 | 3 | 3 12 11 2 | 4 7 5 0 | 5 3 6 2 | 6 1 6 2 |
| 4 | 4 | 4 17 3 2 | 5 16 6 2 | 6 18 0 3 | 8 2 0 2 |
| 5 | 5 | 6 1 7 1 | 7 5 8 1 | 8 12 6 3 | 10 2 6 3 |
| 6 | 6 | 7 5 11 0 | 8 14 10 0 | 10 7 1 0 | 12 3 0 3 |
| 7 | 7 | 8 10 3 0 | 10 3 11 2 | 12 1 7 1 | 14 3 7 0 |
| 8 | 8 | 9 14 7 0 | 11 13 1 1 | 13 16 1 2 | 16 4 1 1 |
| 9 | 9 | 10 18 10 2 | 13 2 2 3 | 15 10 7 2 | 18 4 7 1 |
| 10 | 10 | 12 3 2 2 | 14 11 4 2 | 17 5 1 3 | 20 5 1 2 |
| 20 | 20 | 24 6 5 0 | 29 2 9 0 | 34 10 3 2 | 40 10 2 3 |
| 30 | 30 | 36 9 7 1 | 43 14 1 2 | 51 15 5 1 | 60 15 4 1 |
| 40 | 40 | 48 12 9 3 | 58 5 6 0 | 69 0 7 0 | 81 0 5 3 |
| 50 | 50 | 60 16 0 0 | 72 16 10 2 | 86 5 8 3 | 101 5 7 0 |
| 60 | 60 | 72 19 2 2 | 87 8 3 0 | 103 10 10 2 | 121 10 8 2 |
| 70 | 70 | 85 2 5 0 | 101 19 7 2 | 120 16 0 1 | 141 15 10 0 |
| 80 | 80 | 97 5 7 1 | 116 11 0 0 | 138 1 2 0 | 162 0 11 1 |
| 90 | 90 | 109 8 9 3 | 131 2 4 3 | 155 6 3 3 | 182 6 0 3 |
| 100 | 100 | 121 12 0 0 | 145 13 9 0 | 172 11 5 2 | 202 11 2 1 |
| 200 | 200 | 243 4 0 0 | 291 7 6 2 | 345 2 11 1 | 405 2 4 2 |
| 300 | 300 | 364 16 0 0 | 437 1 3 2 | 517 14 4 3 | 607 13 6 3 |
| 400 | 400 | 486 8 0 0 | 582 15 0 2 | 690 5 10 1 | 810 4 9 0 |
| 500 | 500 | 608 0 0 0 | 728 8 10 0 | 862 17 3 3 | 1012 15 11 1 |

## *A* TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe*, &c.

| *Annual Rent of Lands.* | 12 *Years lapſed* *Fines Payable.* | 13 *Years lapſed.* *Fines Payable.* | 14 *Years lapſed.* *Fines. Payable.* | 15 *Years lapſed.* *Fines Payable.* | 16 *Years lapſed.* *Fines Payable.* |
|---|---|---|---|---|---|
| *l.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* | *l. s. d. q.* |
| 1 | 2 7 2 2 | 2 14 8 0 | 3 3 0 0 | 3 12 3 2 | 4 2 7 3 |
| 2 | 4 14 4 3 | 5 9 4 0 | 6 5 11 3 | 7 4 6 3 | 8 5 3 2 |
| 3 | 7 1 7 1 | 8 4 0 0 | 9 8 11 3 | 10 16 10 1 | 12 7 11 1 |
| 4 | 9 8 9 3 | 10 18 8 0 | 12 11 11 3 | 14 9 1 3 | 16 10 7 1 |
| 5 | 11 16 0 0 | 13 13 4 0 | 15 14 11 2 | 18 1 5 0 | 20 13 3 0 |
| 6 | 14 3 2 2 | 16 8 0 0 | 18 17 11 2 | 21 13 8 2 | 24 15 10 3 |
| 7 | 16 10 5 0 | 19 2 8 0 | 22 0 11 2 | 25 5 11 3 | 28 18 6 2 |
| 8 | 18 17 7 1 | 21 17 4 0 | 25 3 11 1 | 28 18 0 3 | 33 1 2 1 |
| 9 | 21 4 9 3 | 24 12 0 0 | 28 6 11 1 | 32 10 4 1 | 37 3 10 0 |
| 10 | 23 12 0 1 | 27 6 8 0 | 31 9 11 1 | 36 2 10 0 | 41 6 5 3 |
| 20 | 47 4 0 2 | 54 13 4 0 | 62 19 10 1 | 72 5 8 0 | 82 12 11 3 |
| 30 | 70 16 0 3 | 82 0 0 0 | 94 9 9 2 | 108 8 6 1 | 123 19 5 3 |
| 40 | 94 8 1 0 | 109 6 8 0 | 125 19 8 3 | 144 11 4 1 | 165 5 11 2 |
| 50 | 118 0 1 1 | 136 13 4 0 | 157 9 8 0 | 180 14 2 1 | 206 12 5 2 |
| 60 | 141 12 1 2 | 164 0 0 0 | 188 19 7 1 | 216 17 0 1 | 247 18 11 1 |
| 70 | 165 4 1 3 | 191 6 8 0 | 220 9 6 2 | 252 19 10 1 | 289 5 5 1 |
| 80 | 188 16 2 0 | 218 13 4 0 | 251 19 5 3 | 289 0 8 2 | 330 11 11 0 |
| 90 | 212 8 2 1 | 246 0 0 0 | 283 9 4 3 | 325 3 6 2 | 371 18 5 0 |
| 100 | 236 0 2 3 | 273 6 8 0 | 314 19 4 1 | 361 8 4 2 | 413 4 10 3 |
| 200 | 472 0 5 2 | 546 13 3 0 | 629 18 8 1 | 722 16 9 1 | 826 9 10 0 |
| 300 | 708 0 8 1 | 819 19 10 2 | 944 18 0 1 | 1084 5 1 3 | 1239 14 8 2 |
| 400 | 944 0 11 0 | 1093 6 6 0 | 1259 17 4 2 | 1445 13 6 2 | 1652 19 7 1 |
| 500 | 1180 1 1 1 | 1366 13 1 1 | 1574 16 8 2 | 1807 1 11 0 | 2066 4 6 1 |

## *A* TABLE *ſhewing by Inſpection the Fines payable for any Number of Years lapſed or expired in a Church or College-Leaſe*, &c.

| *Annual Rent of Lands.* | 17 *Years lapſed.* *Fines Payable.* | | | | 18 *Years lapſed.* *Fines Payable.* | | | | 19 *Years lapſed.* *Fines Payable.* | | | | 20 *Years lapſed.* *Fines Payab'e.* | | | | 21 *Years Leaſe.* *Fines Payable.* | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *l.* | *l.* | *s.* | *d.* | *q* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q.* | *l.* | *s.* | *d.* | *q.* |
| 1 | 4 | 14 | 2 | 2 | 5 | 7 | 1 | 2 | 6 | 1 | 6 | 1 | 6 | 17 | 6 | 0 | 7 | 15 | 6 | 0 |
| 2 | 9 | 8 | 5 | 0 | 10 | 14 | 2 | 3 | 12 | 3 | 0 | 1 | 13 | 14 | 11 | 3 | 15 | 11 | 0 | 0 |
| 3 | 14 | 2 | 7 | 3 | 16 | 1 | 4 | 1 | 18 | 4 | 6 | 2 | 20 | 12 | 5 | 3 | 23 | 6 | 6 | 0 |
| 4 | 18 | 16 | 10 | 1 | 21 | 8 | 5 | 2 | 24 | 6 | 0 | 3 | 27 | 9 | 11 | 3 | 31 | 2 | 0 | 0 |
| 5 | 23 | 11 | 0 | 3 | 26 | 15 | 7 | 0 | 30 | 7 | 6 | 3 | 34 | 7 | 5 | 3 | 38 | 17 | 6 | 0 |
| 6 | 28 | 5 | 3 | 2 | 32 | 2 | 8 | 2 | 36 | 9 | 1 | 0 | 41 | 4 | 11 | 2 | 46 | 13 | 0 | 0 |
| 7 | 32 | 19 | 6 | 0 | 37 | 9 | 9 | 3 | 42 | 10 | 7 | 1 | 48 | 2 | 5 | 2 | 54 | 8 | 6 | 0 |
| 8 | 37 | 13 | 8 | 2 | 42 | 16 | 11 | 1 | 48 | 12 | 1 | 1 | 54 | 19 | 11 | 2 | 62 | 4 | 0 | 0 |
| 9 | 42 | 7 | 11 | 0 | 48 | 4 | 0 | 2 | 54 | 13 | 7 | 2 | 61 | 17 | 5 | 2 | 69 | 19 | 6 | 0 |
| 10 | 47 | 2 | 1 | 3 | 53 | 11 | 2 | 0 | 60 | 15 | 1 | 3 | 68 | 14 | 11 | 1 | 77 | 15 | 0 | 0 |
| 20 | 94 | 4 | 3 | 1 | 107 | 2 | 4 | 0 | 121 | 10 | 3 | 1 | 137 | 9 | 10 | 3 | 155 | 10 | 0 | 1 |
| 30 | 141 | 6 | 5 | 0 | 160 | 13 | 6 | 0 | 182 | 5 | 5 | 0 | 206 | 4 | 10 | 0 | 233 | 5 | 0 | 2 |
| 40 | 188 | 8 | 6 | 3 | 214 | 4 | 8 | 1 | 243 | 0 | 6 | 3 | 274 | 19 | 9 | 2 | 311 | 0 | 0 | 3 |
| 50 | 235 | 10 | 8 | 1 | 267 | 15 | 10 | 1 | 303 | 15 | 8 | 1 | 343 | 14 | 8 | 3 | 388 | 15 | 0 | 3 |
| 60 | 282 | 12 | 10 | 0 | 321 | 7 | 0 | 1 | 364 | 10 | 10 | 0 | 412 | 9 | 8 | 0 | 466 | 10 | 1 | 0 |
| 70 | 329 | 14 | 11 | 2 | 374 | 18 | 2 | 1 | 425 | 5 | 11 | 2 | 481 | 4 | 7 | 2 | 544 | 5 | 1 | 1 |
| 80 | 376 | 17 | 1 | 1 | 428 | 9 | 4 | 1 | 486 | 1 | 1 | 1 | 549 | 19 | 6 | 3 | 622 | 0 | 1 | 1 |
| 90 | 423 | 19 | 3 | 0 | 482 | 0 | 6 | 1 | 546 | 16 | 3 | 0 | 618 | 14 | 6 | 1 | 699 | 15 | 1 | 2 |
| 100 | 471 | 1 | 4 | 2 | 535 | 11 | 8 | 2 | 607 | 11 | 4 | 2 | 687 | 9 | 5 | 2 | 777 | 10 | 1 | 3 |
| 200 | 942 | 2 | 9 | 1 | 1071 | 3 | 5 | 0 | 1215 | 2 | 9 | 1 | 1374 | 18 | 11 | 1 | 1555 | 0 | 3 | 2 |
| 300 | 1413 | 4 | 1 | 3 | 1606 | 15 | 1 | 1 | 1822 | 14 | 1 | 3 | 2062 | 8 | 4 | 3 | 2332 | 10 | 5 | 1 |
| 400 | 1884 | 5 | 6 | 2 | 2142 | 6 | 10 | 0 | 2430 | 5 | 6 | 2 | 2749 | 17 | 10 | 1 | 3110 | 0 | 7 | 0 |
| 500 | 2355 | 6 | 11 | 0 | 2677 | 18 | 6 | 1 | 3037 | 16 | 11 | 0 | 3437 | 7 | 3 | 3 | 3887 | 10 | 8 | 3 |

*The Use of this Table.*

*Example* 1.] What Fine is to be paid, to make up a Lease 21 Years, when 9 Years of 21 are lapsed (or let slip) supposing the Rent *l.*147 *per annum*?

| | *per ann.* | *Fine.* |
|---|---|---|
| In the Table under 9 Years lapsed against | *l.* 100 is | *l.*145 : 13 : 9 |
| against | 40 ... | 58 : 5 : 6 |
| | 7 ... | 10 : 3 : 11½ |
| | Sums, *Rent* 147 | *Fine* 214 : 3 : 2½ |

There are two Books that afford us a Table of this kind: The first was written by the Reverend *John Newton* D. D. one of the King's Chaplains, *Anno* 1668. which contains only Fines, or Value of 1 *l. per ann.* Rent, all in Decimals. The other is said to be approved of by a very great Author, (who perhaps never saw it) which is very tedious in its Use; as I shall shew by the Example above, which is performed by my Table: To find the Fine payable to make up 9 Years lapsed 21 Years; Rent 147 *l. per ann.* his Answer is 1 : 1 : 2 : 5 Purchase, taken out of one Table; which he values by another thus:

| *per ann.* | *Year.* | *Qr.* | *Mon.* | *Tenths.* |
|---|---|---|---|---|
| *l.* 147 for 1 Year = | *l.* 147 : | — : | — | |
| 100 for 1 Quarter = | *l.* 25 : | — : | — | |
| 2 Months = | | 16 : | 13 : | 4 |
| 5 Tenths of a Mon. | | 4 : | 3 : | 4 |
| *l.* 40 for 1 Quarter ..... | | 10 : | — : | — |
| 2 Months ..... | | 6 : | 13 : | 4 |
| 5 Tenths of a Month | | 1 : | 13 : | 4 |
| 7 for 1 Quarter ..... | | 1 : | 15 : | — |
| 2 Months ..... | | 1 : | 3 : | 4 |
| 5 Tenths of a Month | | — : | 5 : | 10 |

Sums.

| *l. per ann.* | *Fine.* |
|---|---|
| 147. | *l.* 214 : 7 : 6 |

But 'tis plain the Use of my Table (as above) is much more brief, and will be allow'd more accurate.

*Example* 2.] To make up a Lease of 14 Years any Numbers of Years less than 21: As suppose I would add 5 Years to 14 in a Lease of *l.* 40 *per ann.* take 5 from 7 there rests 2; then take the Fine payable for 2 Years lapsed = *l.* 8 : 10 : 1 : 1 from the Annual Rent, (here 40) and the Remainer is = *l.* 31 : 9 : 10 : 3 = the Sum to be paid to add 5 Years.

*Example* 3.] To find the Value of a Church or College full Leaſe of Lands of 21 Years: as ſuppoſe *l.* 80 *per ann.* The laſt Column in the Table againſt *l.* 80 gives you *l.* 622 : — : 1 : 1, the Anſwer.

I need not tell the Reader, That in caſe of Improvement by the Seller, he may advance the Rent to the Buyers in proportion.

## SECT. IX. *To Extract the Square and Cube Roots of Decimals and Mixt Numbers.*

### 1ſt, *For the Square Root.*

FOR the Square Root. What is the Square Root of .000976541?

To what is ſaid under the Extraction of the Square Root for Intire Numbers, it is only neceſſary to add for Decimals, that you begin to point over Seconds place of the Decimal given, and ſo proceed over every other towards the right hand: and if the Decimal Places given are not an even Number, you muſt make them ſo, by adding a Cypher towards the right hand.

. . . . .
.00097 65410 (.03124 = Root.
. . . .
6) 76
62) 1554
624) 31010
6034 reſts.

2. That for every Decimal Place you would have in the Root, you muſt (by adding Cyphers towards the right hand, if need be) make twice ſo many Decimal Places in that given to have its Root extracted.

There are ſeveral Ways of Extracting the Roots, but I chuſe that natural one, according to the Algebraic Canon, as in whole Numbers foregoing.

### 2*dly*, *For the Cube Root.*

*Example* 1.] *Of a Mixt Number.* What is the Cube Root of 32.934168093464? Anſwer 3.2054.

In theſe Caſes you muſt point over the Thirds place of the Decimal, *&c.* over every Third toward the right hand adding Cyphers, if the Point fall not over the place next the right hand.

*Example* 2.] What is the Cube Root of 91? See the Work.

In Examples of this kind, where the whole Number is not a compleat Cube, you muſt add ſo many times 3 Cyphers towards the right hand, as you intend to have Decimal Places in the Root:

as

as here are 6 Cyphers for the 2 Decimal Places in the Root. And if you desire more, it is but putting 3 Cyphers towards the right hand of the Remainer 481151, and proceed to make that Root as exact as you please.

3. If you have the Square or Cube Roots of a Vulgar Fraction to Extract, you may reduce it to a Decimal, and then proceed as in the Example above.

4. And if the Decimals have Cyphers before next the Point, keep to the Rule under the last Example of the Cube Root above for the Points over: as .014672, point thus .014672; .0014672, point thus, .00146720; and .00014672 point thus, .000146720, *&c.* and extract as in Whole Numbers: For in this Case of the Cube of Decimals, their Places must always be, or be made, 3, 6, or 9, *&c* by putting Cyphers toward the right hand of the Decimal whose Root is required.

Having already shewn how to extract the Square, Cube, *&c.* Roots of Intire Numbers, and Fractions Vulgar and Decimal; I shall in the Use of Logarithms give a much more easy and short Way of Extraction, and also shew the Use of Logarithms in resolving Questions of Interest: on both which accounts, Logarithms (among many other of their Uses) are extremely to be valued.

```
. . .
91.000000 (4.49
-----------
27000
-----
 12
 48
-----
 492
-----
  64
 192
 192
-----
21184
-------
5816000
-------
  132
 5808
-------
 58212
-------
   729
 10692
 52272
---------
5334849
---------
481151 rests.
```

*Here ends Decimal Arithmetic.*

CHAP.

# CHAP. IV.

## *Duodecimal Arithmetic.*

I. *Umeration.*] I have shew'd under *Duodecimals*, in the Definition of Numbers, *Chap.* 1. what these Fractions are; as in *Foot. I II III IIII* 1 : 1 : 1 : 1 : 1. This is read, 1 Foot, 1 Prime (or 12th of a Foot) 1 Second (or 12th of a Prime) 1 Third (or 12th of a Second) 1 Fourth (or 12th of a Third) *&c.* which is sufficient to know how to read or write Duodecimals. I shall proceed to

II. *Addition.*] This is no more of consequence to what is above, but to divide the Sum of each Series by 12, putting down the Remainer, and carrying the Quotes to the next. But that next the left hand is done as all other things that have but one Denomination. The Example makes it clear.

| *Feet.* I | II | III | IIII | IIIII |
|---|---|---|---|---|
| 321 : | 11 : | 10 : | 9 : | 8 : 7 |
| 210 : | 10 : | 9 : | 8 : | 7 : 6 |
| 109 : | 9 : | 8 : | 7 : | 6 : 5 |
| 98 : | 8 : | 7 : | 6 : | 5 : 4 |
| 87 : | 7 : | 6 : | 5 : | 4 : 3 |
| 76 : | 6 : | 5 : | 4 : | 3 : 2 |
| 65 : | 5 : | 4 : | 3 : | 2 : 1 |
| *Sum* 971 : | 0 : | 4 : | 9 : | 1 : 4 |

III. *Substraction of Duodecimals.*] A Joiner having lined several Rooms with Cedar, as above; finds the Deductions for the Apertures, as Chimneys, Windows, Beauvets, Arches, Vistoes, *&c.* to be 89 : 9 : 8 : 7 : 6 : 5

How many Feet must he be paid for?

Substracting the lesser from the greater, adding 12 where the upper is too little, or 10 in that to the left, the Remainder is the Answer 881 : 2 : 8 : 1 : 6 : 11

IV. *Multiplication.*] This is the principal Rule to be insisted on in this kind of Arithmetic, as being much the oftenest in use, and consequently the most to be observed. *Ex-*

*Example* 1.] Multiply 9 *F.* 10′ by 8 : 8′ Feet. See the Work in the Margin: Where I ſay, 8 times 10 Primes is 80 or 6′ to carry, and 8″ to put down; then 8 times 9 is 72, and 6 is 78 Primes, which divided mentally by 12 is 6 *F.* 6′ which I put down as you ſee. Then I multiply 9 : 10′ by the 8 *F*, ſaying 8 times 10 is 80 Primes, which is 8′ and 6 Foot to carry; then 8 times 9 is 72, and 6 carried is 78: the Sum of which 2 Lines or Products is 85 : 2′ : 8″ = the Anſwer.

| *Foot.* | ′ | ″ |
|---|---|---|
| 9 | 10 | |
| 8 | 8 | |
| 6 : | 6 : | 8 |
| 78 : | 8 | |
| | ′ | ″ |
| Foot 85 : | 2 : | 8 = Anſwer. |

By which Example you may perceive the Neceſſity of having in your mind the Product of any 2 Numbers under 13, or not exceeding 12, by 12, as in the Multiplication-Table.

*Example* 2.] Multiply 16 : 10 : 6 by 9 : 3. See the Operation. Here 'tis plain I multiply, 1ſt, the Seconds, then the Primes, and then the Integers by the 3, carrying the Quote of each Product divided by 12 to that towards the left hand; placing the Remainer in each Diviſion under the Degrees of the Factors, as the Example indicateth.

| *F.* | ′ | ″ |
|---|---|---|
| 16 : | 10 : | 6 |
| 9 : | 3 : | — |
| 4 : | 2 : | 7 : 6 |
| 151 : | 10 : | 6 |
| *Prod.* 156 : | 1 : | 1 : 6 |

*Examp.* 3.] Multiply 372 : 11′ : 4″ by 25 : 6′ : 3″. I have inſerted the whole Work. As,

1ſt, I multiply 4″ by 3 produceth 12, ſo I carry 1; 11′ by 3 is 33, and 1 is 34, I put down 10, and carry 2; then I multiply 372 by the 3, adding the 2 makes 1118″, all which I put down as you ſee. Then I multiply 372 : 11 : 4 by 6′, and it produceth 2237′ : 8″ : 0‴, as you ſee.

| | | I | II | III | IIII |
|---|---|---|---|---|---|
| | 372 | 11 | 4 | | |
| | 25 | 6 | 3 | | |
| 1ſt *Product* | .... | ..... | 1118 | 10 | |
| 2d *Product* | | 2237 | 8 | | |
| 3d *Product* | | 283 | 4 | | |
| *Sum* | | 2520 | 1130 | 110 | |
| *Quote* | ..... | 94 | 12 | | |
| *Sum* | ..... | 2614 | | | |
| *Quote* | 217 | 110 | | | |
| *Prod.* 372 *by* 25 = | 9300 | | | | |
| *Add.* | | | | | |
| *Prod. or Anſw.* = | 9517 | 10 | 2 | 10 | |

*And this I take to be the moſt eaſy and natural Way of Working, when the Integers are large Numbers.*

3*dly*, I multiply 372 : 11 : 4 by 25 : as 4 times 25 is 100, or $8^{l} : 4^{ll}$; put 4 down, and carry 8; 11 times 5 is 55, and 8 is $63^{l}$, put 3 down, and carry 6; 11 times 2 is $22^{l}$, and 6 is 28, or $283^{l} : 4^{ll}$.

4*thly*, I sum up those 3 Lines makes $2520^{l} : 1130^{ll} : 10^{lll}$.

5*thly*, I divide the 1130 Seconds mentally by 12, produceth $94^{l}$, which makes $2614^{l} : 2^{ll}$.

6*thly*, I divide, as before, $2614^{l}$ by 12, produceth 217 Integers, and $10^{l}$ remains.

7*thly*, I multiply the 372 by the 25 at once, as taught in Decimals, and the Product is 9300: So the Sum or Answer is 9517 *Integ.* $10^{l} : 2^{ll} : 10^{lll} : 0^{llll}$.

*Notes to be observed as appear by the above Examples.*

If you multiply 2ds by 2ds, the Product is 4ths; Primes by 2ds place gives 3ds, Primes by Primes gives Seconds, and Integers by Primes gives Primes, and by 2ds gives 2ds, *&c.* And accordingly it is proper to place them Degrees towards the right hand, that so the Degrees of each part of the Operation may fall under the like in the given Duodecimals.

So also 2 Places given in order, as Integers and Primes, to be multiplied by 2 Places or Denominations, produce 3 Places; 3 Places by 2 produce 4 Places; and 3 Places by 3 produceth 5 Denominations, or Degrees in the Product; *i. e.* 1 less than the Sum of the Exponents or Places, or Names in both the Factors given.

V. *Division of Duodecimals.*] Divide 156 *F.* $1^{l} : 1^{ll} : 6^{lll} : 9^{llll}$ by 9. See the Operation: Where what rests is always reduced into the next Denomination or Degree, and the respective Figures of each Degree added.

| *F.* | *l ll lll llll* |
|---|---|
| 9 (156 : 1 : 1 : 6 : 9 | (17 : 4 : 1 : 6 : 1 = Quote. |
| 66 | |
| 37 | |
| 13 | |
| 54 | |
| 9 | |

There is rarely occasion ('tis to be supposed) to divide by a Number of more than 1 Degree or Name: yet in case there should,

should, it may be done as in the Margin, where 85 : 2′ : 8″ is divided by 8 : 8′. Where I say, the Eights in 85 are 9 times, which put in the Quote, and say 9 times 8′ is 72, (which is 6 to carry, and put down 0) 9 times 8 Integers is 72, and 6 is 78, which put down, and deducted, the Remainer is 7 : 2′ or 86′; to which bring down the 8″, and say 8 in 86 is 10 times; 10 times 8′ is 80, or 6′ : 8, put the 8 down, and carry the 6, saying 10 times 8 is 80, and 6 is 86, which deducted, 0 remains: which proves the Truth of the first Example in Multiplication.

```
F.  1        1   11      1
8 : 8) 85 : 2 : 8 (9 : 10 = the Quote.
       78 : 0
       -------
        7 : 2
       86 : 8
       86 : 8
       -------
            0 rests.
```

And by the same Rules the Example in the Margin, or any other, is performed. But it may be observed, that in this Division more than one Place of Figures may, and often must be put in the Quote at once.

```
    1        1   11  111      1   11
9 : 3) 156 : 1 : 1 : 6 (16 : 10 : 6
       148 : 0          = Quote.
       -------
         8 : 1
        97 : 1
        92 : 6
       -------
         4 : 7
           55 : 6
           55 : 6
       -------
            0 rests.
```

# CHAP. V.

## *Sexageſimal Arithmetic.*

THESE Fractions, or 60th Parts, are by ſome called Aſtronomicals, becauſe uſed in the Menſuration of Time and Motion in Aſtronomy: as $1^{h}$: $1^{I}$: $1^{II}$: $1^{III}$: $1^{IIII}$, that is, 1 Hour, 1 Minute (or 60th part of an Hour) 1 Second (or 60th part of a Minute) 1 Third (or 60th part of a Second) 1 Fourth (or 60th of a Third) *&c.* of Time. And in Motion or Meaſure in the Heavens, Earth, or Sea, $1^{o}$: $1^{I}$: $1^{II}$ is 1 Degree (or 60, but according to ſome 'tis *69 Engliſh* Miles upon the Terraqueous Globe) $1^{I}$ is 1 Minute (or 60th part of a Degree) $1^{II}$ is 1 Second (or 60th of a Minute) $1^{III}$ is the 60th of a Second, *&c.* See *Sexageſimal Numbers*, at the beginning of *Chap.* 1.

*Addition* is only to ſum up each Column, and divide them by 60, carrying the Quotes, *&c.* And *Subſtraction* is ſo eaſy, that neither of them need an Example. But

*In Multiplication*

I think the beſt way is to work as *per* Margin; as being a methodical and eaſy Way, which does not charge the Memory: and I am ſure 'tis more brief than any Way I have ſeen. The Rule is plain; for having multiplied the Denominations one in another, and placed the whole Products as you ſee, I add them; and find the Sum as under the ſecond black Line.

| *h.* | *I* | *II* | |
|---|---|---|---|
| 57 : | 29 : | 52 | } Multiply. |
| 6 : | 12 : | 10 | |
| | | 570 : | 290 : 520 |
| | 684 : | 348 : | 624 |
| 342 : | 174 : | 312 | |
| 342 : | 858 : | 1230 : | 914 : 520 |
| 14 : | 20 : | 15 : | 8 |
| *Anſwer* = 356 : | 38 : | 45 : | 22 : 40 |

Then I begin at the right hand to divide by 60, and find $520^{IIII}$ to be $8^{III}$ : $40^{IIII}$, placed as you ſee; and ſo I proceed with the reſt, not making a Digit more than what is down. *Ex-*

*Example* 2.] $75^{h} : 33^{I} : 52^{II} : 27^{III}$ by $32 : 35^{I} : 24^{II} : 42^{III}$. See the Operation just in the same Method with the last Example above.

| | *h.* | *I* | *II* | *III* | *IIII* | *IIIII* | *IIIIII* |
|---|---|---|---|---|---|---|---|
| Multiply } | 75 | 33 | 52 | 27 | | | |
| | 32 | 35 | 24 | 42 | | | |
| *Multiplication of Sexagesimals.* | | | | 3150 | 1386 | 2184 | 1134 |
| | | | 1800 | 792 | 1248 | 648 | |
| | | 2625 | 1155 | 1820 | 945 | | |
| | 2400 | 1056 | 1664 | 864 | | | |
| *Sums Totals* | 2400 | 3681 | 4619 | 6626 | 3579 | 2832 | 1134 |
| *Quotes* | = 62 | 78 | 111 | 60 | 47 | 18 | |
| *Product* = | 2462 | 39 | 50 | 26 | 26 | 30 | 54 *Answ.* |

Here it may be noted, that I multiply $27^{III}$, $52^{II}$, &c. by $42^{III}$, without making 2 Lines, (as is taught in Multiplication of Decimals.)

2*dly*, I begin at $1134^{IIIIII}$ to divide the Line of Totals by 60, (or all except Units place by 6) so the Quote is 18, and Remainer 54; then I divide $^{28}_{1}{}^{3}_{8}{}^{2}$ by 60, and 47 is the Quotient, and 30 the Remainer, &c.

*I shall give the Reader the Proof of this by Division*, for a Conclusion; *viz.*

*Division of Sexagesimals.*

| *I* *II* *III* | *I* *II* *III* *IIII* *IIIII* *IIIIII* | *I* *II* *III* |
|---|---|---|
| $32 : 35 : 24 : 42$ ) | $2462 : 39 : 50 : 26 : 26 : 30 : 54$ ( | $75 : 33 : 52 : 27$ = Quote. |
| | 2444 : 15 : 52 : 30 = the Divisor multiplied in the 75 in the (Quote, and divided by 60. | |
| | 18 : 23 : 57 : 56 the Difference between the 4 first places in (the two last Lines. | |
| | 1103 : 57 : 56 : 26 ditto with the Integ. reduc'd to Primes, &c. | |
| | 1075 : 28 : 35 : 6 the Divisor multiplied in $33^{I}$. | |
| | 28 : 29 : 21 : 20 the Difference between the 2 last. | |
| | 1709 : 21 : 20 : 30 ditto with the $28^{I}$ reduced to $^{II}$, &c. | |
| | 1694 : 41 : 24 : 24 the Divisor multip. in $52^{II}$ in the Quote. | |
| | 14 : 39 : 56 : 6 the Difference between the 2 last. | |
| | 879 : 56 : 6 : 54 ditto with the $14^{II}$ reduced to $^{III}$. | |
| | 879 : 56 : 6 : 54 the Divisor mult. in 27 in Quote. | |

o Remains.

# CHAP. VI.

## *Political Arithmetic.*

THIS Specie of Arithmetic has nothing new in it, as to the Nature of the Numbers themſelves, nor as to the Manner of Operation; but only in the Application or Subject about which the Numbers are employ'd, which is Political (or relating to Polity or Government:) As to diſcover the Riches and Strength of a Nation, by eſtimating its Income from Land, Trade, Commerce, Art and Labour, Exports, Imports, its Number of People, Males, Females, Fighting Men, *&c.*

And there have been ſeveral Pieces written on this Subject, as Mr. *Grant*'s Obſervations on the Bills of Mortality of *London* in 1676, done at the requeſt of the Royal Society; Sir *William Petty*'s Political Arithmetic in 1687, and his Book of the Uſe of Duplicate Proportion; the Learned Dr *Haley*'s Obſervations on the Bills of Mortality of *Breſlaw* in 1692, whereby he computes the Value of Annuitys for Lives at different Ages thereof, *&c.* as mentioned a little farther; and Dr. *Davenant* and *Gregory King* Eſq; on Revenues and Trade, *&c.* I ſhall give ſome Examples of this Way of Computing: As,

*Prop.* 1. To find the Number of People in *England*, and how many are Males, Females, and Fighting Men.

Firſt, I find the Number of Houſes within the Weekly Bills of *London*, by conſidering the Number of thoſe Families who pay to the Poor in each Pariſh, and of the Poor themſelves; or by the Poll-Books, King's-Tax-Books, *&c.* Which ſuppoſe I find 108000 (as they are thereabout) theſe at 5 in each Family, one with another, makes 540000 People in the ſaid Compaſs. Now admit that by a Poll-Tax, *&c.* it has been found that the Heads in *London* are to thoſe in all the reſt of *England* as 1 to 9; there muſt at this rate be 4860000 Souls in *England*. And it having been found by many

Years

Years Obſervation upon the Weekly Bills, that there are 14 Males to 13 Females; ſo that of the ſaid 4860000 Perſons, there muſt be 2520000 Males, and 2340000 Females: for

27. 14 :: 4860000. 2520000. Or 27. 13 :: 4860000. 2340000

And hence to find the Fighting Men, it has been computed, That 37.4 of 100 Souls are above 16, and under 60; therefore

100. 37.4 :: 2520000. 942480 = the Fighting Men.

*Prop.* 2. To find the Number of People by the Coals imported: I find, ſuppoſe, by the Coal-Meters Books 404200 Chaldron imported in one Year, *com. Ann.* It may eaſily, by conſidering the Medium of the ſeveral Claſſes of Houſekeepers, be found that each Houſe, one with onother, burneth about $3\frac{3}{4}$ Chaldron *per ann.* So that by Diviſion I find 107786 Houſes, which is but 214 ſhort of the Number of Houſes in the Weekly Bills, found as in the laſt Propoſition.

*Prop.* 3. To find the Number of Houſes within the Bills, by the Yards Square of the Ground. I find (ſuppoſe, as *per* Mr. *Grant*) 54 Houſes in every hundred Yards Square, and 2000 ſuch Squares built upon within the Bills of Mortality; which multiplied together gives 108000 Houſes, as *per* the firſt Method.

From theſe and ſuch-like Reaſonings by Political Arithmetic, the Vitality *per Cent.* of ſeveral Ages of Perſons is found.

As that 64 of 100 born are alive at 6 Years old.

40 of 100 ——— at 16
25 of 100 ——— at 26
16 of 100 ——— at 36
10 of 100 ——— at 46
6 of 100 ——— at 56
3 of 100 ——— at 60
1 of 100 ——— at 76

And that the Income of *England per ann.* is, from Trade *l.* 9000000
Land ——— 14000000
Art and Labour 20000000

So that the whole Annual Income is 43000000 *l. Sterl. per ann.*

The Income of *France l.* 81000000
Of *Holland l.* 18000000

Alſo that the Exports of *England* in *Engliſh* Commodities are about *l.* 5000000 *per ann.*

The Current Coin of *England* is about *l.*7000000.

The

The Expence for the aboveſaid Number of People at 8 *l. per* Head *per ann.* is *l.* 38,880000 *per ann.* So that *England* increaſeth in Riches at that rate, *l.* 4120000 *per ann.*

The Number of Acres of the Land of *England* has been by the very Judicious eſteemed (of the ſeveral ſorts of Land) as follows: *viz.*

| | *N° of Acres.* |
|---|---|
| Of Arable Land —— | 9,000000 |
| Paſture and Meadow | 12,000000 |
| Wood and Coppice — | 3,000000 |
| Foreſts, Parks, and Commons——— | 3,000000 |
| Houſes, Orchards, and Church-yards | 1,000000 |

So that by the above ſaid Eſtimate there is 8 $\frac{1}{2}\frac{2}{8}$ Acres for each Head in *England*, and about *l.* 1 : 8 : 10 each in Caſh.

| | *N° of Acres.* |
|---|---|
| In the laſt Column — | 28,000000 |
| Heaths and Barren Lands——— | 10,000000 |
| Rivers, Lakes and Ponds——— | 500000 |
| Roads and Waſte-Land | 500000 |
| Sum = the Acres in *England*——— | 39,000000 |
| In *Scotland* | 24,000000 |
| *Ireland* | 29,000000 |

| | *N° of Acres.* |
|---|---|
| Arces in *Great Britain* and *Ireland* | 92,000000 |
| In laſt Column—— | 2652,000000 |
| *Mogul*'s Empire — | 741,000000 |
| *China* and *Tonquin*— | 702,000000 |
| *Tartary*——— | 2730,000000 |
| *Georgia* ——— | 117,000000 |
| *Madagaſcar* ——— | 78,000000 |
| *Sumatra*, and Iſles thereabout—— | 80,000000 |
| *Japan*——— | 117,000000 |
| All *Africa*——— | 4680,000000 |
| *America*——— | 3510,000000 |
| Iſles *Wight*, *Man*, *Guernſey*, *Jerſey*; & in the *Weſt-Indies* | 50,000000 |

| | *N° of Acres.* |
|---|---|
| There is computed to be in *France* | 117,000000 |
| The 17 Provinces — | 13,000000 |
| *Spain* and *Portugal* — | 78,000000 |
| *Italy*, *Venice*, and Iſles thereabout | 66,000000 |
| *Germany*——— | 120,000000 |
| *Sweden*, *Norway*, and *Denmark*—— | 156,000000 |
| *Poland* about——— | 116,000000 |
| *Muſcovy*——— | 468,000000 |
| *Turkey* in *Europe*—— | 192,000000 |
| *Turkey* in *Aſia*——— | 234,000000 |
| *Arabia* ——— | 585,000000 |
| *Perſia*——— | 507,000000 |

So that the Acres on the whole Earth are computed——— 15549,000000

Area of the Earth and Sea, as the Superficies of a Sphere in Acres——— 127675,955000

So

So that the Area of the Sea in Acres is —— 112126,000000
And the People of the whole World are computed at —— 300,000000

But the People in the World, in proportion as the Acres of *England* is to the People of *England*, would be 1937,600000. For

39,000000. 4,860000 : : 15549,000000. 1937,600000

This Difference above 300,000000 (which is about $\frac{5}{6}$ of the whole 1937, *&c.*) is caused by the vast quantitys of Barren Ground, as $\frac{9}{10}$ of *Africa*, $\frac{2}{3}$ *America*, $\frac{5}{7}$ *Tartary*, $\frac{2}{3}$ *Russia*, $\frac{7}{8}$ *Arabia*, &c. which are probably desert and unpeopled.

The very Ingenious and Accurate Dr. *Haley*, in his said judicious Remarks upon the *Breslaw* Bills of Mortality, (wherein both the Ages and Sexes of all that died were monthly delivered and compared with the Number of Births, for the Years 1687, 1688, 1689, 1690, and 1691) hath calculated a Table, (published in the *Philosophical Transactions* in 1692, and in *Miscellanea Curiosa*, Vol. 1.) and hath shew'd these Uses thereof; as, 1*st*, In finding the Proportion of Men able to bear Arms in any Multitude, from 18 to 56 Years of Age. 2*dly*, The different Degrees of Vitality in all Ages; as at what Number of Years it is an even Lay, that a Person of any Age shall die: for instance, that a Man of 30 Years of Age liveth between 27 and 28 Years; That 'tis 80 to 1, that a Person of 25 Years does not die in a Year; That 'tis $5\frac{1}{2}$ to 1, that one of 40 lives 7 Years. 3*dly*, He computes the Value of Annuitys for Lives, (as in the curious little Table after the Tables of Compound Interest foregoing) and the Price of Insurances; and that one half of those who are born, do not live above 17 Years, *&c.*

Mr. *Grant* (besides what he computeth, as above, of the Vitality *per Cent.*) saith, That the People of *London* are about one 14th of the People of *England*. And

Sir *William Petty* saith, That there are more People living between the Ages of 16 and 26 than any other Age; and thence infers, That the Square Root of every Person's Age under 16 (whose Square Root is 4) sheweth the Proportion of Probability of such Person's living to 70 Years: *i. e.* It is 4 times more likely that one of 16 Years of Age lives to be 70, than a Child of 1 Year; 'Tis thrice as probable that one of 9 Years lives to 70, than that a new-born Child does; That the odds is 5 to 4, that one of 25 dieth before 1 of 16 Years old; That 'tis 6 to 5, that one of 36 dieth before 1 of 25 Years: These Proportions being the Roots of 25,

16,

16, and 36 = the Ages, as is above ſaid. He alſo ſays, That the Shipping of *Europe* is about 2000000 Tons, of which *England* hath 500000, the *Dutch* 900000, the *French* 100000, *Spain*, *Portugal*, and *Italy* 250000, *&c.*

And beſides what is ſaid above of the Number of Acres of each kind of Land in *England*, Meſſieurs *King* and *Davenant* ſay farther, That the Increaſe of the People of *England* is 9000 *per annum*, Allowances being made for War, Plague, Shipping, and the Plantations. They reckon the Souls of *London* 530000, in the Citys and Market-Towns in *England* 870000, the Villages and Hamlets 410000. The Annual Produce by Cattel in Butter, Cheeſe, and Milk, about *l.* 2500000; the Value of the Wool yearly ſhorn, about *l.* 2000000: of Horſes bred yearly *l.* 250000; Value of Fleſh yearly ſpent as Food *l.* 3350000; of the Tallow and Hides about *l.* 600000; Hay yearly conſumed by Horſes *l.* 1300000; by other Cattel 1000000, *&c.*

# CHAP. VII.

## *Logarithmical Arithmetic.*

IN this kind of Arithmetic the Work of Multiplication, Diviſion, and Extraction of Roots, are performed not by the Numbers themſelves, but by Artificial Numbers adapted to thoſe given; ſo that their Addition performs Multiplication; Subſtraction does the buſineſs of Diviſion; and Dividing by 2, 3, 4, *&c.* of theſe Artificial Numbers, gives the Square, Cube, Biquadrate, *&c.* Roots of the reſpective Natural Numbers.

2. There are to every Natural or Common Number, an Artificial one proportioned; which being formed into a Table, where each Natural Number has its own Artificial ſtanding right againſt it, 'tis called a Table of Artificial Numbers or Logarithms.

3. Of theſe Tables ſome have the Logarithms of all Natural Numbers from 1 to 1000, ſome to 10000, and ſome to 100000.

And

And again, theſe Logarithms conſiſt of Places from 5 to 14; but one and the ſame Table has always but one Number of Places: and the more extenſive any Table is, the more it is univerſally uſeful; and therefore I ſhall by and by give Rules for enlarging a Table.

4. There may be ſeveral kinds of Logarithms contrived; for any Series of Numbers in Arithmetical Progreſſion are the Logarithms of thoſe right againſt them in Geometrical Proportion: As

| | |
|---|---|
| 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024 | Natural Numbers in Geomet. Progr. |
| 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. | Their Logarithms in Arith. Progreſ. |

And the aforeſaid Uſes of Logarithms will appear even in this ſmall Example of a Table: As,

1. *In Multiplication.* As 32 by 16. Againſt theſe in the upper Series, you have 5 and 4 in the lower (or that of Logarithms) therefore add 5 to 4 maketh 9, which ſee in the Logarithms, and over it is 512 = the Product of 32 by 16, done by only adding their reſpective Logarithms 5 and 4. So 32 by 32 is 1024; 8 by 16 = 128, *&c.*

2. *In Diviſion.* To divide 1024 by 64: Under 1024 ſtands the Logarithm 10; and under 64 ſtands the Log. = 6, therefore take 6 from 10, and the Remainer is the Log. = 4, over which ſtands the Quotient 16. And ſo 512 divided by 16, quotes 32, and proves the firſt Example in Multiplication, *&c.*

3. *In the Extraction of the Square Root.* Admit you would have the Square Root of 1024: Under that you have the Logar. 10. the half of which is 5, over which Log. is 32 = the Square Root of 1024, by only taking half its Logarithm; ſo the Root of 256 is 16, *&c.*

4. *In Extracting the Cube Root:* If you would have the Cube Root of 512, under that is the Log. 9. a 3d of which is 3; over which Log. ſtands the Anſwer 8 = the Cube-Root ſought, *&c.*

5. By this very ſmall Specimen it may be obſerved how neceſſary (becauſe exceeding uſeful) a Table of Artificial Numbers muſt needs be, when they are made ſuch a univerſal Series, as to be fitted for all Numbers from 1 to 100000; and this was the happy Contrivance or Thought of the Famous Lord *Neper*, a *Scotiſh* Baron, *Anno* 1614. whoſe Memory will always be precious, eſpecially to all Trigonometrical Calculators. But the firſt Table, as aboveſaid, was made

by that Learned Mathematician of our own Country, Mr. *Henry Briggs*, ſometime *Savilian* Profeſſor of Geometry in the Univerſity of *Oxford*, for Natural Numbers to 100000, and the Log. to 14 places; which new Invention was extremely approved of and promoted by *Kepler* and other Foreigners.

6. Theſe Logarithms being a Series of Numbers in Arithmetical Proportion to anſwer 1, 2, 3, 4, *&c.* to 10 or 100 Thouſand, muſt needs be Decimals, except the Index or firſt Figure towards the left hand, ſeparated from the Logarithm by a Point: for theſe ſhew how many Places the Natural Number conſiſts of, if it be not a Decimal; but if a Decimal, the Index is marked with a Note of Negation, as —, put commonly under the Index or Characteriſtic: which Index, in caſe of Logarithms of Decimals, does ſhew how many Places from the Point the firſt ſignificant Figure ſtands.

| *The Index of the Log. of all Intire Numbers.* | *In Decimals the Index of* |
|---|---|
| from | |
| 1, to 10 excl. is 0 | .1, .2, .3, *&c.* is 1 |
| 10, to 100 .... is 1 | .01, .02, .03, *&c.* 2 |
| 100, to 1000 ... is 2 | .001, .002, *&c.* 3 |
| 1000, to 10000 is .. 3 | .0001, .0002, *&c.* 4 |
| 10000, to 100000 = 4 *&c.* | .00001, *&c.* 5 *&c.* |

And 'tis plain, that the Indexes of Logarithms of intire Numbers are always 1 leſs in Value, than the Number of Places in thoſe of whole Numbers.

*Note*, That for taking the Logarithm out of my Table, *&c.* you may ſee *Sect.* 2, & 4. of this Chap.

7. Indefatigable muſt the Trouble and Pains be ſuppoſed, which Mr. *Briggs* had in his firſt Table; as muſt be granted, when the Method is underſtood of making but one commonly uſed, conſiſting of half the Number of Places. Which is this: They firſt found ſo many Mean Proportionals between 10 and 1, by the continual adding Cyphers, about 28, and half ſo many afterward towards the right hand of the 10, and then extracting the Roots, till at laſt the Root was ſo ſmall, as to have as many Cyphers next the Point in the Decimal, as they intended the Log. ſhould conſiſt of Places; which could not be done under 27 very long Extractions: theſe ſeveral

veral Means they put down in order, from the extreme 10 downward.

2*dly*, They found Logarithms to theſe Extreme and Means, by putting down againſt the Extreme 10.000, *&c.* the Log. 1.000, *&c.* (as is beforeſaid) and proceeding to take half that Log. continually, half of that half, *&c.* and placing them orderly againſt their reſpective Mean Proportionals found as before.

3*dly*, Then when they had a mind to make the Log. of any Number, they by extracting the Square Root, found ſo many continual Means between the Number given and 1, that the laſt Mean was the leaſt Root that had ſo many Cyphers next the Point, as the Logarithm was to have Places.

4*thly*, Then to find Logarithms to theſe Means; the Proportion is

As the ſignificant Figures of the Mean Proportional found between 10 and 1, (of the like place with the laſt continual Mean in theſe you are upon)
Is to its reſpective Logarithm (omitting the Cyphers)
So is the laſt Mean in the Series you are now a working,
To its Logarithm.

5*thly*, This Logarithm of the laſt continual Mean between the Number you are inveſtigating the Log. of, and 1, being found, is to be doubled, and that, *&c.* Products doubled continually; putting the ſeveral Products againſt the reſpective Means laſt found, till at laſt the Product falling againſt the Number whoſe Log. is ſought, gives you that Logarithm, by which ſeveral others are eaſily found: for the Log. of 4 multiplied by 2, gives the Log. of 16; $\frac{1}{2}$ Log. of 4 is that of 2: ſo have you the Logarithms of all the Powers of 2; and the Log. of 10 (or 1.0) leſs the Log. of 2. leaveth the Log. of 5. So you have the Log. of all the Powers of 5, and Product by 2 and 5.

I did intend to have inſerted here the Whole of finding the Log. of any Number between 1 and 10; but there being lately ſo many briefer Ways of finding the Logarithms, publiſhed by Mr. *Mercator*, Mr. *Gregory*, Mr. *Speidal*, and eſpecially by Dr. *Haley*, in *Philoſophical Tranſactions*, N°216. that I thought it not worth while to give an Example of the old tedious Way above-mentioned.

It could hardly be expected, that in one Treatiſe handling ſo many different Parts of Arithmetic, I ſhould inſert a Table of Logarithms; which alone, with its Uſe, makes a conſiderable Volume: And eſpecially conſidering there are ſo many Tables already publiſhed; for beſides Mr. *Briggs*, Original, there are Tranſcripts by

*Norwood, Gunter, Wingate, Moor, Newhouſe, Leyburn, Sturmy, Taylor, Harris, Ward,* &c. *Engliſhmen*; beſides thoſe of *Kepler, Ulacq, Urſin,* &c. Foreigners. But I have however (the better to make this Book anſwer to the Title) inſerted a Table to 10000, and Rules to make it to 100000000.

## SECT. II. *To Add, Subſtract, Multiply, and Divide Logarithms; or to perform the Work of Multiplication, Diviſion, to Square and Cube, and Extraction of Roots of Intire Numbers by the Logarithms.*

BUT it muſt firſt be obſerved, That

I. *To take any Logarithm in my Table belonging to any Natural Number:*

If the Natural Number conſiſt of three Places only, you have them in the Column under *Natural Numbers*; and in the next Column toward the right hand are their reſpective Logarithms under (o).

But if the Number given conſiſt of four places, look for all but Units place under *Natural Numbers* as before, right againſt which, and under the Digit in Units place, is your Logarithm required. And in like manner for any Logarithm under 100, ſee for that in Tens place under *Natural Numbers*; and for that in Units at the head of the Column, and in the common Angle of meeting ſtands the Log. ſought.

II. *To find the Natural Number to a Logarithm.*

This you will eaſily do by the help of your Index, and eſpecially by conſidering the Nature of the Numbers which increaſe from the left hand towards the right; as the Digits at the head of the Columns, and thoſe in the left hand Columns reſpectively increaſe where four Places is in the Natural Number. But downwards the Logarithms are to ſuch Natural Numbers as increaſe by Ten: Thus the Logarithm 8123785 I find under 2, and againſt 649; ſo that it is the Logarithm of 6492: but if it were the Logarithm 8122447, I find that under (o) and againſt 649; ſo that 6490 is the Natural Number ſought. And the ſame Logarithm is alſo for 649: As all are Logarithms for the Natural Number expreſſed in the Table, or for that Number multiplied by 1 or more Tens; but you are to be guided by your Index what places your Number is.

So

*So much for the Logarithm of Integers:* And for thoſe of Fractions and Mixt Numbers, ſee *Sect.* 3. following, and more of the above, *Sect.* 4.

I. *Multiplication of Intire Numbers by the Logarithms.*

*Example* 1.] Multiply 91 by 93.

*Rule.* To the Logarithm of 91, which is = 1.9590414
Add the Logarithm of 93 ..... = 1.9684829

Product = 8463; the Nat. Numb. to the Log. 3.9275243 Sum.

*Example* 2.] Multiply 4 by 177.

*Rule.* To the Log. of 177, which is = 2.2479733 } Add.
Add the Log. of 4, which is = 0.6020600 }

Product 708; its Logarithm = 2.8500333 Sum.

Theſe two Examples are ſufficient to ſhew how to multiply any Intire Number by another; by the Logarithms, where I need not give many words to ſhew that againſt the Natural Numbers given ſtand their Logarithms; and againſt the Sum of thoſe Logarithms ſtand the reſpective Natural Numbers 8463 and 708: *i. e.* In my Table you have the Log. 9275243 under 3, and againſt 846 (the Units place, where there are 4) being always over the Logarithm. See *Sect.* 4. and the Rules above for finding the Logarithms, and their Natural Numbers.

II. *Diviſion by Logarithms.*

*Example* 1.] Divide 8463 by 93.

*Rule.* From the Logarithm of 8463, which is = 3.9275243
Deduct the Log. of 93, which is = 1.9684829

Reſts the Log. of the Quote 91, which is = 1.9590414

*Example* 2.] Divide 708 by 177.

From the Logarithm of 708, which is = 2.8500333
Deduct the Log. of 177, which is = 2.2479733

Reſts the Log. of the Quote 4; which is = 0.6020600

III. *To Square any Intire Number, and Extract the Square Root by the Logarithms.*

1. To ſquare (or involve in itſelf) any whole Number.

*Exam-*

*Example.*] What is the Square of 99?
*Rule.* Multiply the Logarithm of 99, which is = 1.9956352
by ..... 2

9801, the Square fought; being the Natural Number to the Product ——— 3.9912704

2. To evolve, or extract the Square Root of any *Intire Number.*
*Example.*] What is the Square Root of 9801?
*Rule.* Take half of the Logarithm of 9801. Log is 3.9912704
Half that Log. of 9801 is = 1.9956352
Which half is the Log. of 99 = the Root fought. And this proves the laft Queftion.

## IV. *To Cube, or Extract the Cube Root of any Intire Number, by the Logarithms.*

1. To Cube any *Whole Number.*
*Example.*] What is the Cube of 21?
*Rule.* Multiply the Log. of 21, which is ..... = 1.3222193
by ..... 3

And the Cube required is 9261 = the Natural Number of the Log. ——— 3.9666579

2. To extract the Cube Root of any *Intire Number.*
*Example.*] What is the Root of 9261?
The Log. of 9261 is as before = 3.9666579
$\frac{1}{3}$ of that Logarithm is = 1.3222193
Whofe Natural Number is 21 = the Cube Root required.

3. If you defire the Biquadrate of any Number, multiply its Log. by 4, gives the Log. of the Anfwer: And if you would have the Biquadrate or Surfolid Roots, *&c.* of a Number, divide its Logarithm by 4 or 5, *&c.* Examples are needlefs; but note, that if the Index is lefs than the Divifor, take fo often as the Divifor can be had in 2 places.

## SECT. III. *To perform the Work of Multiplication, Divifion, Square or Cube,* &c. *or Extract the Roots of Fractions and Mixt Numbers, by the Logarithms.*

BEFORE I give Examples of Logarithms, it will be neceffary to give a Rule for *Adding or Subftracting Negative Indices or Characteriftics.*

*Note,*

*Note*, That when the Index is mark'd under thus (—) it is ſo much leſs than nothing as thoſe beforegoing, or 2, 3, 1, *&c.*

1. The Rules for adding and ſubſtracting the Indices of Logarithms are numerous, and not eaſy to retain in mind: I have therefore ſuppoſed (which taking the Reaſon of the thing along with it, eaſily ſolves all, without charging the Memory) when the Index is negative, that it is a Perſon owing ſo many *l.* 100 more than he is worth, as the Index-Digit expreſſeth.

| *Addition of Indices.* | *Subſtraction.* |
|---|---|
| Thus, 1. Add the Index 3 | From —— 2 |
| to —— 1 | Take —— 1 |
| Sum = 2 | Reſts 3 |
| *Exam.* 2. Add —— 3 | 2*dly*, From —— 2 |
| to —— 1 | Take —— 1 |
| Sum = 2 | Reſts 3 |
| *Exam.* 3. Add —— 2 | 3*dly*, From —— 5 |
| to —— 3 | Take —— 2 |
| Sum = 5 | Reſts 3 |
| | *Theſe prove the Truth of thoſe in Addition.* |

This being done, the adding to or taking from the upper Index according to the Nature of each, makes the Sum or Remainer plainly appear.

1. So in the firſt Example of Addition above, I conſider that if I add *l.* 100 to a Perſon who is 3 worſe than nothing, the Sum muſt be 2; that is, I then make him worth but 2 leſs than nothing.

2. In the ſecond Example, if I add 1 leſs than (0) to; that is, if I take 1 from a Perſon who hath 3, the Sum is but 2; for that is all that is left. And if I add 3 leſs than nothing to him who before was 2 worſe than (0) I leave him 5 leſs than (0), as in Example 3.

*So in Subſtraction of Indices:*

3. If (as in the firſt Example) I take 1 from him who already is 2 worſe, or leſs than (0), I leave him 3 leſs than nothing; and ſo 3 is the Remainer.

4. If

4. If (as in the 2d Example) I take $\underline{1}$ (or 1 leſs than nothing) from him that hath 2 in poſſeſſion already, I then leave him worth 3, becauſe I take 1 leſs than nothing from him: therefore I add 1 to him, that is, I make 2 into 3, which is therefore the Remainer. (See the words *Negative Arithmetic* in Algebra.) And all this conſider'd right, is but one ſingle, ſhort, and eaſy Rule.

## II. *To find the Logarithm of a Mixt Number, or of a Decimal, by the Table.*

Prop. 1. *To find the Logarithm of a Mixt Number, as of* 9.177.

*Rule.* Take the Log. of 9177, except the Index; and to that Logarithm prefix an Index according to the Number of Places in the Integral Part of the Number given, as by the little Table under the 6th Head of the firſt Sect. of this Chapter, and you have the Anſwer.

*Examp.* Thus the Log. of 9177 (without the Index) is 9627007; and the proper Index for 1 Place being 0, therefore the Anſwer is 0.9627007.

So alſo the Log. of 91.77 is = 1.9627007, and of 917.7 'tis = 2.9627007.

Prop. 2. *To find the Logarithm of a Decimal, as of* .5

*Rule.* Take the Log. of 5, except the Index, and prefix to that Log. an Index agreeable to the Decimal, as *per* the ſaid little Table under the 6th Head of the 1ſt Sect. of this Chapter.

*Examples.* The Log. of 5 is = 6989700, without its Index: to which Log. if you put its proper Index $\underline{1}$, (there being no Cyphers between the Point and the 5) the Anſwer is $\underline{1}$.6989700. And by the ſame Rule the Log. of .05 is $\underline{2}$.6989700; of .005 = $\underline{3}$.6989700; and of .000579 the Log. is $\underline{4}$.7626785; being the Log. of 579, with the Index of .000579 put before it.

## III. *To find Mixt Numbers or Decimals to Logarithms by the Table.*

Prop. 1. *To find a Mixt Number to a Logarithm.*

*Rule.* Regard not the Index; but look for the Logarithm among thoſe which would have the greateſt Indexes, which having found, take the Natural Number ſtanding againſt it, and point off for Integers according to the Index given.

*Example.* What is the Natural Number to the Log. 1.9627007?

My Table extending to 10000, the greateſt Index is 3; among which I find my Logarithm given, and right againſt it I find 9177.

9177. Now the Index given being 1, requires that 2 of the 4 Places be Integers, ſo that the Anſwer is 91.77: And alſo the Natural Number to 2.9627007 muſt be 917.7, *&c.* as *per* the Rule for Indexes above.

Prop. 2. *To find the Decimal to a Logarithm.*

*Rule.* Look for your Logarithm in that part of the Table where 'tis to be found, as before in the laſt, and put down the Natural Number that ſtands right againſt it: then according to the Greatneſs of the Index given, make the Point of the Decimal.

*Example.* What is the Natural Number to 4.7626785?

Againſt this Logarithm I find 5790. Now becauſe my Index given is 4, I place 5 in the 4th place from the Point, and ſay therefore that .000579 is the true Decimal ſought.

## IV. *To Multiply Mixt Numbers and Decimals, by adding their Logarithms.*

I have ſo fully prepared the Reader for this, that I only need to give Examples.

*Example* 1. A Mixt Number by a Mixt, as 9.336 by 12.09

To the Logarithm of 9.336, which is 0.9701608
Add the Log. of 12.09, which is —— 1.0824263

The Sum is the Log. of 112.9 the Product, *viz.* 2.0525871

*Example* 2. A Decimal by a Decimal, as .09 by .123

To the Log. of .09, which is —— 2.9542425
Add the Log. of 123, which is —— 1.0899051

The Sum is the Log. of the Product .01107, which is 2.0441476

*Example* 3. A Whole Number by a Decimal, as 765 by .00345

To the Log. of 765, which is 2.8836614
Add the Log. of .00345, which is 3.5378191

The Sum is the Log. of the Product 2.639, *viz.* 0.4214805

*Example* 4. A Mixt Number by a Decimal, as 1.23 by .00009

To the Log. of 1.23, which is —— 0.0899051
Add the Log. of .00009, which is 5.9542425

The Sum is the Log. of the Product .0001107 = 4.0441476

*Note*, That *whenſoever* 1 *is carried to the Index, that* 1 *is affirmative*, and muſt be added to the lower Index, and that Sum to the upper, as *per* the Rules and Examples of adding Indices at the beginning of this Section; and more particularly in the three laſt Examples.

### V. *To divide Mixt Numbers, Decimals*, &c. *by Subſtracting their Logarithms.*

In the following Scheme you have all the Caſes that can happen under this Head: which will be eaſily performed, only obſerving the firſt and ſecond Examples of ſubſtracting Indices, and the Rule at the beginning of this Section.

*The*

*The 9 Cases of Division, done by Logarithms.*

| *Cases.* | *Numbers given, or Dividends and Divisors, &c.* | *Their Logarithms: Substract that of the Divisor.* |
|---|---|---|
| 1. | Divide 237.3 | —— 2.3752977 |
| | by 25.32 | —— 1.4034637 |
| | Quote = 9.376 | Rests 0.9718340 |
| 2. | Divide 11.5 | —— 1.0606978 |
| | by 932 | —— 2.9694159 |
| | Quote .01234 | Rests 2.0912819 |
| 3. | Divide .1234 | —— 1.0913151 |
| | by .321 | —— 1.5065050 |
| | Quote .3846 | Rests 1.5848101 |
| 4. | Divide .463 | —— 1.6655810 |
| | by 3214 | —— 3.5070459 |
| | Quote = .000144 | Rests 4.1585351 |
| 5. | Divide 2 | —— 0.3010300 |
| | by 3.214 | —— 0.5070459 |
| | Quote .6222 | Rests 1.7939841 |
| 6. | Divide 5 | —— 0.6989700 |
| | by 365 | —— 2.5622929 |
| | Quote = .0137 | Rests 2.1366771 |
| 7. | Divide 1.26 | —— 0.1003705 |
| | by .002 | —— 3.3010300 |
| | Quote = 630 | Rest 2.7993405 |
| 8. | Divide .5 | —— 1.6989700 |
| | by 7.5 | —— 0.8750612 |
| | Quote = .0667 | Rests 2.8239088 |
| 9. | Divide 999 | —— 2.9995655 |
| | by .00013 | —— 4.1139433 |
| | Quo. 7684615.384 | Rests 6.8856222 |

## VI. *To Square any Mixt Number or Decimal, or to Extract the Square Root thereof.*

1. *To ſquare ſuch Numbers.*

*Rule.* Multiply the Logarithm of the Roots given by 2; but if the Index be negative, out of the Product of that by the 2 deduct what Tens are carried from the place next toward the right hand, and put down the Remainer for the Index of the Product (or Log. of the Square.)

*Examples.*

### I. For Squaring Decimals, &c. by Logarithms.

| *The Square Roots given.* | *The Logarithms of thoſe Roots.* | *Double the laſt Col. or the Log. of the Squares.* | *The Squares ſought.* |
|---|---|---|---|
| 987.6. | 2.9945811 | 5.9891622 | 975353.76 |
| 98.76. | 1.9945811 | 3.9891622 | 9753.5376 |
| 9.876. | 0.9945811 | 1.9891622 | 97.5353 |
| .9876. | $\underline{1}$.9945811 | $\underline{1}$.9891622 | .9753 |
| .09876. | $\underline{2}$.9945811 | $\underline{3}$.9891622 | .009753 |
| .009876. | $\underline{3}$.9945811 | $\underline{5}$.9891622 | .0009753 |
| .0009876. | $\underline{4}$.9945811 | $\underline{7}$.9891622 | .0000009753 |
| .00009876. | $\underline{5}$.9945811 | $\underline{9}$.9891622 | .0000●0009753 |

### Or, II. For the Square Roots; theſe Titles obſerve.

| *Squ. Roots ſought (or the Natural N° to the half Logarithms.)* | *$\frac{1}{2}$ the Logarithms of the Squares; or thoſe of the Roots.* | *Logarithms of the Squares given.* | *Squares given, whoſe Roots are required.* |
|---|---|---|---|

2. *Obſerve in Extracting the Square Root*, that is to ſay, in taking half of the Log. of the Square; if the Index be an odd Number, add 1 to it in your mind, and ſo proceed: As in taking half $\underline{3}$.989, &c. I ſay half of 4 is 2; of 19, = 9: of 18, = 9, &c.

VII. *To*

## VII. *To Cube, or Extract the Cube Root of Decimals, &c.*

### *Examples.*

I. For the Cubing of Decimals by Logarithms.

| *The Cube Roots given.* | *Logarithms of those Roots.* | *Three times the Logarithms.* | *The Cube Numbers sought.* |
|---|---|---|---|
| 3.1 | 0.4913617 | 1.4740851 | 29.791 |
| .31 | 1.4913617 | 2.4740851 | .029791 |
| .031 | 2.4913617 | 5.4740851 | .00002979 1 |
| .0031 | 3.4913617 | 8.4740851 | .00000002979 1 |
| .00031 | 4.4913617 | 11.4740851 | .00000000002979 1 |
| .000031 | 5.4913617 | 14.4740851 | .00000000000002979 1 |

Or, II. For the Cube Root obſerve theſe Titles of the Columns.

| *Cube Roots ſought.* | *$\frac{1}{3}$ of thoſe Logarithms, being thoſe of the Roots.* | *Logarithms of thoſe Cube Numbers.* | *Cube Numbers given; whoſe Roots are required.* |
|---|---|---|---|

1. You ſee by the Examples, that to Cube any Number (as thoſe next the left hand) is to multiply their Logarithms (as thoſe in the 2d Column) by 3; and the Products are thoſe in the 3d Column, whoſe Natural Numbers required are thoſe in the 4th Column. And here note, That what Tens you carry, you muſt abate out of 3 times the Index; as taught for the Square Root.

2. For the Examples of Extracting the Cube Root: The Numbers in the Column next the right hand are Examples of Cube Numbers given, the next Column are their Logarithms, and the third Column are third parts of thoſe Logarithms. Where note, That if your Index cannot be divided by 3 without a Remainer, you muſt add ſo many Units to it, as is neceſſary for that end: And ſo many Units as you borrow'd there, you muſt pay Tens to the

the next Figure, and then take a 3d of the whole Log. The Examples make all plain.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Powers.*

*Rule* 1. Multiply the Log. of the Root given by 4, 5, or 6; obſerving the Rules given before, abating what you carry, out of the Product of the Index reſpectively.

Or *for finding the Biquadrate, Surſolid, or Squared Cube Roots.*

*Rule* 2. Divide the Log. of the Power by 4, 5, or 6; obſerving the Rule of adding ſo many to the Index, that you can divide it without a Remainer, and paying ſo many Tens as you added or borrow'd Units, to the next Figure.

| *Examples to Rule* 1. | *Logarithm of the Powers.* | *Whoſe Nat. Nº are the Anſ. or Powers requir'd* |
|---|---|---|
| Thus *the Biquadrate* of .3; its Log. being = 1.4771212; 4 times that —— | = 3.9084848 | = .0081 |
| The *Surſolid Power* of .3; its Log. being 1.4771212; 5 times that —— | = 3.3856060 | = .00243 |
| The *Squared Cube* of .3; its Log. being 1.4771212; 6 times that —— | = 4.8627272 | = .000729 |

| *And according to the laſt Rule.* | *Log. of the Roots.* | *Roots ſought.* |
|---|---|---|
| The *Biquadrate Root* of .0081, its Log. being 3.9084848; $\frac{1}{4}$ that is —— | = 1.4771212 | = .3. |
| The *Surſolid Root* of .00243, its Log. being 3.3856060; $\frac{1}{5}$ that is —— | = 1.4771212 | = .3. |
| The *Squared Cube Root* of .000729, its Log. being 4.8627272; $\frac{1}{6}$ of that is | = 1.4771212 | = .3. |

Which latter Examples prove the Truth of the former.

## VIII. *To find the Logarithms of Vulgar Fractions, and of the Operations thereby.*

*Prop.* To find the Log. of $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$.

*Rule.* From the Log. of the Numerator deduct the Log. of the Denominator, and the Remainer is the Log. for the Anſwer.

*Exam-*

*Example.* From the Log. of 3456, which is = 3.5385737
Take the Log. of 4567, which is = 3.6596310

Remains the Log. of $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$, which is = 1.8789427

*Prop.* 2. To find the Log. of a Compound Fraction.
*Rule.* Reduce the Compound to a Simple, and work as above.
*Example.* To find the Log. of $\frac{4}{5}\frac{5}{6}\frac{6}{7}$ of $\frac{5}{1}\frac{7}{1}\frac{6}{1}$.
The Compound in a Simple being $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$, the Log. is as before;
Or Log. of $\frac{4}{5}\frac{5}{6}\frac{6}{7}$ of $\frac{5}{1}\frac{7}{1}\frac{6}{1}$ = 1.8789427.

*Prop.* 3. To find the Logarithm of a Mixt Number.
*Rule.* Reduce it to an Improper Fraction, and find the Logarithm as *per* the firſt Prop. This needs no Example.
*Prop.* 4. To reduce a Vulgar Fraction to a Decimal.
*Rule.* Find the Log. of the Vulgar Fraction as *per* the firſt: then againſt that Log. in the Table you have the Decimal.
*Example.* To reduce $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ to a Decimal. The Log. of this Vulgar Fraction, as before, is = 1.8789427; and the Natural Number to that, *per* the Table, is .7567 = the Anſwer.

And for reducing ſuch Fractions as this given, whoſe Terms have ſo many places, I take this to be the moſt eaſy and ſpeedy Way of reducing them to Decimals: and if your Table of Logarithms were to 100000, or where the Numerator is a place leſs than the Denominator, the Rule would in moſt Caſes be exact enough.
*Prop.* 5. What is the Product of one Vulgar Fraction by another, giving the Anſwer in a Decimal Fraction?
*Rule.* Add the Log. of the one Fraction to the Log. of the other found as *per* the 1ſt Prop. and look for the total Log. in the Table, againſt which ſtands the Decimal required.
*Example.* What is the Product of $\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ by $\frac{7}{9}\frac{6}{8}\frac{5}{7}\frac{6}{6}$?

$\frac{3}{4}\frac{4}{5}\frac{5}{6}\frac{6}{7}$ its Log. as before = 1.8789427
$\frac{7}{9}\frac{6}{8}\frac{5}{7}\frac{6}{6}$ its Log. ..... = 2.8890803

The Product for Anſwer is .05862, whoſe Log. is 2.7680230

And thus 'tis plain, that knowing by the Rules delivered how to find the Logarithms of Vulgar and Decimal Fractions, they may be multiplied or divided ſimply or promiſcuouſly. For,
6. Suppoſe it were required to divide .05862 by $\frac{7}{9}\frac{6}{8}\frac{5}{7}\frac{6}{6}$:

If

If from the Log. of .05862, which is = 2.7680230
You take the Log. $\frac{765}{9876}$, which is = 2.8890803

The Remainer is the Log. of .7567, or of $\frac{3456}{4567}$, *viz.* } = 1.8789427

Which proves the Operation above.

7. By what is above ſaid, it will follow, that the Powers or Roots of Vulgar Fractions are eaſily found by multiplying the Log. of the Vulgar Fraction aſſigned as a Root, by 2, 3, 4, 5, 6, *&c.* to obtain the Power; or by dividing the Log. of the Vulgar Fraction aſſigned as a Power, by 2, 3, *&c.* to obtain the reſpective Roots. And this I take alſo to be the eaſieſt and briefeſt Way of finding the Powers or Roots of Vulgar Fractions; eſpecially where the Terms conſiſt of 3, 4, or 5 places, and your Table of Logarithms is large enough.

## SECT. IV. *The Explanation of my Table of Logarithms, whoſe Method ſomewhat differs from* Briggs, Norwood, *and ſome others.* 2dly, *Of the Arithmetical Complements of Logarithms.* 3dly, *Of Parts Proportional and Differences: And,* 4thly, *How to enlarge the Tables of Logarithms ſeveral Ways.*

1. THE eaſieſt Tables for a Learner to uſe, are thoſe that have the Indices put down, and Logarithms to the Natural Numbers which are intirely placed ſucceſſively. But I would here explain to him another kind of Table placed without the Indices, and which have not the Units place of the Natural Number with the reſt, but over the reſpective Logarithm (this is the beſt Method, and which I have therefore obſerved in my following Table of Logarithms) thus.

| *Numb.* | 0 | 1 | 2 | 3 | 4 | *&c.* to 9 |
|---|---|---|---|---|---|---|
| 522 | 7176705 | 7177537 | 7178369 | 7179200 | 7180032 | |
| 523 | 7185017 | 7185847 | 7186677 | 7187507 | 7188337 | |
| 524 | 7193313 | 7194142 | 7194970 | 7195799 | 7196627 | |
| 525 | 7201593 | 7202420 | 7203247 | 7204074 | 7204901 | |

In

In caſe ſuch a Table as this is to be uſed; you muſt, for example to find the Logarithm of 5244, put down, *1ſt*, 3 for your Index, as *per* the Rules in the ſixth Paragraph of the firſt Sect. of this Chapter.

*2dly*, Having found 524, (the three firſt Figures of your Natural Number given toward the left hand) caſt your eye in the Line toward the right hand, till you come under the Figure in Units place of the Number given, (here 4) where againſt 524, and under 4, you have the Log. required: for the Log. of 5244 is 3.7196627.

So the Log. of 5223 is 3.7179200; and that of 523 = 2.7185017: the Log. of the laſt Natural Number having all its places found together under *Number*; the Log. againſt 523, and under 0, is that ſought, *&c.*

Theſe Logarithms in Mr. *Briggs*'s firſt and large Table, alſo in *Norwood* and ſome others, ſtand thus; (tho they take up much room.)

| *N°* | *Log.* | *N°* | *Log.* | *N°* | *Log.* |
|---|---|---|---|---|---|
| 522 | 2.7176705 | 5221 | 3.7177537 | 5231 | 3.7185847 |
| 523 | 2.7185017 | 5222 | 3.7178369 | 5232 | 3.7186677 |
| 524 | 2.7193313 | 5223 | 3.7179200 | 5233 | 3.7187507 |
| 525 | 2.7201593 | 5224 | 3.7180032 | 5234 | 3.7188337, *&c* |

II. *The Arithmetical Complement* of a Logarithm, is that which will make it 10.0000000; ſo that to find the Complement, the Logarithm given muſt be deducted from the 10.0, *&c.*

Now the beſt way to ſubſtract this (not according to the common way, but) to take every Digit of the Subtrahend from 9, except that in Units place, which take from 10: And to do this, you may begin at either end to ſubſtract.

10.0000000
*Log. given* = 3.7204901

*Its Compl. Arith.* = 6.2795099

The Complement Arith. is often uſed in Analogies wrought by Artificial Numbers, to prevent Trouble and Confuſion: For inſtead of ſubſtracting a Logarithm from the Sum of two others, it is uſual to add the Complement Arithmetical, and from the Sum of the three Logarithms to abate Radius or 10.0 from the Index of that Sum; which gives the ſame Anſwer as tho you had deducted the Logarithm of the firſt Number.

To inſtance in a common Queſtion in the Rule of Proportion: If I give *l.* 25 : 10 : — for 48 Yards of Cloth; what will 15 coſt at that rate?

Yards 48, its Logarithm = 1.6812412

*l.* 25.5, its Logarithm = 1.4065402
Yards 15, its Logarithm = 1.1760912 } Add.

Sum 2.5826314

From which deducting the first Log. the Remainer is that of *l.*7.968, *viz.* —— = 0.9013902

This by the Complement Arithmetical is done thus:

48. Compl. Arith. = 8.3.1187588
25.5 : : its Log. . . = 1.4065402
15, its Log. . . . . = 1.1760912

Sum abating 10 in the Index, the Log. of 7.968 the Answer as before —— = 0.9013902

These kinds of Operations are much used in Trigonometry, with the Logarithms of Natural Sines, Tangents, *&c.*

## III. *Of Differences and Proportional Parts.*

1. By the *Differences* is meant the Differences of the Logarithms of 2 Numbers immediately and numerically succeeding one another, or the Differences of the Products of such Numbers multiplied by 10.

2. By *Proportional Parts* you are to understand one or more tenth Parts of those Differences. And these are apply'd in Practice thus:

IV. *When you would find the Logarithm of any Number that is greater than what is contained in the subsequent Table of Logarithms*, and you would do this *by the help of the Table of Differences and Parts Proportional.*

Suppose you were to find the Logarithm of 51716: see the Operation.

I find in the Table the Log. of 5171 or 51710, which is —— = 7135745
And of the next Number 5172 or 51720 = 7136585

The Difference of these Logarithms is = 840 *Diff.*
Which found in the Table of Differences (following that of the Logarithms) right against that 840 and under 6 (the Digit in Units place 504 *Add.*
7136249 *Sum.*

of

of the Number given) you find the Proportional Part 504; which added to the Logarithm of 5171, the Sum is that of 51716, which with its proper Index is 4.7136249. You may prove the Truth of this by dividing 51716 by 7, the Quote is = 7388; to the Log. of which if you add that of 7, the Sum is the Log. of 51716, as above.

2. *To prove the Truth of any Logarithms in the Table, without the Table of Proportional Parts*: as thoſe of 7000 to 7010 excluſive.

The Difference between the Logarithms of 700 and 701, or 7000 and 7010, is 6200.

| *Tenths of the Difference.* | *Are theſe Proportional Parts.* | *Log. of 7000 repeated.* | *Sums of the laſt Logarithms and Proport. Parts reſpectively.* | *The Natural Numbers to the laſt Logarithms.* |
|---|---|---|---|---|
| 1 | 620 | 8450981 | 8451601 | 7001 |
| 2 | 1240 | 8450981 | 8452221 | 7002 |
| 3 | 1860 | 8450981 | 8452841 | 7003 |
| 4 | 2480 | 8450981 | 8453461 | 7004 |
| 5 | 3100 | 8450981 | 8454081 | 7005 |
| 6 | 3720 | 8450981 | 8454701 | 7006 |
| 7 | 4340 | 8450981 | 8455321 | 7007 |
| 8 | 4960 | 8450981 | 8455941 | 7008 |
| 9 | 5580 | 8450981 | 8456561 | 7009 |

By this it appears, that the Difference between the Logarithms of 7000 and 7010 being divided into 10 parts, 1 Tenth is 620, the Part to add to the Logarithm of 7000 to give the Logarithm of 7001; 2 Tenths added to the Logarithm of 7000, gives the Logarithm of 7002; 3 Tenths of the Difference 6200, which is 1860, added to 8450981 (the Log. of 7000) giveth 8452841 = the Logarithm of 7003, *&c.*

And if you would have the Proportional Part immediately to make the Log. of 7000 that of 7008, it is only multiplying a Tenth of the Difference (which here is 620) by 8, and the Product is the Proportional Part 4960; which added to 8450981 = the Log. of 7000, the Sum is 8455941 = the Log. of 7008 required. And thus the 9 Logarithms between 7000 and 7010 excluſive, are examined and proved: And ſo may any other without a Table of Differences and Proportional Parts, or by even calculating or examining thoſe at the ſame time.

3. *To augment the Table of Logarithms*, ſo as to make that which extends only to ſhew the Log. of any Natural Number under 10000, ſubſervient to give that of any Number under 100000, *&c.* without the Table of Parts Proportional. For example;

1. To find the Log. of 64736.

The Log. of 6473 or 64730 is = 8111056
Of 6474 or 64740 is = 8111727
The Difference = 671
A Tenth of that Difference (being the Proportional Part for 1, that is to make the Log. of 64731) — 67.1
Which multiply by 6 (the Place of Units in the given Number) produceth — 402.6
Or becauſe the .6 is above one half, you may make the Proportional Part for 6 (in Units place of the Number given) — 403
Which added to the Log. above of 64730, *viz.* — 8111056
The Sum is the Log. of 64736 or Anſwer = 4.8111459

For as 10. 671 :: 6. 403 *ferè.*

*i. e.* As the Number of Cyphers added to the Natural Number in your Table with a 1 before ſuch (0) towards the left hand;
Is to the Proportional Part, or Difference of the 2 Logarithms:
So is the Figures towards the right hand exceeding the Places for which your Table is calculated,
To the Proportional Part to be added to the Log. to anſwer thoſe Figures.

2. To find (by a Table which has only the Logarithms to Numbers under 10000) what the Logarithms are for all Numbers under 1000000: For inſtance what is the Log. of 647365?

Log. of 647300 is = 8111056
of 647400 is = 8111727
Difference = 671
$\frac{1}{10}$ thereof, there being 100 difference between the Nat. N° of which theſe are Logs. — 6.71
This multiplied by 65 (the 2 places toward the right hand exceeding the Compaſs of the Table) — 65
Produceth = 436.15
Which added to Log. 647300 — 8111056
Sum = 8111492 = the Log. of 647365

For 100. 671 :: 65. 436.15 the .15 may be omitted. 3. To

3. To find (by a Table which has only the Logarithms to Natural Numbers not exceeding 10000) what the Logarithms are for all Numbers not exceeding 10000000000. For example,

I would know the Logarithm of 296820514.

| | *Nat. N°.* | *Logarithms.* |
|---|---|---|
| As before, the Log. of the moſt places towards the left hand of the Number given, contained in my Table is of 2966, which is alſo of | 2966000000 | = 4721711 |
| The Log. of the next Number is of 2967, or of | 2967000000 | = 4723175 |
| The Difference of theſe Logarithms is | | = 1464 |
| Which Difference divided by 1000000 that each Unit may have its Proportional Part of that Difference, that for 1 is | | = .001464 |
| But becauſe the Number given requires 820514 of thoſe Parts for its Part Proportional, therefore I multiply the Difference (as *per* the former Rules) by | | = 820514 |
| And the Product is the Part Proport. for 820514 | | = 1201.232496 |
| Which (omitting the Decimal as uſeleſs) added to the Log. of 2966000000 as above, which is | | 4721711 |
| The Sum is the Logarithm ſought, *viz.* | | 4722912 |

Which having a proper Index put before it, according to the Number given, as *per* the 5th Rule of the firſt Sect. of this Chapter:

The Log. of 296820514 is found = 9.4722912

I have in this laſt Example made this matter ſo very plain, that I need add nothing for that end; only to obſerve,

1*ſt*, That the Log. of any Number is the ſame, tho that Number have never ſo many Cyphers added to its right hand. And therefore in

*Example* 1. of this Head, I call the Log. of 6473, which is 8111056, the Log. of 64730.

In *Example* 2. I call the ſame Logarithm that of 647300. And

In *Example* 3. I call the Log. of 2966 (which is 4721711) the Log. of 29660000000, making up the reſt of the Places toward the right hand (for which I have no Logarithms in my Table) with Cyphers.

2*dly*, (As in the laſt Example) whatever Number of Cyphers thoſe are, I cut off ſo many Places of the Difference of the Logarithms, *&c.*

And

And thus I have endeavoured to make this Part (which has been omitted by moſt others) very intelligible, by ſhewing how to prove the Truth of Logaritnms, or find thoſe to Numbers that are large by the Differences and Parts Proportional, as they are found in Tables for that purpoſe: And have alſo moſt eaſily and plainly ſhew'd how Logarithms of large Numbers are found without ſuch Tables of Differences and Proportional Parts. It remains next to ſhew,

1. *How to find Natural Numbers to large Logarithms by the Tables of Differences and Parts Proportional.*
2. *How to do the ſame without ſuch Tables of Differences and Parts Proportional.*

4. *To find the Natural Numbers to Logarithms beyond the Compaſs of your Table, by the Differences and Parts Proportional.*

I would know the Natural Number to the Logarithm 4.8111459 See the Operation.

The Logarithm given = 8111459

The next Log. in the Table leſs than that given is the Log. of 6473, (which are the four firſt Places towards the left hand of the Number required) *viz.* } = 8111056

The next Log. in the Table greater than the laſt (being that of 6474) is } = 8111727

The Difference of the 2 firſt Logarithms = 403

Difference of the 2 laſt Logarithms = 671

Now to find the 5th Figure of the Number ſought, or that in Units place, (the Index of the Log. given ſhewing that it hath but 5 places) look for the 2d Difference under [*Differences*] in the Table of Proportional Parts, and you'll find the next it (greater) to be 672. Then look in the ſame Line towards the right hand, and you'll find the 1ſt Difference 403; and at the head of that Column ſtands (6) which put in Units place, gives 64736 for the Number ſought. This proves the Truth of the 1ſt Example, laſt above-ſaid.

5. *To find the Natural Numbers (tho very large) to any Log. given, without the Tables of Parts Proportional.*

What is the Natural Number to the Logarithm 9.4722912?

I find the Logarithms, and take the Differences as above thus.

The

The Log. given (without the Index) is 4722912 = 1ſt Differ. 1201
The next leſs in the Table (which is the Log. of 2966) is ——— 4721711
The next Log. (greater than the laſt) is ——— 4723175 = 2d Differ. 1464

Now whereas the Index ſhews the Natural Number required to have 10 places, and there being only the firſt 4 found, there wants 6 places, which with a Unit towards the left hand is = 1000000: therefore I ſay, As the 2d Difference is to 1000000 : :

So is the 1ſt Difference to the reſt of the Nat. N° required, *i. e.*

1464. 1000000 : : 1201. 820514.

Which 820514 placed towards the right hand of the 2966 found in the Table as above, is together 29668205 14 = the Anſwer.

But the Logarithms of 7 places being too few to find ſo large a Natural Number by, makes the Differences of too few places; and conſequently the 4th Proportional in the laſt Analogy will fall ſomewhat ſhort, if done by a Table having only 7 places. However, the Example ſhews the true Method of proceeding in this Caſe, which is all here intended.

This laſt Example was deſigned to prove the Truth of the third; where .232496 being omitted, if that Decimal be annexed to the 1201, the third in the laſt Analogy, it will then be divided by the 1464, and produce the true Quotient 820514 exactly.

## Sect. V. *The Uſe of Logarithms in ſolving Queſtions of Compound Intereſt.*

Queſt. 1. *WHAT is the Amount of* l. 10 in 130 Years, at 5 *per Cent?*

*Rule.*] Multiply the Log. of 1.05 (which is 1 *l.* and its Intereſt for a Year, which in the ſubſequent Queſtions I call the Ratio) by 130, (the N° of Years) and the Product is the Log. of the Amount of *l.*1 in 130 Years.

To this Log. add that of *l.*10 = the Principal; and the Sum is the Log. of the Anſwer, *viz.* of *l.* 5684.409 = the Amount required.

*Example.*

1.05, its Log. is = 0.0211893
Multiply by Years 130

| | *l.* | |
|---|---|---|
| Amount of 1*l.* in 130 Years. | = 568.4, its Log. | 2.7546090 |
| To which add the Log. of the Princip. 10 *l.* | | 1.0 |
| Sum or Log. of 5684.409 | | 3.7546090 |

Queſt.

Queſt. 2. *What is the preſent Worth* of *l.*5684 : 8 : 2 ¼ (or of *l.*5684.409) due at the end of 130 Years?

*Rule.*] Find the Logarithm of the Amount of *l.*1 for 130 Years, as before.

Subſtract the Log. of that Amount from the Log. of the Sum whoſe preſent Worth is required, and the Remainer is the Logarithm of the preſent Worth, as *per* Marg.

So I find *l.*5684.409 to be received 130 Years hence, is worth *l.*10 in preſent Money; which proves the 1ſt.

*Example.*

| | |
|---|---|
| Ratio = 1.05, its Log. = | 0.0211893 |
| Multiply by ..... | 130 |
| Product, or Amount of *l.*1 in 130 Years = 568.4; its Log. —— | 2.7546090 |
| Which laſt, deduct from the Log. of *l.*5684.409 | 3.7546090 |
| And the remaining Log. is that of *l.*10, the Anſwer —— | 1.0000000 |

---

Queſt 3. *What is the Amount of the Leaſe or Annuity* of *l.*25 *per ann.* in 21 Years, according to its utmoſt Improvement at 5 *per Cent?*

*Rule.*] Find the Amount of *l.*1 for 21 Years, as above.

2. Then from the Log. of the Amount leſs 1, deduct the Log. of the Ratio leſs 1; and the Remainer is the Amount of *l.*1 Annuity in 21 Years.

3. Add to the laſt Log. that of 25 the Annuity, and the Sum is the Anſwer, or *l.*893, the Amount of 25 *per ann.* in 21 Years.

*Example.*

| | |
|---|---|
| Ratio 1.05, its Log. = | 0.0211893 |
| Multiply by Years ..... | 21 |
| Product, or Amount of *l.*1 in 21 Years 2.786; its Log. —— | 0.4449753 |
| 1.786 = the Amount leſs 1; its Log. —— | 0.2518815 |
| 1.05 = Ratio, leſs 1 = .05; its Log. —— | 2.6989700 |
| Which laſt deducted, there reſts the Log. of *l.*35.72, the Amount of *l.*1 *per an.* in 21 Yrs. | 1.5529115 |
| To which adding the Log. of 25 the Annu. | 1.3979400 |
| The Sum is the Log. of *l.*893 the Amount of the Annuity for Anſw. | 2.9508515 |

Queſt.

Queſt. 4. *What is the preſent Worth of a Leaſe, or Annuity,* of *l.* 25 *per annum,* to continue 21 Years at 5 *per Cent.* Intereſt?

*Rule.*] Find the Amount of *l.*1 in 21 Years, as in the laſt Queſtion.

2*dly,* To the Log. of that Amount add the Log. of the Ratio, leſs 1.

3*dly,* Subſtract that Sum from the Log. of the Amount, leſs 1; and the Remainer is the Log. of the preſent Worth of *l.*1 Annuity to continue 21 Years.

4*thly,* To the laſt Log. add that of the Annuity given, and the Sum is the Log. of the preſent Worth required.

*Example.*

| | |
|---|---|
| 2.786 = the Amount of *l.*1 in 21 Years (as above) its Log. | 0.4449753 |
| .05 = Ratio leſs 1, its Log. | 2.6989700 |
| Sum = ..... | 1.1439453 |
| 1.786 = Amount, leſs 1, its Log. | 0.2518815 |
| Reſts the Log. of 12.821 *l.* which is the preſent Worth of *l.* 1 *per ann.* to continue 21 Years | 1.1079362 |
| To which add the Log. of *l.*25 the Annuity | 1.3979400 |
| And the Sum is the Log. of the Anſwer *l.* 320.527, which is | 2.5058762 |

---

Queſt. 5. *What Annuity or Leaſe will the Sum* of *l.* 320.527, or *l.*320 : 10 : 6½ purchaſe, to continue 21 Years, allowing the Purchaſer 5 *per Cent?*

*Rule.*] Find the Amount of *l.*1 in 21 Years, as in the 1ſt, *&c.* Queſtions.

2*dly,* To the Log. of that Amount add the Log. of the Ratio leſs 1, as in the laſt.

3*dly,* From that Sum deduct the Logarithm of the Amount leſs 1, and the Remainer is the Log. of the Annuity that *l.*1 will purchaſe for 21 Years.

4*thly,* To the Log. of the Annuity of *l.*1 add the Log. of the Purchaſe-Money, and the Sum is the Log. of the Annuity ſought here *l.*25 *per ann.* as *per* the Work in the Margin.

*Example.*

| | |
|---|---|
| 2.786 the Amount of *l.*1 in 21 Years, its Log. | 0.4449753 |
| .05 = Ratio leſs 1, its Log. | 2.6989700 |
| Sum ..... = | 1.1439453 |
| 1.786 = Amount, leſs 1, deduct its Log. | 0.2518815 |
| Reſts the Log. of the Annuity that *l.* 1 will purchaſe for 21 Years, *viz.* of .078, which Log. is | 2.8920638 |
| 320.527 the Purchaſe-Money, its Log. add = | 2.5058762 |
| *l.*25 *per ann.* the Annuity required, whoſe Log. is the Sum of the 2 laſt Logs. | 1.3979400 |

Queſt. 6. What is an Eſtate worth in Fee, which contains Arable Land *l.*60 *per ann.* Underwood which once in 15 Years is worth *l.* 200, and there is alſo Timber which 'tis computed after 30 Years Growth will be worth *l.*1000 ?

*Rule.*] The Arable Land at 20 Years Purchaſe is worth ——— } *l.* 1200 : — : —

2*dly*, *l.*200 (the firſt for the Underwood) due 15 Years hence at 6 *per Cent.* by the Logarithms (as *per* the 2d Queſtion) is found thus : 1.06 its Log. = 0.0253058
Multiply by Years . . . . . 15

2.397 = Amount of *l.*1 in 15 Years, its Log. ——— } 0.3795870
Which taken from the Log. of *l.* 200 ——— } 2.3010300

The Remainer is that of the preſent Worth ——— } 1.9214430 = Log. of *l.* 83 : 9 : —

3*dly*, By the ſame Rule the preſent Value of *l.*200 due 30 Years hence is ——— } 34 : 16 : —

4*thly*, By ditto Rule the Value of the 3d 200 due 45 Years hence, is ——— } 14 : 10 : —

5*thly*, The Value of the 4th *l.*200 due 60 Years hence, is *per* ditto Rule ——— } 6 : 1 : —

6*thly*, The Value of *l.*1000 for the Timber due 30 Years hence, is in preſent ——— } 174 : 2 : —

The Sum of which ſix Articles is the Anſwer near enough (for 'tis needleſs in this Purchaſe to take notice of the 5th 200 *l.*) ——— } 1512 : 18 : —

Queſt.

Queſt. 7. *What Annuity payable Quarterly, will l.* 320.527 *purchaſe,* to continue 21 Years at 5 *per Cent?*

*Rule.*] Firſt take a fourth of the Log. of the Ratio.

2*dly*, Multiply that 4th by the Quarters in the Years given, and that gives the Log. of the Amount of *l.*1 for thoſe Quarters.

3*dly*, To the Log. of that Amount add that of the Rate *per* Quarter leſs 1.

4*thly*, From that Sum deduct the Log. of the Amount for the Quarters, leſs 1; and the Remainer is the Log. of the Annuity that *l.* 1 will purchaſe for 21 Years payable quarterly.

5*thly*, To the laſt Log. add that of the Purchaſe-Money, and the Sum is the Log. of the Anſwer.

*Example.*

| | |
|---|---|
| 1.05 the Ratio, its Log. | 0.0211893 |
| A 4th of that Log. wch is the Log. of 1.012, the Rate *per* Quarter | 0.0052974 |
| Which Log. multiply by the Quarters in 21 Yrs. | ..... 84 |
| And that produceth the Log. of 2.786 = the Amount of *l.*1 for 84 Quarters, *viz.* —— | 0.4449816 |
| To which Log. add that of .012 = Rate per Quarter, leſs 1 | 2.0791812 |
| Sum ..... | 2.5241628 |
| 1.786 = the Amount, leſs 1, its Log. deduct | 0.2518815 |
| Reſts the Log. of .01872 *l* = the Annuity that *l.*1 will purchaſe, payable quarterly, *viz.* | 2.2722813 |
| To which Log. add that of *l.* 320.527 the Purch. Mony, which is | 2.5058762 |
| And the Sum is the Log. of *l.*6, the Quarterly Payment requir'd | 0.7781575 |

By this Example it is obvious, that the *l.* 320.527 will purchaſe but 6*l. per* Quarter, at the Rate of 24 *l. per ann.* Whereas by the 5th Queſtion it appears, that Sum will purchaſe *l.* 25 *per ann.* if payable yearly. So that there is *l.*1 *per ann.* difference in that Purchaſe between receiving the Rent quarterly and yearly, the Purchaſer having the Uſe of the Quarters Rents.

Queſt. 8. *What is the preſent Worth of* l. 24 per ann. *payable by equal Quarterly Payments*, to continue 21 Years?

*Rule.*] Firſt find the Amount of *l.*1 for 84 Quarters, as above.

2*dly*, To that Log. add the Log. of the Rate *per* Quarter (found as above) leſs 1.

3*dly*, Deduct that Sum from the Log. of the Amount leſs 1, and the Remainer is the Log. of the preſent Worth of *l* 1 *per* Quarter, to continue 84 Quarters.

4*thly*, To that Log. add that of *l.*6 *per* Quarter, (as the 4th of 24 *per ann.*) and the Sum is the Log. of the preſent Worth of the Annuity given, payable quarterly.

*This proves the Truth* of the laſt Queſtion.

*Example.*

| | |
|---|---|
| Rate *per* Quarter found as above, is | 1.012 |
| 2.786 = the Amount of *l.*1 for 84 Qurs (found as above) its Log. | 0.4449816 |
| .012 = the Rate *per* Quarter leſs 1, its Log. Add | 2.0791812 |
| Sum of thoſe 2 Logs. | 2.5241628 |
| 1.786 = the Amount above, leſs 1, deduct its Logarithm | 0.2518815 |
| Reſts the Log. of *l.* 53.42 = preſent Worth of *l.* 1 *per* Quarter, to continue 21 Years | 1.7277187 |
| 6 *l. per* Quarter (as *per* the Annuity given) its Log. Add | 0.7781575 |
| And the Sum is the Log. of 320.527 the Anſw. | 2.5058762 |

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*Queſt* 9. In an Account of Conſequence 'tis found that *l.* 5684.409 is the Amount of 10 *l.* Compound Intereſt at 5 *per Cent.* what Time was required for that Increaſe?

*Rule.*] From the Log. of the Amount deduct that of the Principal put out.

2*dly*, Divide the remaining Log. by the Log of the Ratio, and the Quotient is the Years required. See the Operation.

*Example.*

| | |
|---|---|
| *l.*5684.409 its Log. | 3.7546090 |
| 10 *l.* the Principal, its Log. Deduct | 1. |
| 1.05, its Log. .0211893 the Diviſor | 2.7546090 (130 ... 635670 |
| So the Anſwer is 130 Years. | |

Queſt.

*Queſt.* 10. In a certain Account *l.* 5684.409 is charged as the Amount of 10 *l.* in 130 Years Compound Intereſt: But upon a Tryal the Simple Intereſt being only allowed according to the Rate agreed on, whence the ſaid Amount is computed; now it not appearing by any Bond, or Entry, what Rate or Intereſt was reckoned, the Queſtion is how that may be found out?

*Rule.* From the Log. of the Amount deduct that of the Principal put forth.

2*dly*, Divide that remaining Log. by the Number of Years, and the Quote is the Log. of *l.*1 and its Intereſt for a Year. Therefore ſay,

3*dly*, As *l.*1 is to *l.*1 and its Intereſt found :: So is 100 to 100 and its Intereſt. Whence taking 100, the reſt muſt be the Rate of Intereſt required.

*Example.*

5684.409 the Amount, its Log. ——— 3.7546090
10 = the Principal, deduct its Log. ——— 1.
Divide by 130) reſts ... 2.7546090
Quoteth the Log. of *l.* 1.05, which is ——— 0.0211893

Therefore
As 1. 1.05 :: 100. 105
And 105 leſs 100 = 5 the Intereſt required *per Cent.*

---

*Queſt.* 11. A Gentleman bought a Leaſe of *l.* 25 *per ann.* to continue 21 Years for *l.* 320 527: At what Rate was Intereſt allowed him *per Cent?*

To do this by the Logarithms, take the Log. of *l.*320.527, which is ——— 2.5058762
Whence deduct the Log. of the Annuity given 25, which is ——— 1.3979400

And the Remainer is the Log. of 12 821 (or the preſent Worth of *l.* 1 *per ann.* to continue 21 Years; *which note.* ——— 1.1079362

Now to find what Intereſt this is reckon'd at, I make tryal (as *per Queſt.* 4.) and find *at 6 per Cent.* that the preſent Worth of *l.*1 *per ann.* to continue 21 Years is 11.764: which being leſs than the 12.821 (the preſent Worth of *l.*1 *per ann.* as above) I know that 6 *per Cent.* is too much, (for the more the preſent Worth is, the leſs is the Rate of Intereſt, and the contrary.)

Therefore I make tryal at 4 *per Cent.* and (by the Rules under the 4th Queſtion) I find the preſent Worth of *l.*1

*per*

*per ann.* at 4 *per Cent.* is (to continue 21 Years) 14.031: which being more than the above 12.821, I know that 4 *per Cent.* is too little. So that it neither being at 6 nor 4, but finding the Rate to lie between them, I ſuppoſe it 5 *per Cent.* And trying by the Rules under the ſaid 4th Queſtion I find the Log. of the preſent Worth of *l.* 1 *per ann.* to continue 21 Years is that of 12.821, the ſame as above, *viz.* ——— 1.1079362

And conſequently I know that 5 *per Cent.* is the Rate ſought.

*Or thus to find the Rate:*

Having found the preſent Worth at 4 and 6 *per Cent.* as above, proceed thus.

1. Take the Difference between the Years Purchaſe given (or found by the Logarithms) which Years Purchaſe as above are ——— } 12.821

And the Years Purchaſe at 4 *per Cent.* found ——— 14.031

Which Difference is ——— = 1.210

2. The Years Purchaſe at 4 *per Cent.* are found = 14.031

Ditto at 6 *per Cent.* as above are = 11.764

2d Difference = 2.267

3. Say 2.267. 1.210 :: 2. 1

*i. e.* As the 2d Difference is to the firſt: ſo is 2 the Difference between the Rates *per Cent.* (ſuppoſed) as before, to 1 = the Proportion to be added to the leſſer Rate 4 to give the Rate ſought for; in which caſe the 1ſt place in the 4th Proportional is ſufficient, where (0) is next the Point in the Decimal, if the Diviſion were carry'd on farther.

Queſt. 12. *What is the preſent Value of the Reverſion of an Annuity or Leaſe* of *l.* 50 *per ann.* to continue 21 Years, to commence after the Expiration of 7 Years, at 5 *per Cent. per ann.* allowed to the Purchaſer.

*Rule.*] Firſt add the Years (the 7 to the 21) make 28.

2*dly*, Find the preſent Worth of the Annuity to continue 28 Years by the 4th Queſtion, which you'll find ——— } *l.* 745 : — : —

3*dly*, Find

| | |
|---|---|
| *3dly*, Find the preſent Worth of the *l.* 50 *per ann.* (by the Rules under the ſaid 4th Queſt.) to continue 7 Years, which is | *l.* 283 : 6 : — |
| *4thly*, Deduct the latter from the former, and the Remainer is the preſent Worth of the Reverſion required | *l.* 461 : 14 : — |

13. *When it is required to purchaſe an Eſtate either in Fee or for Years mentioned, with Money due at any time to come:* as ſuppoſe I would purchaſe an Annuity to continue 14 Years, with *l.* 1000 due to me at the end of 5 Years; the Annuity to commence preſently at 5 *per Cent.*

| | |
|---|---|
| *Rule.*] Firſt by the 2d Queſtion find the preſent Worth of *l.*1000 due at the End of 5 Years, which is | *l.* 783: 10 : — |
| *2dly*, By the 5th Queſtion find what Annuity to continue 14 Years *l.*783 : 10 will purchaſe; which for Anſwer you'll find at 5 *per Cent.* | *l.* 79 : 2 *per ann.* 14 Years. |

I need not tell the Reader, that in caſe the *Purchaſe had been required in Fee, to divide the preſent Worth of* l.1000 = l.783.5 *by the Number of Years Purchaſe agreed on*; as at 20 it will be *l.* 39 : 3 : 6 *per ann.* for ever.

14. *When you would Fine off Rent:* As ſuppoſe a Leaſe is *l.* 90 *per ann.* and the Tenant is willing to fine off *l.* 40 *per ann.* for his Leaſe of 21 Years at 5 *per Cent.* in this Caſe there is nothing to be done but by the 4th Queſt. find the preſent Worth of *l.* 40 *per ann.* to continue 21 Years; which preſent Worth *l.* 512 : 16 : 11 muſt be given the Landlord as a Fine, and *l.* 50 *per ann.* muſt be paid Annual Rent. On the contrary,

15. *When you would Rent off a Fine:* As ſuppoſe a Leaſe for 21 Years is to be lett at *l.*50 *per ann.* with a Fine of *l.* 512.846; but the Leaſee not having ſo much ready Money to ſpare, he is willing to pay an Equivalent Rent, and no Fine. To anſwer this, you need only to conſider by the Rules under the 5th Queſtion, what Annuity to continue 21 Years at 5 *per Cent.* the ſaid Fine = *l.*512.846 will

will purchaſe; which you'll find *l.* 40 *per ann.* which added to the *l.* 50, ſhews that he muſt pay *l.* 90 *per ann.* if he pays no Fine.

Theſe 15 Queſtions and Propoſitions contain the Fundamental Rules for ſolving all Queſtions of Compound Intereſt whatever; and whoſoever doth thoroughly underſtand them, may be able to give a true Anſwer to any Queſtion relating to Selling or Purchaſing Eſtates, *&c.* without the help of Tables for that purpoſe; provided he has well learnt the Logarithms.

*The End of Logarithmical Arithmetic, and its Uſe.*

# A TABLE

A

# TABLE

OF THE

# LOGARITHMS

TO ALL

# NUMBERS,

Not exceeding 10000, whether they be Intire, Broken, or mixt Numbers.

And the Differences of *Logarithms*, and the Parts proportional, whereby the *Logarithm* of any Number is produced to 100000.

Particularly useful in Extracting the Square, Cube, *&c.* Roots, and solving Questions in Compound Interest, *&c.*

| Natural Numbers. | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 1 | 0000000 | 0413927 | 0791812 | 1139433 | 1461280 |
| 2 | 3010300 | 3222193 | 3424227 | 3617278 | 3802112 |
| 3 | 4771212 | 4913617 | 5051500 | 5185139 | 5314789 |
| 4 | 6020600 | 6127838 | 6232493 | 6334684 | 6434527 |
| 5 | 6989700 | 7075702 | 7160033 | 7242759 | 7323937 |
| 6 | 7781512 | 7853298 | 7923917 | 7993405 | 8061800 |
| 7 | 8450980 | 8512583 | 8573325 | 8633229 | 8692317 |
| 8 | 9030900 | 9084850 | 9138138 | 9190781 | 9242793 |
| 9 | 9542425 | 9590414 | 9637878 | 9684829 | 9731278 |
| 100 | 0000000 | 0004341 | 0008677 | 0013009 | 0017337 |
| 101 | 0043214 | 0047511 | 0051805 | 0056094 | 0060379 |
| 102 | 0086002 | 0090257 | 0094509 | 0098756 | 0102999 |
| 103 | 0128372 | 0132587 | 0136794 | 0141003 | 0145205 |
| 104 | 0170333 | 0174507 | 0178677 | 0182843 | 0187005 |
| 105 | 0211893 | 0216027 | 0220157 | 0224284 | 0228406 |
| 106 | 0253059 | 0257154 | 0261245 | 0265333 | 0269416 |
| 107 | 0293838 | 0297895 | 0301948 | 0305997 | 0310043 |
| 108 | 0334237 | 0338257 | 0342273 | 0346284 | 0350293 |
| 109 | 0374265 | 0378247 | 0382226 | 0386202 | 0390173 |
| 110 | 0413927 | 0417873 | 0421816 | 0425755 | 0429691 |
| 111 | 0453230 | 0457140 | 0461048 | 0464952 | 0468852 |
| 112 | 0492180 | 0496056 | 0499928 | 0503797 | 0507663 |
| 113 | 0530784 | 0534626 | 0538464 | 0542299 | 0546130 |
| 114 | 0569048 | 0572856 | 0576661 | 0580462 | 0584260 |
| 115 | 0606978 | 0610753 | 0614525 | 0618293 | 0622058 |
| 116 | 0644580 | 0648322 | 0652061 | 0655797 | 0659530 |
| 117 | 0681859 | 0685569 | 0689276 | 0692980 | 0696681 |
| 118 | 0718820 | 0722499 | 0726175 | 0729847 | 0733517 |
| 119 | 0755470 | 0759118 | 0762762 | 0766404 | 0770043 |
| 120 | 0791812 | 0795430 | 0799045 | 0802656 | 0806265 |
| 121 | 0827854 | 0831441 | 0835026 | 0838608 | 0842187 |
| 122 | 0863598 | 0867157 | 0870712 | 0874264 | 0877814 |
| 123 | 0899051 | 0902580 | 0906107 | 0909631 | 0913151 |
| 124 | 0934217 | 0937718 | 0941216 | 0944711 | 0948204 |
| 125 | 0969100 | 0972573 | 0976043 | 0979511 | 0982975 |
| 126 | 1003705 | 1007151 | 1010593 | 1014033 | 1017471 |
| 127 | 1038037 | 1041455 | 1044871 | 1048284 | 1051694 |
| 128 | 1072100 | 1075491 | 1078880 | 1082266 | 1085650 |
| 129 | 1105897 | 1109262 | 1112625 | 1115985 | 1119343 |
| 130 | 1139433 | 1142773 | 1146110 | 1149444 | 1152776 |
| 131 | 1172713 | 1176027 | 1179338 | 1182647 | 1185954 |
| 132 | 1205739 | 1209028 | 1212314 | 1215598 | 1218880 |
| 133 | 1238516 | 1241780 | 1245042 | 1248301 | 1251558 |
| 134 | 1271048 | 1274288 | 1277525 | 1280760 | 1283993 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 1 | 1760912 | 2041200 | 2304489 | 2552725 | 2787536 |
| 2 | 3979400 | 4149733 | 4313637 | 4471580 | 4623980 |
| 3 | 5440680 | 5563025 | 5682017 | 5797836 | 5910646 |
| 4 | 6532125 | 6627578 | 6720978 | 6812412 | 6901961 |
| 5 | 7403627 | 7481880 | 7558748 | 7634280 | 7708520 |
| 6 | 8129133 | 8195439 | 8260748 | 8325089 | 8388491 |
| 7 | 8750613 | 8808136 | 8864907 | 8920946 | 8976271 |
| 8 | 9294189 | 9344984 | 9395192 | 9444827 | 9493900 |
| 9 | 9777236 | 9822712 | 9867717 | 9912261 | 9956352 |
| 100 | 0021661 | 0025980 | 0030295 | 0034605 | 0038912 |
| 101 | 0064660 | 0068937 | 0073209 | 0077478 | 0081742 |
| 102 | 0107239 | 0111473 | 0115704 | 0119931 | 0124154 |
| 103 | 0149403 | 0153597 | 0157787 | 0161973 | 0166155 |
| 104 | 0191163 | 0195317 | 0199467 | 0203613 | 0207755 |
| 105 | 0232524 | 0236639 | 0240750 | 0244857 | 0248960 |
| 106 | 0273496 | 0277572 | 0281644 | 0285712 | 0289777 |
| 107 | 0314085 | 0318123 | 0322157 | 0326188 | 0330214 |
| 108 | 0354297 | 0358298 | 0362295 | 0366289 | 0370279 |
| 109 | 0394141 | 0398105 | 0402066 | 0406023 | 0409977 |
| 110 | 0433623 | 0437551 | 0441476 | 0445398 | 0449315 |
| 111 | 0472749 | 0476642 | 0480532 | 0484418 | 0488301 |
| 112 | 0511525 | 0515384 | 0519239 | 0523091 | 0526939 |
| 113 | 0549958 | 0553783 | 0557605 | 0561423 | 0565237 |
| 114 | 0588055 | 0591846 | 0595634 | 0599419 | 0603200 |
| 115 | 0625820 | 0629578 | 0633334 | 0637085 | 0640834 |
| 116 | 0663259 | 0666985 | 0670708 | 0674428 | 0678145 |
| 117 | 0700379 | 0704073 | 0707765 | 0711453 | 0715138 |
| 118 | 0737183 | 0740847 | 0744507 | 0748164 | 0751818 |
| 119 | 0773679 | 0777312 | 0780941 | 0784568 | 0788192 |
| 120 | 0809870 | 0813473 | 0817073 | 0820669 | 0824263 |
| 121 | 0845763 | 0849336 | 0852906 | 0856473 | 0860037 |
| 122 | 0881361 | 0884905 | 0888446 | 0891984 | 0895519 |
| 123 | 0916669 | 0920185 | 0923696 | 0927206 | 0930712 |
| 124 | 0951693 | 0955180 | 0958664 | 0962146 | 0965624 |
| 125 | 0986437 | 0989896 | 0993353 | 0996806 | 1000257 |
| 126 | 1020905 | 1024337 | 1027766 | 1031192 | 1034616 |
| 127 | 1055102 | 1058506 | 1061909 | 1065308 | 1068705 |
| 128 | 1089031 | 1092410 | 1095785 | 1099159 | 1102529 |
| 129 | 1122698 | 1126050 | 1129400 | 1132746 | 1136091 |
| 130 | 1156105 | 1159432 | 1162756 | 1166077 | 1169396 |
| 131 | 1189257 | 1192559 | 1195858 | 1199154 | 1202448 |
| 132 | 1222159 | 1225435 | 1228709 | 1231981 | 1235250 |
| 133 | 1254813 | 1258064 | 1261314 | 1264561 | 1267806 |
| 134 | 1287223 | 1290450 | 1293676 | 1296890 | 1300110 |

| Natural Numbers | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 135 | 1303338 | 1306553 | 1309767 | 1312978 | 1316187 |
| 136 | 1335389 | 1338581 | 1341771 | 1344958 | 1348144 |
| 137 | 1367206 | 1370374 | 1373541 | 1376705 | 1379867 |
| 138 | 1398791 | 1401937 | 1405080 | 1408222 | 1411361 |
| 139 | 1430148 | 1433271 | 1436392 | 1439511 | 1442628 |
| 140 | 1461280 | 1464381 | 1467480 | 1470577 | 1473671 |
| 141 | 1492191 | 1495270 | 1498347 | 1501422 | 1504494 |
| 142 | 1522883 | 1525941 | 1528996 | 1532049 | 1535100 |
| 143 | 1553360 | 1556396 | 1559430 | 1562462 | 1565491 |
| 144 | 1583625 | 1586640 | 1589653 | 1592663 | 1595672 |
| 145 | 1613680 | 1616674 | 1619666 | 1622656 | 1625644 |
| 146 | 1643528 | 1646502 | 1649474 | 1652443 | 1655411 |
| 147 | 1673173 | 1676127 | 1679078 | 1682027 | 1684975 |
| 148 | 1702617 | 1705550 | 1708482 | 1711411 | 1714339 |
| 149 | 1731863 | 1734776 | 1737688 | 1740598 | 1743506 |
| 150 | 1760913 | 1763807 | 1766699 | 1769590 | 1772478 |
| 151 | 1789769 | 1792645 | 1795518 | 1798389 | 1801259 |
| 152 | 1818436 | 1821292 | 1824146 | 1826999 | 1829850 |
| 153 | 1846941 | 1849752 | 1852588 | 1855421 | 1858253 |
| 154 | 1875207 | 1878026 | 1880844 | 1883659 | 1886473 |
| 155 | 1903317 | 1906118 | 1908917 | 1911714 | 1914510 |
| 156 | 1931246 | 1934029 | 1936810 | 1939590 | 1942367 |
| 157 | 1958996 | 1961762 | 1964525 | 1967287 | 1970047 |
| 158 | 1986571 | 1989319 | 1992065 | 1994809 | 1997552 |
| 159 | 2013971 | 2016702 | 2019431 | 2022158 | 2024883 |
| 160 | 2041200 | 2043913 | 2046625 | 2049335 | 2052044 |
| 161 | 2068259 | 2070955 | 2073650 | 2076344 | 2079035 |
| 162 | 2095150 | 2097830 | 2100508 | 2103185 | 2105860 |
| 163 | 2121876 | 2124540 | 2127201 | 2129862 | 2132521 |
| 164 | 2148438 | 2151085 | 2153732 | 2156376 | 2159018 |
| 165 | 2174839 | 2177471 | 2180100 | 2182727 | 2185355 |
| 166 | 2201081 | 2203696 | 2206310 | 2208922 | 2211533 |
| 167 | 2227165 | 2229764 | 2232363 | 2234959 | 2237554 |
| 168 | 2253093 | 2255677 | 2258260 | 2260841 | 2263421 |
| 169 | 2278867 | 2281436 | 2284003 | 2286570 | 2289134 |
| 170 | 2304489 | 2307043 | 2309596 | 2312146 | 2314696 |
| 171 | 2329961 | 2332500 | 2335038 | 2337574 | 2340108 |
| 172 | 2355284 | 2357809 | 2360331 | 2362853 | 2365373 |
| 173 | 2380461 | 2382971 | 2385479 | 2387986 | 2390491 |
| 174 | 2405492 | 2407988 | 2410481 | 2412974 | 2415465 |
| 175 | 2430380 | 2432861 | 2435341 | 2437819 | 2440296 |
| 176 | 2455127 | 2457593 | 2460059 | 2462523 | 2464986 |
| 177 | 2479733 | 2482186 | 2484637 | 2487085 | 2489536 |
| 178 | 2504200 | 2506639 | 2509077 | 2511513 | 2513948 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 135 | 1319393 | 1322597 | 1325798 | 1328998 | 1332194 |
| 136 | 1351326 | 1354507 | 1357685 | 1360861 | 1364034 |
| 137 | 1383027 | 1386184 | 1389339 | 1392492 | 1395643 |
| 138 | 1414498 | 1417632 | 1420765 | 1423895 | 1427022 |
| 139 | 1445742 | 1448854 | 1451964 | 1455072 | 1458177 |
| 140 | 1476763 | 1479853 | 1482941 | 1486026 | 1489110 |
| 141 | 1507564 | 1510632 | 1513698 | 1516762 | 1519824 |
| 142 | 1538149 | 1541195 | 1544240 | 1547282 | 1550322 |
| 143 | 1568519 | 1571544 | 1574568 | 1577589 | 1580608 |
| 144 | 1598678 | 1601683 | 1604685 | 1607686 | 1610684 |
| 145 | 1628630 | 1631614 | 1634595 | 1637575 | 1640553 |
| 146 | 1658376 | 1661340 | 1664301 | 1667260 | 1670218 |
| 147 | 1687920 | 1690863 | 1693805 | 1696744 | 1699682 |
| 148 | 1717264 | 1720188 | 1723110 | 1726029 | 1728947 |
| 149 | 1746412 | 1749316 | 1752218 | 1755118 | 1758016 |
| 150 | 1775365 | 1778250 | 1781132 | 1784013 | 1786892 |
| 151 | 1804126 | 1806992 | 1809856 | 1812718 | 1815578 |
| 152 | 1832698 | 1835545 | 1838390 | 1841233 | 1844075 |
| 153 | 1861084 | 1863912 | 1866739 | 1869563 | 1872386 |
| 154 | 1889285 | 1892095 | 1894903 | 1897709 | 1900514 |
| 155 | 1917304 | 1920096 | 1922886 | 1925674 | 1928461 |
| 156 | 1945143 | 1947917 | 1950690 | 1953460 | 1956229 |
| 157 | 1972806 | 1975562 | 1978317 | 1981070 | 1983821 |
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| 162 | 2108534 | 2111203 | 2113876 | 2116544 | 2119211 |
| 163 | 2135178 | 2137833 | 2140487 | 2143139 | 2145789 |
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| 166 | 2214142 | 2216750 | 2219356 | 2221960 | 2224563 |
| 167 | 2240148 | 2242740 | 2245331 | 2247920 | 2250507 |
| 168 | 2265999 | 2268576 | 2271151 | 2273724 | 2276296 |
| 169 | 2291697 | 2294258 | 2296818 | 2299377 | 2301934 |
| 170 | 2317244 | 2319790 | 2322335 | 2324879 | 2327421 |
| 171 | 2342641 | 2345173 | 2347703 | 2350232 | 2352759 |
| 172 | 2367891 | 2370408 | 2372923 | 2375437 | 2377950 |
| 173 | 2392995 | 2395497 | 2397998 | 2400498 | 2402996 |
| 174 | 2417954 | 2420442 | 2422929 | 2425414 | 2427898 |
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| 193 | 2855573 | 2857823 | 2860071 | 2862318 | 2864565 |
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| 196 | 2922561 | 2924776 | 2926990 | 2929203 | 2931415 |
| 197 | 2944662 | 2946866 | 2949069 | 2951271 | 2953471 |
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| 217 | 3364597 | 3366598 | 3368598 | 3370597 | 3372595 |
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| 237 | 3747483 | 3749316 | 3751147 | 3752977 | 3754807 |
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| 261 | 4166405 | 4168069 | 4169732 | 4171394 | 4173056 |
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| 240 | 3811151 | 3812956 | 3814761 | 3816565 | 3818368 |
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| 467 | 6697816 | 6698745 | 6699674 | 6700602 | 6701530 |
| 468 | 6707096 | 6708023 | 6708950 | 6709876 | 6710802 |
| 469 | 6716356 | 6717281 | 6718206 | 6719130 | 6720054 |
| 470 | 6725596 | 6726519 | 6727442 | 6728365 | 6729287 |
| 471 | 6734817 | 6735738 | 6736659 | 6737574 | 6738500 |
| 472 | 6744018 | 6744937 | 6745856 | 6746775 | 6747693 |
| 473 | 6753200 | 6754117 | 6755034 | 6755951 | 6756867 |
| 474 | 6762362 | 6763277 | 6764192 | 6765107 | 6766022 |
| 475 | 6771505 | 6772418 | 6773332 | 6774244 | 6775157 |
| 476 | 6780629 | 6781540 | 6782452 | 6783362 | 6784273 |
| 477 | 6789734 | 6790643 | 6791552 | 6792461 | 6793370 |
| 478 | 6798819 | 6799727 | 6800634 | 6801541 | 6802448 |
| 479 | 6807886 | 6808792 | 6809697 | 6810602 | 6811507 |
| 480 | 6816934 | 6817838 | 6818741 | 6819645 | 6820548 |
| 481 | 6825963 | 6826865 | 6827766 | 6828668 | 6829569 |
| 482 | 6834973 | 6835873 | 6836773 | 6837673 | 6838572 |
| 483 | 6843965 | 6844863 | 6845761 | 6846659 | 6847556 |
| 484 | 6852938 | 6853834 | 6854730 | 6855626 | 6856522 |
| 485 | 6861892 | 6862787 | 6863681 | 6864575 | 6865469 |
| 486 | 6870828 | 6871721 | 6872613 | 6873506 | 6874398 |

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| 487 | 6875290 | 6876181 | 6877073 | 6877964 | 6878855 |
| 488 | 6884198 | 6885088 | 6885978 | 6886867 | 6887757 |
| 489 | 6893089 | 6893977 | 6894864 | 6895752 | 6896640 |
| 490 | 6901961 | 6902847 | 6903733 | 6904616 | 6905505 |
| 491 | 6910815 | 6911699 | 6912584 | 6913468 | 6914352 |
| 492 | 6919651 | 6920534 | 6921416 | 6922298 | 6923180 |
| 493 | 6928469 | 6929350 | 6930231 | 6931111 | 6931991 |
| 494 | 6937269 | 6938148 | 6939027 | 6939906 | 6940785 |
| 495 | 6946052 | 6946929 | 6947806 | 6948683 | 6949560 |
| 496 | 6954817 | 6955692 | 6956568 | 6957443 | 6958318 |
| 497 | 6963564 | 6964438 | 6965311 | 6966185 | 6967058 |
| 498 | 6972293 | 6973165 | 6974037 | 6974909 | 6975780 |
| 499 | 6981005 | 6981876 | 6982746 | 6983616 | 6984485 |
| 500 | 6989700 | 6990569 | 6991437 | 6992305 | 6993173 |
| 501 | 6998377 | 6999244 | 7000111 | 7000977 | 7001843 |
| 502 | 7007037 | 7007902 | 7008767 | 7009632 | 7010496 |
| 503 | 7015680 | 7016543 | 7017406 | 7018269 | 7019132 |
| 504 | 7024305 | 7025167 | 7026028 | 7026890 | 7027751 |
| 505 | 7032914 | 7033774 | 7034633 | 7035493 | 7036352 |
| 506 | 7041505 | 7042363 | 7043221 | 7044079 | 7044937 |
| 507 | 7050080 | 7050936 | 7051792 | 7052649 | 7053505 |
| 508 | 7058637 | 7059492 | 7060347 | 7061201 | 7062055 |
| 509 | 7067178 | 7068031 | 7068884 | 7069737 | 7070589 |
| 510 | 7075702 | 7076553 | 7077405 | 7078256 | 7079107 |
| 511 | 7084206 | 7085059 | 7085908 | 7086758 | 7087607 |
| 512 | 7092700 | 7093548 | 7094396 | 7095244 | 7096091 |
| 513 | 7101174 | 7102020 | 7102866 | 7103713 | 7104559 |
| 514 | 7109631 | 7110476 | 7111321 | 7112165 | 7113010 |
| 515 | 7118072 | 7118915 | 7119759 | 7120601 | 7121444 |
| 516 | 7126497 | 7127339 | 7128180 | 7129021 | 7129862 |
| 517 | 7134905 | 7135745 | 7136585 | 7137425 | 7138264 |
| 518 | 7143298 | 7144136 | 7144974 | 7145812 | 7146650 |
| 519 | 7151674 | 7152510 | 7153347 | 7154183 | 7155019 |
| 520 | 7160033 | 7160869 | 7161703 | 7162538 | 7163373 |
| 521 | 7168377 | 7169211 | 7170044 | 7170877 | 7171710 |
| 522 | 7176705 | 7177537 | 7178369 | 7179200 | 7180032 |
| 523 | 7185017 | 7185847 | 7186677 | 7187507 | 7188337 |
| 524 | 7193313 | 7194142 | 7194970 | 7195799 | 7196627 |
| 525 | 7201593 | 7202420 | 7203247 | 7204074 | 7204901 |
| 526 | 7209857 | 7210683 | 7211508 | 7212334 | 7213159 |
| 527 | 7218106 | 7218930 | 7219754 | 7220578 | 7221401 |
| 528 | 7226339 | 7227162 | 7227984 | 7228806 | 7229628 |
| 529 | 7234557 | 7235378 | 7236198 | 7237019 | 7237839 |
| 530 | 7242759 | 7243578 | 7244397 | 7245216 | 7246035 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 487 | 6879746 | 6880637 | 6881528 | 6882418 | 6883308 |
| 488 | 6888646 | 6889535 | 6890423 | 6891312 | 6892200 |
| 489 | 6897527 | 6898414 | 6899301 | 6900188 | 6901074 |
| 490 | 6906390 | 6907275 | 6908161 | 6909046 | 6909930 |
| 491 | 6915235 | 6916119 | 6917002 | 6917885 | 6918768 |
| 492 | 6924062 | 6924944 | 6925826 | 6926707 | 6927588 |
| 493 | 6932872 | 6933752 | 6934631 | 6935511 | 6936390 |
| 494 | 6941663 | 6942541 | 6943419 | 6944297 | 6945174 |
| 495 | 6950437 | 6951313 | 6952189 | 6953065 | 6953941 |
| 496 | 6959193 | 6960067 | 6960942 | 6961816 | 6962690 |
| 497 | 6967931 | 6968804 | 6969676 | 6970549 | 6971421 |
| 498 | 6976652 | 6977523 | 6978394 | 6979264 | 6980135 |
| 499 | 6985355 | 6986224 | 6987093 | 6987963 | 6988831 |
| 500 | 6994041 | 6994908 | 6995776 | 6996643 | 6997510 |
| 501 | 7002709 | 7003575 | 7004441 | 7005307 | 7006172 |
| 502 | 7011361 | 7012225 | 7013089 | 7013953 | 7014816 |
| 503 | 7019995 | 7020857 | 7021719 | 7022582 | 7023444 |
| 504 | 7028612 | 7029472 | 7030333 | 7031193 | 7032054 |
| 505 | 7037212 | 7038071 | 7038929 | 7039788 | 7040647 |
| 506 | 7045793 | 7046652 | 7047509 | 7048366 | 7049223 |
| 507 | 7054360 | 7055216 | 7056072 | 7056927 | 7057782 |
| 508 | 7062910 | 7063764 | 7064617 | 7065471 | 7066324 |
| 509 | 7071442 | 7072294 | 7073146 | 7073998 | 7074850 |
| 510 | 7079957 | 7080808 | 7081659 | 7082509 | 7083359 |
| 511 | 7088456 | 7089305 | 7090154 | 7091003 | 7091851 |
| 512 | 7096939 | 7097786 | 7098633 | 7099480 | 7100327 |
| 513 | 7105404 | 7106250 | 7107096 | 7107941 | 7108786 |
| 514 | 7113854 | 7114698 | 7115542 | 7116385 | 7117229 |
| 515 | 7122287 | 7123129 | 7123971 | 7124813 | 7125655 |
| 516 | 7130703 | 7131544 | 7132385 | 7133225 | 7134065 |
| 517 | 7139104 | 7139943 | 7140782 | 7141620 | 7142459 |
| 518 | 7147488 | 7148325 | 7149162 | 7150000 | 7150837 |
| 519 | 7155856 | 7156691 | 7157527 | 7158363 | 7159198 |
| 520 | 7164207 | 7165042 | 7165876 | 7166710 | 7167544 |
| 521 | 7172543 | 7173376 | 7174208 | 7175041 | 7175873 |
| 522 | 7180863 | 7181694 | 7182525 | 7183356 | 7184186 |
| 523 | 7189167 | 7189996 | 7190826 | 7191655 | 7192484 |
| 524 | 7197455 | 7198283 | 7199111 | 7199938 | 7200766 |
| 525 | 7205727 | 7206554 | 7207380 | 7208206 | 7209032 |
| 526 | 7213984 | 7214809 | 7215633 | 7216458 | 7217282 |
| 527 | 7222225 | 7223048 | 7223871 | 7224694 | 7225517 |
| 528 | 7230450 | 7231272 | 7232093 | 7232914 | 7233736 |
| 529 | 7238660 | 7239480 | 7240300 | 7241120 | 7241939 |
| 530 | 7246854 | 7247672 | 7248491 | 7249309 | 7250127 |

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| 531 | 7250945 | 7251763 | 7252581 | 7253398 | 7254215 |
| 532 | 7259116 | 7259933 | 7260749 | 7261565 | 7262380 |
| 533 | 7267272 | 7268087 | 7268901 | 7269716 | 7270531 |
| 534 | 7275413 | 7276226 | 7277039 | 7277852 | 7278664 |
| 535 | 7283538 | 7284349 | 7285161 | 7285972 | 7286784 |
| 536 | 7291648 | 7292458 | 7293268 | 7294078 | 7294888 |
| 537 | 7299743 | 7300551 | 7301360 | 7302168 | 7302977 |
| 538 | 7307823 | 7308630 | 7309437 | 7310244 | 7311051 |
| 539 | 7315888 | 7316693 | 7317499 | 7318304 | 7319109 |
| 540 | 7323938 | 7324749 | 7325546 | 7326350 | 7327153 |
| 541 | 7331973 | 7332775 | 7333578 | 7334380 | 7335182 |
| 542 | 7339993 | 7340794 | 7341595 | 7342396 | 7343197 |
| 543 | 7347998 | 7348798 | 7349598 | 7350397 | 7351196 |
| 544 | 7355989 | 7356787 | 7357585 | 7358383 | 7359181 |
| 545 | 7363965 | 7364762 | 7365558 | 7366355 | 7367151 |
| 546 | 7371926 | 7372722 | 7373517 | 7374312 | 7375107 |
| 547 | 7379873 | 7380667 | 7381461 | 7382254 | 7383048 |
| 548 | 7387806 | 7388598 | 7389390 | 7390182 | 7390974 |
| 549 | 7395723 | 7396514 | 7397305 | 7398096 | 7398886 |
| 550 | 7403627 | 7404416 | 7405206 | 7405995 | 7406784 |
| 551 | 7411516 | 7412304 | 7413092 | 7413880 | 7414668 |
| 552 | 7419391 | 7420177 | 7420964 | 7421750 | 7422537 |
| 553 | 7427251 | 7428037 | 7428822 | 7429607 | 7430392 |
| 554 | 7435098 | 7435881 | 7436665 | 7437449 | 7438232 |
| 555 | 7442930 | 7443712 | 7444495 | 7445277 | 7446059 |
| 556 | 7450748 | 7451529 | 7452310 | 7453091 | 7453871 |
| 557 | 7458552 | 7459332 | 7460111 | 7460890 | 7461670 |
| 558 | 7466342 | 7467120 | 7467898 | 7468676 | 7469454 |
| 559 | 7474118 | 7474895 | 7475672 | 7476448 | 7477225 |
| 560 | 7481880 | 7482656 | 7483431 | 7484206 | 7484981 |
| 561 | 7489629 | 7490403 | 7491177 | 7491950 | 7492724 |
| 562 | 7497363 | 7498136 | 7498908 | 7499681 | 7500453 |
| 563 | 7505087 | 7505855 | 7506626 | 7507398 | 7508168 |
| 564 | 7512791 | 7513561 | 7514331 | 7515100 | 7515870 |
| 565 | 7520484 | 7521253 | 7522022 | 7522790 | 7523558 |
| 566 | 7528164 | 7528932 | 7529699 | 7530466 | 7531232 |
| 567 | 7535831 | 7536596 | 7537362 | 7538128 | 7538893 |
| 568 | 7543483 | 7544248 | 7545012 | 7545777 | 7546541 |
| 569 | 7551123 | 7551886 | 7552649 | 7553412 | 7554178 |
| 570 | 7558749 | 7559510 | 7560279 | 7561034 | 7561795 |
| 571 | 7566361 | 7567122 | 7567882 | 7568642 | 7569402 |
| 572 | 7573960 | 7574719 | 7575479 | 7576237 | 7576996 |
| 573 | 7581546 | 7582304 | 7583062 | 7583819 | 7584577 |
| 574 | 7589119 | 7589875 | 7590632 | 7591388 | 7592144 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 531 | 7255033 | 7255850 | 7256667 | 7257483 | 7258300 |
| 532 | 7263196 | 7264012 | 7264827 | 7265642 | 7266457 |
| 533 | 7271344 | 7272158 | 7272972 | 7273786 | 7274599 |
| 534 | 7279477 | 7280290 | 7281101 | 7281914 | 7282726 |
| 535 | 7287595 | 7288406 | 7289216 | 7290027 | 7290838 |
| 536 | 7295697 | 7296506 | 7297316 | 7298125 | 7298934 |
| 537 | 7303785 | 7304593 | 7305400 | 7306208 | 7307015 |
| 538 | 7311857 | 7312663 | 7313470 | 7314276 | 7315082 |
| 539 | 7319914 | 7320719 | 7321524 | 7322329 | 7323133 |
| 540 | 7327957 | 7328760 | 7329564 | 7330367 | 7331170 |
| 541 | 7335985 | 7336787 | 7337588 | 7338390 | 7339191 |
| 542 | 7343997 | 7344798 | 7345598 | 7346398 | 7347198 |
| 543 | 7351995 | 7352794 | 7353593 | 7354392 | 7355191 |
| 544 | 7359979 | 7360776 | 7361574 | 7362371 | 7363168 |
| 545 | 7367948 | 7368744 | 7369540 | 7370335 | 7371131 |
| 546 | 7375902 | 7376696 | 7377491 | 7378285 | 7379079 |
| 547 | 7383841 | 7384634 | 7385427 | 7386220 | 7387013 |
| 548 | 7391766 | 7392558 | 7393350 | 7394141 | 7394932 |
| 549 | 7399677 | 7400467 | 7401257 | 7402047 | 7402837 |
| 550 | 7407573 | 7408362 | 7409151 | 7409939 | 7410728 |
| 551 | 7415455 | 7416283 | 7417030 | 7417817 | 7418604 |
| 552 | 7423323 | 7424109 | 7424895 | 7425680 | 7426466 |
| 553 | 7431176 | 7431961 | 7432745 | 7433530 | 7434314 |
| 554 | 7439015 | 7439799 | 7440582 | 7441365 | 7442147 |
| 555 | 7446841 | 7447622 | 7448404 | 7449187 | 7449967 |
| 556 | 7454652 | 7455432 | 7456212 | 7456992 | 7457772 |
| 557 | 7462449 | 7463228 | 7464006 | 7464785 | 7465564 |
| 558 | 7470232 | 7471009 | 7471787 | 7472564 | 7473341 |
| 559 | 7478001 | 7478777 | 7479553 | 7480329 | 7481105 |
| 560 | 7485756 | 7486531 | 7487306 | 7488080 | 7488854 |
| 561 | 7493498 | 7494271 | 7495044 | 7495817 | 7496590 |
| 562 | 7501225 | 7501997 | 7502769 | 7503541 | 7504312 |
| 563 | 7508939 | 7509710 | 7510480 | 7511251 | 7512021 |
| 564 | 7516639 | 7517409 | 7518178 | 7518947 | 7519716 |
| 565 | 7524326 | 7525094 | 7525862 | 7526629 | 7527397 |
| 566 | 7531999 | 7532766 | 7533532 | 7534298 | 7535065 |
| 567 | 7539659 | 7540424 | 7541189 | 7541954 | 7542719 |
| 568 | 7547305 | 7548069 | 7548832 | 7549596 | 7550359 |
| 569 | 7554937 | 7555700 | 7556462 | 7557224 | 7557987 |
| 570 | 7562556 | 7563318 | 7564079 | 7564840 | 7565600 |
| 571 | 7570162 | 7570922 | 7571682 | 7572441 | 7573201 |
| 572 | 7577755 | 7578513 | 7579272 | 7580030 | 7580788 |
| 573 | 7585334 | 7586091 | 7586848 | 7587605 | 7588362 |
| 574 | 7592900 | 7593656 | 7594412 | 7595168 | 7595923 |

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| 576 | 7604225 | 7604979 | 7605733 | 7606486 | 7607240 |
| 577 | 7611758 | 7612511 | 7613263 | 7614016 | 7614768 |
| 578 | 7619278 | 7620030 | 7620781 | 7621532 | 7622283 |
| 579 | 7626786 | 7627536 | 7628286 | 7629035 | 7629785 |
| 580 | 7634280 | 7635029 | 7635777 | 7636526 | 7637274 |
| 581 | 7641761 | 7642509 | 7643256 | 7644003 | 7644750 |
| 582 | 7649230 | 7649976 | 7650722 | 7651468 | 7652214 |
| 583 | 7656686 | 7657430 | 7658175 | 7658920 | 7659664 |
| 584 | 7664128 | 7664872 | 7665616 | 7666359 | 7667102 |
| 585 | 7671559 | 7672301 | 7673043 | 7673785 | 7674527 |
| 586 | 7678976 | 7679717 | 7680458 | 7681199 | 7681940 |
| 587 | 7686381 | 7687121 | 7687860 | 7688600 | 7689339 |
| 588 | 7693773 | 7694512 | 7695250 | 7695988 | 7696727 |
| 589 | 7701153 | 7701890 | 7702627 | 7703364 | 7704101 |
| 590 | 7708520 | 7709256 | 7709992 | 7710728 | 7711463 |
| 591 | 7715875 | 7716610 | 7717344 | 7718079 | 7718813 |
| 592 | 7723217 | 7723951 | 7724684 | 7725417 | 7726150 |
| 593 | 7730547 | 7731279 | 7732011 | 7732743 | 7733475 |
| 594 | 7737864 | 7738596 | 7739326 | 7740057 | 7740788 |
| 595 | 7745170 | 7745899 | 7746629 | 7747359 | 7748088 |
| 596 | 7752463 | 7753191 | 7753920 | 7754648 | 7755376 |
| 597 | 7759743 | 7760471 | 7761198 | 7761925 | 7762652 |
| 598 | 7767012 | 7767738 | 7768464 | 7769190 | 7769916 |
| 599 | 7774268 | 7774993 | 7775718 | 7776443 | 7777167 |
| 600 | 7781513 | 7782236 | 7782960 | 7783683 | 7784407 |
| 601 | 7788745 | 7789467 | 7790190 | 7790912 | 7791634 |
| 602 | 7795965 | 7796686 | 7797408 | 7798129 | 7798850 |
| 603 | 7803173 | 7803893 | 7804613 | 7805333 | 7806053 |
| 604 | 7810369 | 7811088 | 7811807 | 7812526 | 7813245 |
| 605 | 7817554 | 7818272 | 7818989 | 7819707 | 7820424 |
| 606 | 7824726 | 7825443 | 7826159 | 7826876 | 7827592 |
| 607 | 7831887 | 7832602 | 7833318 | 7834033 | 7834748 |
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| 609 | 7846173 | 7846886 | 7847599 | 7848312 | 7849024 |
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| 612 | 7867514 | 7868224 | 7868933 | 7869643 | 7870352 |
| 613 | 7874605 | 7875313 | 7876021 | 7876730 | 7877438 |
| 614 | 7881684 | 7882391 | 7883098 | 7883805 | 7884512 |
| 615 | 7888751 | 7889457 | 7890163 | 7890869 | 7891575 |
| 616 | 7895807 | 7896512 | 7897217 | 7897922 | 7898626 |
| 617 | 7902852 | 7903555 | 7904259 | 7904963 | 7905666 |
| 618 | 7909885 | 7910587 | 7911290 | 7911992 | 7912695 |

| Natural Numbers | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
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| 576 | 7607993 | 7608746 | 7609500 | 7610253 | 7611005 |
| 577 | 7615520 | 7616272 | 7617024 | 7617775 | 7618527 |
| 578 | 7623034 | 7623784 | 7624535 | 7625285 | 7626035 |
| 579 | 7630534 | 7631284 | 7632033 | 7632782 | 7633531 |
| 580 | 7638022 | 7638770 | 7639518 | 7640266 | 7641014 |
| 581 | 7645497 | 7646244 | 7646991 | 7647737 | 7648484 |
| 582 | 7652959 | 7653705 | 7654450 | 7655195 | 7655941 |
| 583 | 7660409 | 7661153 | 7661897 | 7662641 | 7663385 |
| 584 | 7667845 | 7668588 | 7669331 | 7670074 | 7670816 |
| 585 | 7675269 | 7676011 | 7676752 | 7677494 | 7678235 |
| 586 | 7682680 | 7683421 | 7684161 | 7684901 | 7685641 |
| 587 | 7690079 | 7690818 | 7691557 | 7692296 | 7693035 |
| 588 | 7697465 | 7698203 | 7698940 | 7699678 | 7700416 |
| 589 | 7704838 | 7705575 | 7706311 | 7707048 | 7707784 |
| 590 | 7712199 | 7712934 | 7713670 | 7714405 | 7715140 |
| 591 | 7719547 | 7720282 | 7721016 | 7721750 | 7722483 |
| 592 | 7726884 | 7727616 | 7728349 | 7729082 | 7729814 |
| 593 | 7734207 | 7734939 | 7735670 | 7736402 | 7737133 |
| 594 | 7741519 | 7742249 | 7742979 | 7743710 | 7744440 |
| 595 | 7748818 | 7749547 | 7750276 | 7751005 | 7751734 |
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| 615 | 7892281 | 7892986 | 7893691 | 7894397 | 7895102 |
| 616 | 7899331 | 7900035 | 7900739 | 7901444 | 7902148 |
| 617 | 7906370 | 7907073 | 7907776 | 7908479 | 7909182 |
| 618 | 7913397 | 7914099 | 7914801 | 7915503 | 7916205 |

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| 621 | 7930916 | 7931615 | 7932314 | 7933014 | 7933712 |
| 622 | 7937904 | 7938602 | 7939300 | 7939998 | 7940696 |
| 623 | 7944880 | 7945578 | 7946274 | 7946971 | 7947668 |
| 624 | 7951846 | 7952542 | 7953238 | 7953933 | 7954629 |
| 625 | 7958800 | 7959495 | 7960190 | 7960884 | 7961578 |
| 626 | 7965743 | 7966437 | 7967131 | 7967824 | 7968517 |
| 627 | 7972675 | 7973368 | 7974060 | 7974753 | 7975445 |
| 628 | 7979596 | 7980288 | 7980979 | 7981671 | 7982362 |
| 629 | 7986506 | 7987197 | 7987887 | 7988577 | 7989267 |
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| 631 | 8000294 | 8000982 | 8001670 | 8002358 | 8003045 |
| 632 | 8007171 | 8007858 | 8008545 | 8009232 | 8009919 |
| 633 | 8014037 | 8014723 | 8015409 | 8016095 | 8016781 |
| 634 | 8020893 | 8021578 | 8022262 | 8022947 | 8023632 |
| 635 | 8027737 | 8028421 | 8029105 | 8029789 | 8030472 |
| 636 | 8034571 | 8035254 | 8035937 | 8036619 | 8037302 |
| 637 | 8041394 | 8042076 | 8042758 | 8043439 | 8044121 |
| 638 | 8048207 | 8048887 | 8049568 | 8050248 | 8050929 |
| 639 | 8055009 | 8055688 | 8056368 | 8057047 | 8057726 |
| 640 | 8061800 | 8062478 | 8063157 | 8063835 | 8064513 |
| 641 | 8068580 | 8069258 | 8069935 | 8070612 | 8071290 |
| 642 | 8075350 | 8076027 | 8076703 | 8077379 | 8078055 |
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| 644 | 8088859 | 8089533 | 8090207 | 8090881 | 8091555 |
| 645 | 8095597 | 8096270 | 8096944 | 8097617 | 8098290 |
| 646 | 8102325 | 8102997 | 8103670 | 8104342 | 8105013 |
| 647 | 8109043 | 8109714 | 8110385 | 8111056 | 8111727 |
| 648 | 8115750 | 8116420 | 8117090 | 8117760 | 8118430 |
| 649 | 8122447 | 8123116 | 8123785 | 8124454 | 8125123 |
| 650 | 8129134 | 8129802 | 8130470 | 8131138 | 8131805 |
| 651 | 8135810 | 8136477 | 8137144 | 8137811 | 8138478 |
| 652 | 8142476 | 8143142 | 8143808 | 8144474 | 8145140 |
| 653 | 8149132 | 8149797 | 8150462 | 8151127 | 8151791 |
| 654 | 8155777 | 8156441 | 8157105 | 8157769 | 8158433 |
| 655 | 8162413 | 8163076 | 8163739 | 8164402 | 8165064 |
| 656 | 8169038 | 8169700 | 8170362 | 8171024 | 8171686 |
| 657 | 8175654 | 8176315 | 8176976 | 8177636 | 8178297 |
| 658 | 8182259 | 8182919 | 8183579 | 8184239 | 8184898 |
| 659 | 8188854 | 8189513 | 8190172 | 8190831 | 8191489 |
| 660 | 8195439 | 8196097 | 8196755 | 8197413 | 8198071 |
| 661 | 8202015 | 8202672 | 8203328 | 8203987 | 8204642 |
| 662 | 8208580 | 8209236 | 8209892 | 8210548 | 8211203 |

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| 619 | 7920413 | 7921114 | 7921815 | 7922516 | 7923216 |
| 620 | 7927418 | 7928118 | 7928817 | 7929517 | 7930217 |
| 621 | 7934411 | 7935110 | 7935809 | 7936507 | 7937206 |
| 622 | 7941394 | 7942091 | 7942789 | 7943486 | 7944183 |
| 623 | 7948365 | 7949061 | 7949757 | 7950454 | 7951150 |
| 624 | 7955324 | 7956020 | 7956715 | 7957410 | 7958105 |
| 625 | 7962273 | 7962967 | 7963662 | 7964356 | 7965050 |
| 626 | 7969211 | 7969904 | 7970597 | 7971290 | 7971983 |
| 627 | 7976137 | 7976829 | 7977521 | 7978213 | 7978905 |
| 628 | 7983053 | 7983744 | 7984435 | 7985125 | 7985816 |
| 629 | 7989957 | 7990647 | 7991337 | 7992027 | 7992716 |
| 630 | 7996851 | 7997540 | 7998228 | 7998917 | 7999605 |
| 631 | 8003734 | 8004421 | 8005109 | 8005796 | 8006484 |
| 632 | 8010505 | 8011292 | 8011978 | 8012665 | 8013351 |
| 633 | 8017466 | 8018152 | 8018837 | 8019522 | 8020208 |
| 634 | 8024316 | 8025001 | 8025685 | 8026369 | 8027053 |
| 635 | 8031156 | 8031839 | 8032522 | 8033205 | 8033888 |
| 636 | 8037984 | 8038666 | 8039348 | 8040031 | 8040712 |
| 637 | 8044802 | 8045483 | 8046164 | 8046845 | 8047526 |
| 638 | 8051609 | 8052289 | 8052969 | 8053649 | 8054329 |
| 639 | 8058405 | 8059085 | 8059763 | 8060442 | 8061121 |
| 640 | 8065191 | 8065869 | 8066547 | 8067225 | 8067903 |
| 641 | 8071967 | 8072643 | 8073320 | 8073997 | 8074674 |
| 642 | 8078731 | 8079407 | 8080083 | 8080759 | 8081434 |
| 643 | 8085485 | 8086160 | 8086835 | 8087510 | 8088184 |
| 644 | 8092229 | 8092903 | 8093577 | 8094250 | 8094924 |
| 645 | 8098962 | 8099635 | 8100308 | 8100980 | 8101653 |
| 646 | 8105685 | 8106357 | 8107029 | 8107700 | 8108371 |
| 647 | 8112398 | 8113068 | 8113739 | 8114409 | 8115080 |
| 648 | 8119100 | 8119769 | 8120439 | 8121108 | 8121778 |
| 649 | 8125792 | 8126460 | 8127129 | 8127797 | 8128465 |
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| 651 | 8139144 | 8139811 | 8140477 | 8141144 | 8141810 |
| 652 | 8145805 | 8146471 | 8147136 | 8147801 | 8148467 |
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| 654 | 8159096 | 8159760 | 8160423 | 8161087 | 8161750 |
| 655 | 8165727 | 8166389 | 8167052 | 8167714 | 8168376 |
| 656 | 8172347 | 8173009 | 8173670 | 8174331 | 8174993 |
| 657 | 8178958 | 8179618 | 8180278 | 8180939 | 8181599 |
| 658 | 8185558 | 8186217 | 8186877 | 8187536 | 8188195 |
| 659 | 8192146 | 8192806 | 8193465 | 8194123 | 8194781 |
| 660 | 8198728 | 8199386 | 8200043 | 8200700 | 8201358 |
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| 662 | 8211859 | 8212514 | 8213170 | 8213825 | 8214480 |

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|---|---|---|---|---|---|
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| 665 | 8228216 | 8228869 | 8229522 | 8230175 | 8230828 |
| 666 | 8234742 | 8235394 | 8236046 | 8236698 | 8237350 |
| 667 | 8241258 | 8241909 | 8242560 | 8243211 | 8243862 |
| 668 | 8247765 | 8248415 | 8249065 | 8249715 | 8250364 |
| 669 | 8254261 | 8254910 | 8255559 | 8256208 | 8256857 |
| 670 | 8260748 | 8261396 | 8262044 | 8262692 | 8263340 |
| 671 | 8267225 | 8267872 | 8268519 | 8269166 | 8269813 |
| 672 | 8273693 | 8274339 | 8274985 | 8275631 | 8276277 |
| 673 | 8280151 | 8280796 | 8281441 | 8282086 | 8282731 |
| 674 | 8286599 | 8287243 | 8287887 | 8288532 | 8289176 |
| 675 | 8293038 | 8293681 | 8294324 | 8294967 | 8295611 |
| 676 | 8299467 | 8300109 | 8300752 | 8301394 | 8302036 |
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| 678 | 8312297 | 8312937 | 8313578 | 8314218 | 8314858 |
| 679 | 8318698 | 8319337 | 8319977 | 8320616 | 8321255 |
| 680 | 8325089 | 8325728 | 8326366 | 8327005 | 8327643 |
| 681 | 8331471 | 8332109 | 8332746 | 8333384 | 8334021 |
| 682 | 8337844 | 8338480 | 8339117 | 8339754 | 8340390 |
| 683 | 8344207 | 8344843 | 8345479 | 8346114 | 8346750 |
| 684 | 8350561 | 8351196 | 8351831 | 8352465 | 8353100 |
| 685 | 8356906 | 8357540 | 8358174 | 8358807 | 8359441 |
| 686 | 8363241 | 8363874 | 8364507 | 8365140 | 8365773 |
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| 688 | 8375884 | 8376516 | 8377147 | 8377778 | 8378409 |
| 689 | 8382192 | 8382822 | 8383453 | 8384083 | 8384713 |
| 690 | 8388491 | 8389120 | 8389750 | 8390379 | 8391008 |
| 691 | 8394780 | 8395409 | 8396037 | 8396666 | 8397294 |
| 692 | 8401061 | 8401688 | 8402316 | 8402943 | 8403571 |
| 693 | 8407332 | 8407959 | 8408586 | 8409212 | 8409838 |
| 694 | 8413595 | 8414220 | 8414846 | 8415472 | 8416097 |
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| 697 | 8432328 | 8432951 | 8433574 | 8434197 | 8434819 |
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| 701 | 8457180 | 8457800 | 8458419 | 8459038 | 8459658 |
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| 703 | 8469553 | 8470171 | 8470789 | 8471406 | 8472024 |
| 704 | 8475727 | 8476343 | 8476960 | 8477577 | 8478193 |
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| 706 | 8488047 | 8488662 | 8489277 | 8489892 | 8490507 |

| Natural Numbers | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 663 | 8218409 | 8219064 | 8219718 | 8220372 | 8221027 |
| 664 | 8224950 | 8225603 | 8226257 | 8226910 | 8227563 |
| 665 | 8231481 | 8232133 | 8232786 | 8233438 | 8234090 |
| 666 | 8238002 | 8238653 | 8239305 | 8239956 | 8240607 |
| 667 | 8244513 | 8245163 | 8245814 | 8246464 | 8247114 |
| 668 | 8251014 | 8251664 | 8252313 | 8252963 | 8253612 |
| 669 | 8257506 | 8258154 | 8258803 | 8259451 | 8260100 |
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| 671 | 8270460 | 8271107 | 8271753 | 8272400 | 8273046 |
| 672 | 8276923 | 8277569 | 8278214 | 8278860 | 8279505 |
| 673 | 8283376 | 8284021 | 8284665 | 8285310 | 8285955 |
| 674 | 8289820 | 8290463 | 8291107 | 8291751 | 8292394 |
| 675 | 8296254 | 8296896 | 8297539 | 8298182 | 8298824 |
| 676 | 8302678 | 8303320 | 8303962 | 8304603 | 8305245 |
| 677 | 8309093 | 8309734 | 8310375 | 8311016 | 8311656 |
| 678 | 8315499 | 8316139 | 8316778 | 8317418 | 8318058 |
| 679 | 8321895 | 8322534 | 8323173 | 8323812 | 8324450 |
| 680 | 8328281 | 8328919 | 8329558 | 8330195 | 8330833 |
| 681 | 8334659 | 8335296 | 8335933 | 8336570 | 8337207 |
| 682 | 8341027 | 8341663 | 8342299 | 8342937 | 8343571 |
| 683 | 8347385 | 8348021 | 8348656 | 8349291 | 8349926 |
| 684 | 8353735 | 8354369 | 8355003 | 8355638 | 8356272 |
| 685 | 8360075 | 8360708 | 8361341 | 8361975 | 8362608 |
| 686 | 8366405 | 8367038 | 8367670 | 8368303 | 8368935 |
| 687 | 8372727 | 8373359 | 8373990 | 8374622 | 8375253 |
| 688 | 8379039 | 8379670 | 8380301 | 8380931 | 8381562 |
| 689 | 8385343 | 8385973 | 8386602 | 8387232 | 8387861 |
| 690 | 8391637 | 8392266 | 8392895 | 8393523 | 8394152 |
| 691 | 8397922 | 8398550 | 8399178 | 8399806 | 8400433 |
| 692 | 8404198 | 8404825 | 8405452 | 8406079 | 8406706 |
| 693 | 8410465 | 8411091 | 8411717 | 8412343 | 8412969 |
| 694 | 8416722 | 8417348 | 8417973 | 8418598 | 8419223 |
| 695 | 8422971 | 8423596 | 8424220 | 8424844 | 8425468 |
| 696 | 8429211 | 8429735 | 8430458 | 8431081 | 8431705 |
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| 698 | 8441664 | 8442286 | 8442907 | 8443529 | 8444150 |
| 699 | 8447877 | 8448498 | 8449119 | 8449739 | 8450360 |
| 700 | 8454081 | 8454701 | 8455321 | 8455941 | 8456561 |
| 701 | 8460277 | 8460896 | 8461515 | 8462134 | 8462752 |
| 702 | 8466463 | 8467081 | 8467700 | 8468318 | 8468935 |
| 703 | 8472641 | 8473258 | 8473876 | 8474493 | 8475110 |
| 704 | 8478810 | 8479426 | 8480043 | 8480659 | 8481275 |
| 705 | 8484970 | 8485586 | 8486201 | 8486817 | 8487432 |
| 706 | 8491122 | 8491736 | 8492351 | 8492965 | 8493580 |

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| 708 | 8500333 | 8500946 | 8501559 | 8502172 | 8502786 |
| 709 | 8506462 | 8507075 | 8507687 | 8508300 | 8508912 |
| 710 | 8512583 | 8513195 | 8513807 | 8514418 | 8515030 |
| 711 | 8518696 | 8519307 | 8519917 | 8520528 | 8521139 |
| 712 | 8524800 | 8525410 | 8526020 | 8526629 | 8527239 |
| 713 | 8530895 | 8531504 | 8532113 | 8532722 | 8533331 |
| 714 | 8536982 | 8537590 | 8538198 | 8538806 | 8539414 |
| 715 | 8543060 | 8543668 | 8544275 | 8544882 | 8545489 |
| 716 | 8549130 | 8549737 | 8550343 | 8550949 | 8551556 |
| 717 | 8555192 | 8555797 | 8556403 | 8557008 | 8557614 |
| 718 | 8561244 | 8561849 | 8562454 | 8563059 | 8563663 |
| 719 | 8567289 | 8567893 | 8568497 | 8569101 | 8569704 |
| 720 | 8573325 | 8573928 | 8574531 | 8575134 | 8575737 |
| 721 | 8579353 | 8579955 | 8580557 | 8581159 | 8581761 |
| 722 | 8585372 | 8585973 | 8586575 | 8587176 | 8587777 |
| 723 | 8591383 | 8591984 | 8592584 | 8593185 | 8593785 |
| 724 | 8597386 | 8597985 | 8598585 | 8599185 | 8599784 |
| 725 | 8603380 | 8603979 | 8604578 | 8605177 | 8605776 |
| 726 | 8609366 | 8609964 | 8610562 | 8611160 | 8611758 |
| 727 | 8615344 | 8615941 | 8616539 | 8617136 | 8617733 |
| 728 | 8621314 | 8621910 | 8622507 | 8623103 | 8623699 |
| 729 | 8627275 | 8627871 | 8628467 | 8629062 | 8629658 |
| 730 | 8633229 | 8633823 | 8634418 | 8635013 | 8635608 |
| 731 | 8639174 | 8639768 | 8640362 | 8640956 | 8641550 |
| 732 | 8645111 | 8645704 | 8646297 | 8646890 | 8647483 |
| 733 | 8651040 | 8651632 | 8652225 | 8652817 | 8653409 |
| 734 | 8656961 | 8657552 | 8658144 | 8658735 | 8659327 |
| 735 | 8662873 | 8663464 | 8664055 | 8664646 | 8665236 |
| 736 | 8668778 | 8669368 | 8669958 | 8670548 | 8671138 |
| 737 | 8674675 | 8675264 | 8675853 | 8676442 | 8677031 |
| 738 | 8680564 | 8681152 | 8681740 | 8682329 | 8682917 |
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| 740 | 8692317 | 8692904 | 8693491 | 8694077 | 8694664 |
| 741 | 8698182 | 8698768 | 8699354 | 8699940 | 8700526 |
| 742 | 8704039 | 8704624 | 8705209 | 8705795 | 8706380 |
| 743 | 8709888 | 8710473 | 8711057 | 8711641 | 8712226 |
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| 745 | 8721563 | 8722146 | 8722728 | 8723311 | 8723894 |
| 746 | 8727388 | 8727970 | 8728552 | 8729134 | 8729716 |
| 747 | 8733206 | 8733788 | 8734369 | 8734950 | 8735531 |
| 748 | 8739016 | 8739597 | 8740177 | 8740757 | 8741338 |
| 749 | 8744818 | 8745398 | 8745978 | 8746557 | 8747137 |
| 750 | 8750613 | 8751192 | 8751771 | 8752349 | 8752928 |

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| 708 | 8503399 | 8504011 | 8504624 | 8505237 | 8505850 |
| 709 | 8509524 | 8510136 | 8510748 | 8511360 | 8511972 |
| 710 | 8515641 | 8516252 | 8516863 | 8517474 | 8518085 |
| 711 | 8521749 | 8522359 | 8522970 | 8523580 | 8524190 |
| 712 | 8527849 | 8528458 | 8529058 | 8529677 | 8530286 |
| 713 | 8533940 | 8534548 | 8535157 | 8535765 | 8536374 |
| 714 | 8540022 | 8540630 | 8541238 | 8541845 | 8542453 |
| 715 | 8546096 | 8546703 | 8547310 | 8547917 | 8548524 |
| 716 | 8552162 | 8552768 | 8553374 | 8553980 | 8554586 |
| 717 | 8558219 | 8558824 | 8559429 | 8560035 | 8560640 |
| 718 | 8564268 | 8564872 | 8565476 | 8566081 | 8566685 |
| 719 | 8570308 | 8570912 | 8571515 | 8572118 | 8572722 |
| 720 | 8576340 | 8575943 | 8577545 | 8578148 | 8578750 |
| 721 | 8582363 | 8582965 | 8583567 | 8584169 | 8584770 |
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| 724 | 8600384 | 8600983 | 8601583 | 8602182 | 8602781 |
| 725 | 8605374 | 8606973 | 8607571 | 8608170 | 8608768 |
| 726 | 8612356 | 8612954 | 8613552 | 8614149 | 8614747 |
| 727 | 8618330 | 8618927 | 8619524 | 8620120 | 8620717 |
| 728 | 8624296 | 8624892 | 8625488 | 8626084 | 8626679 |
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| 732 | 8648076 | 8648669 | 8649262 | 8649855 | 8650447 |
| 733 | 8654001 | 8654593 | 8655185 | 8655777 | 8656369 |
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| 746 | 8730298 | 8730880 | 8731461 | 8732043 | 8732625 |
| 747 | 8736112 | 8736693 | 8737274 | 8737855 | 8738435 |
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| 750 | 8753507 | 8754086 | 8754664 | 8755243 | 8755821 |

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| 753 | 8767950 | 8768526 | 8769103 | 8769680 | 8770256 |
| 754 | 8773713 | 8774289 | 8774865 | 8775441 | 8776017 |
| 755 | 8779469 | 8780045 | 8780620 | 8781195 | 8781770 |
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| 757 | 8790959 | 8791532 | 8792106 | 8792680 | 8793253 |
| 758 | 8796692 | 8797265 | 8797838 | 8798411 | 8798983 |
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| 764 | 8830934 | 8831502 | 8832070 | 8832639 | 8833207 |
| 765 | 8836614 | 8837182 | 8837750 | 8838317 | 8838885 |
| 766 | 8842288 | 8842855 | 8843421 | 8843988 | 8844555 |
| 767 | 8847954 | 8848520 | 8849086 | 8849652 | 8850218 |
| 768 | 8853612 | 8854178 | 8854743 | 8855308 | 8855874 |
| 769 | 8859263 | 8859828 | 8860393 | 8860957 | 8861522 |
| 770 | 8864907 | 8865471 | 8866035 | 8866599 | 8867163 |
| 771 | 8870544 | 8871107 | 8871670 | 8872233 | 8872796 |
| 772 | 8876173 | 8876736 | 8877298 | 8877860 | 8878423 |
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| 777 | 8904210 | 8904769 | 8905328 | 8905887 | 8906445 |
| 778 | 8909796 | 8910354 | 8910912 | 8911470 | 8912028 |
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| 780 | 8920946 | 8921503 | 8922059 | 8922616 | 8923173 |
| 781 | 8926510 | 8927066 | 8927622 | 8928178 | 8928734 |
| 782 | 8932068 | 8932623 | 8933178 | 8933733 | 8934288 |
| 783 | 8937618 | 8938172 | 8938727 | 8939281 | 8939836 |
| 784 | 8943161 | 8943715 | 8944268 | 8944822 | 8945376 |
| 785 | 8948697 | 8949250 | 8949803 | 8950356 | 8950909 |
| 786 | 8954225 | 8954778 | 8955330 | 8955883 | 8956435 |
| 787 | 8959747 | 8960299 | 8960851 | 8961403 | 8961954 |
| 789 | 8965262 | 8965813 | 8966364 | 8966915 | 8967466 |
| 789 | 8970770 | 8971320 | 8971871 | 8972421 | 8972971 |
| 790 | 8976271 | 8976821 | 8977370 | 8977920 | 8978469 |
| 791 | 8981765 | 8982314 | 8982863 | 8983412 | 8983960 |
| 792 | 8987252 | 8987800 | 8988348 | 8988897 | 8989445 |
| 793 | 8992732 | 8993279 | 8993827 | 8994375 | 8994922 |
| 794 | 8998205 | 8998752 | 8999299 | 8999846 | 9000392 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
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| 751 | 8759290 | 8759868 | 8760445 | 8761023 | 8761601 |
| 752 | 8765065 | 8765642 | 8766219 | 8766796 | 8767373 |
| 753 | 8770833 | 8771409 | 8771985 | 8772561 | 8773137 |
| 754 | 8776592 | 8777168 | 8777743 | 8778319 | 8778894 |
| 755 | 8782345 | 8782919 | 8783494 | 8784069 | 8784643 |
| 756 | 8788089 | 8788663 | 8789237 | 8789811 | 8790385 |
| 757 | 8793826 | 8794400 | 8794973 | 8795546 | 8796119 |
| 758 | 8799556 | 8800128 | 8800701 | 8801273 | 8801846 |
| 759 | 8805278 | 8805850 | 8806421 | 8806993 | 8807564 |
| 760 | 8810992 | 8811563 | 8812134 | 8812705. | 8813275 |
| 761 | 8816699 | 8817269 | 8817840 | 8818410 | 8818980 |
| 762 | 8822398 | 8822968 | 8823537 | 8824107 | 8824676 |
| 763 | 8828090 | 8828659 | 8829228 | 8829797 | 8830365 |
| 764 | 8833775 | 8834343 | 8834911 | 8835479 | 8836047 |
| 765 | 8839452 | 8840019 | 8840586 | 8841154 | 8841721 |
| 766 | 8845122 | 8845688 | 8846255 | 8846821 | 8847387 |
| 767 | 8850784 | 8851350 | 8851915 | 8852481 | 8853047 |
| 768 | 8856439 | 8857004 | 8857569 | 8858134 | 8858699 |
| 769 | 8862086 | 8862651 | 8863215 | 8863779 | 8864343 |
| 770 | 8867726 | 8868290 | 8868854 | 8869417 | 8869980 |
| 771 | 8873359 | 8873922 | 8874485 | 8875048 | 8875610 |
| 772 | 8878985 | 8879547 | 8880109 | 8880671 | 8881233 |
| 773 | 8884603 | 8885165 | 8885726 | 8886287 | 8886848 |
| 774 | 8890214 | 8890775 | 8891336 | 8891896 | 8892457 |
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| 776 | 8901415 | 8901974 | 8902533 | 8903092 | 8903651 |
| 777 | 8907004 | 8907562 | 8908121 | 8908679 | 8909238 |
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| 779 | 8918161 | 8918718 | 8919275 | 8919832 | 8920389 |
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| 781 | 8929290 | 8929846 | 8930401 | 8930957 | 8931512 |
| 782 | 8934843 | 8935398 | 8935953 | 8936508 | 8937063 |
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| 785 | 8951462 | 8952015 | 8952567 | 8953120 | 8953673 |
| 786 | 8956987 | 8957539 | 8958092 | 8958644 | 8959195 |
| 787 | 8962506 | 8963057 | 8963608 | 8964160 | 8964711 |
| 788 | 8968017 | 8968568 | 8969118 | 8969669 | 8970219 |
| 789 | 8973521 | 8974071 | 8974621 | 8975171 | 8975721 |
| 790 | 8979019 | 8979568 | 8980117 | 8980667 | 8981216 |
| 791 | 8984519 | 8985058 | 8985606 | 8986155 | 8985703 |
| 792 | 8989993 | 8990541 | 8991089 | 8991636 | 8992184 |
| 793 | 8995469 | 8996017 | 8996564 | 8997111 | 8997658 |
| 794 | 9000939 | 9001486 | 9002032 | 9002579 | 9003125 |

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| 797 | 9014583 | 9015128 | 9015673 | 9016218 | 9016762 |
| 798 | 9020029 | 9020573 | 9021117 | 9021661 | 9022205 |
| 799 | 9025468 | 9026011 | 9026555 | 9027098 | 9027641 |
| 800 | 9030900 | 9031443 | 9031985 | 9032528 | 9033071 |
| 801 | 9036325 | 9036867 | 9037409 | 9037951 | 9038493 |
| 802 | 9041744 | 9042285 | 9042827 | 9043368 | 9043909 |
| 803 | 9047155 | 9047696 | 9048237 | 9048778 | 9049318 |
| 804 | 9052560 | 9053101 | 9053641 | 9054181 | 9054721 |
| 805 | 9057960 | 9058498 | 9059038 | 9059577 | 9060116 |
| 806 | 9063351 | 9063889 | 9064428 | 9064967 | 9065505 |
| 807 | 9068735 | 9069273 | 9069812 | 9070350 | 9070887 |
| 808 | 9074114 | 9074651 | 9075188 | 9075726 | 9076263 |
| 809 | 9079485 | 9080022 | 9080559 | 9081095 | 9081632 |
| 810 | 9084850 | 9085386 | 9085922 | 9086458 | 9086994 |
| 811 | 9090209 | 9090744 | 9091279 | 9091815 | 9092350 |
| 812 | 9095560 | 9096095 | 9096630 | 9097165 | 9097699 |
| 813 | 9100905 | 9101440 | 9101974 | 9102508 | 9103042 |
| 814 | 9106244 | 9106778 | 9107311 | 9107844 | 9108378 |
| 815 | 9111576 | 9112109 | 9112642 | 9113174 | 9113707 |
| 816 | 9116902 | 9117434 | 9117966 | 9118498 | 9119030 |
| 817 | 9122220 | 9122752 | 9123234 | 9123815 | 9124346 |
| 818 | 9127533 | 9128064 | 9128595 | 9129126 | 9129656 |
| 819 | 9132839 | 9133369 | 9133899 | 9134430 | 9134960 |
| 820 | 9138139 | 9138668 | 9139198 | 9139727 | 9140257 |
| 821 | 9143432 | 9143961 | 9144489 | 9145018 | 9145547 |
| 822 | 9148718 | 9149246 | 9149775 | 9150303 | 9150831 |
| 823 | 9153998 | 9154526 | 9155054 | 9155581 | 9156109 |
| 824 | 9159272 | 9159799 | 9160326 | 9160853 | 9161380 |
| 825 | 9164539 | 9165066 | 9165592 | 9166118 | 9166645 |
| 826 | 9169800 | 9170326 | 9170852 | 9171378 | 9171903 |
| 827 | 9175055 | 9175580 | 9176105 | 9176630 | 9177155 |
| 828 | 9180303 | 9180828 | 9181352 | 9181877 | 9182401 |
| 829 | 9185545 | 9186069 | 9186593 | 9187117 | 9187640 |
| 830 | 9190781 | 9191304 | 9191827 | 9192350 | 9192873 |
| 831 | 9196010 | 9196533 | 9197055 | 9197578 | 9198100 |
| 832 | 9201233 | 9201755 | 9202277 | 9202799 | 9203321 |
| 833 | 9206450 | 9206971 | 9207493 | 9208014 | 9208535 |
| 834 | 9211661 | 9212181 | 9212702 | 9213222 | 9213743 |
| 835 | 9216865 | 9217385 | 9217905 | 9218425 | 9218945 |
| 836 | 9222063 | 9222582 | 9223102 | 9223621 | 9224140 |
| 837 | 9227255 | 9227773 | 9228292 | 9228811 | 9229330 |
| 838 | 9232440 | 9232958 | 9233477 | 9233995 | 9234513 |

| Natural Numbers | 5 | 6 | 7 | 8 | 9 |
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| 796 | 9011858 | 9012403 | 9012948 | 9013493 | 9014038 |
| 797 | 9017307 | 9017851 | 9018396 | 9018940 | 9019485 |
| 798 | 9022749 | 9023293 | 9023837 | 9024381 | 9024924 |
| 799 | 9028185 | 9028728 | 9029271 | 9029814 | 9030357 |
| 800 | 9033613 | 9034156 | 9034698 | 9035241 | 9035783 |
| 801 | 9039035 | 9039577 | 9040119 | 9040661 | 9041202 |
| 802 | 9044450 | 9044992 | 9045533 | 9046073 | 9046615 |
| 803 | 9049859 | 9050399 | 9050940 | 9051480 | 9052020 |
| 804 | 9055261 | 9055800 | 9056340 | 9056880 | 9057419 |
| 805 | 9060655 | 9061195 | 9061734 | 9062274 | 9062812 |
| 806 | 9066044 | 9066582 | 9067121 | 9067659 | 9068197 |
| 807 | 9071425 | 9071963 | 9072501 | 9073038 | 9073576 |
| 808 | 9076800 | 9077337 | 9077874 | 9078411 | 9078948 |
| 809 | 9082169 | 9082705 | 9083241 | 9083778 | 9084314 |
| 810 | 9087530 | 9088066 | 9088602 | 9089137 | 9089673 |
| 811 | 9092885 | 9093420 | 9093955 | 9094490 | 9095025 |
| 812 | 9098234 | 9098768 | 9099303 | 9099837 | 9100371 |
| 813 | 9103576 | 9104109 | 9104643 | 9105177 | 9105710 |
| 814 | 9108911 | 9109444 | 9109977 | 9110510 | 9111043 |
| 815 | 9114240 | 9114772 | 9115305 | 9115837 | 9116369 |
| 816 | 9119562 | 9120094 | 9120626 | 9121157 | 9121689 |
| 817 | 9124878 | 9125409 | 9125940 | 9126471 | 9127002 |
| 818 | 9130187 | 9130717 | 9131248 | 9131778 | 9132309 |
| 819 | 9135490 | 9136019 | 9136549 | 9137079 | 9137609 |
| 820 | 9140786 | 9141315 | 9141844 | 9142373 | 9142903 |
| 821 | 9146076 | 9146604 | 9147133 | 9147661 | 9148190 |
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| 823 | 9156636 | 9157163 | 9157691 | 9158218 | 9158745 |
| 824 | 9161907 | 9162433 | 9162960 | 9163487 | 9164013 |
| 825 | 9167171 | 9167697 | 9168223 | 9168749 | 9169275 |
| 826 | 9172429 | 9172954 | 9173479 | 9174005 | 9174530 |
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| 828 | 9182925 | 9183449 | 9183973 | 9184497 | 9185021 |
| 829 | 9188164 | 9188687 | 9189211 | 9189734 | 9190258 |
| 830 | 9193396 | 9193919 | 9194442 | 9194965 | 9195488 |
| 831 | 9198623 | 9199145 | 9199667 | 9200189 | 9200711 |
| 832 | 9203842 | 9204364 | 9204886 | 9205407 | 9205929 |
| 833 | 9209056 | 9209577 | 9210098 | 9210619 | 9211140 |
| 834 | 9214263 | 9214784 | 9215304 | 9215824 | 9216345 |
| 835 | 9219465 | 9219984 | 9220504 | 9221024 | 9221543 |
| 836 | 9224659 | 9225179 | 9225698 | 9226217 | 9226736 |
| 837 | 9229848 | 9230367 | 9230885 | 9231404 | 9231922 |
| 838 | 9235031 | 9235549 | 9236066 | 9236584 | 9237102 |

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| 839 | 9237620 | 9238137 | 9238655 | 9239172 | 9239690 |
| 840 | 9242793 | 9243310 | 9243827 | 9244344 | 9244860 |
| 841 | 9247960 | 9248476 | 9248993 | 9249509 | 9250025 |
| 842 | 9253121 | 9253637 | 9254152 | 9254668 | 9255184 |
| 843 | 9258276 | 9258791 | 9259306 | 9259821 | 9260336 |
| 844 | 9263424 | 9263939 | 9264453 | 9264968 | 9265482 |
| 845 | 9268567 | 9269081 | 9269595 | 9270109 | 9270622 |
| 846 | 9273704 | 9274217 | 9274730 | 9275243 | 9275757 |
| 847 | 9278834 | 9279347 | 9279859 | 9280372 | 9280885 |
| 848 | 9283959 | 9284471 | 9284983 | 9285495 | 9286007 |
| 849 | 9289077 | 9289588 | 9290100 | 9290611 | 9291123 |
| 850 | 9294189 | 9294700 | 9295211 | 9295722 | 9296233 |
| 851 | 9299296 | 9299806 | 9300316 | 9300826 | 9301336 |
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| 853 | 9309490 | 9309999 | 9310508 | 9311017 | 9311526 |
| 854 | 9314579 | 9315087 | 9315596 | 9316104 | 9316612 |
| 855 | 9319661 | 9320169 | 9320677 | 9321185 | 9321692 |
| 856 | 9324738 | 9325245 | 9325752 | 9326259 | 9326767 |
| 857 | 9329808 | 9330315 | 9330822 | 9331328 | 9331835 |
| 858 | 9334873 | 9335379 | 9335885 | 9336391 | 9336897 |
| 859 | 9339932 | 9340437 | 9340943 | 9341448 | 9341953 |
| 860 | 9344984 | 9345489 | 9345994 | 9346499 | 9347004 |
| 861 | 9350032 | 9350536 | 9351040 | 9351544 | 9352049 |
| 862 | 9355073 | 9355576 | 9356080 | 9356584 | 9357087 |
| 863 | 9360108 | 9360611 | 9361114 | 9361617 | 9362120 |
| 864 | 9365137 | 9365640 | 9366143 | 9366645 | 9367148 |
| 865 | 9370161 | 9370663 | 9371165 | 9371667 | 9372169 |
| 866 | 9375179 | 9375680 | 9376182 | 9376683 | 9377184 |
| 867 | 9380191 | 9380692 | 9381193 | 9381693 | 9382194 |
| 868 | 9385197 | 9385697 | 9386198 | 9386698 | 9387198 |
| 869 | 9390198 | 9390697 | 9391197 | 9391697 | 9392196 |
| 870 | 9395193 | 9395692 | 9396191 | 9396690 | 9397189 |
| 871 | 9400182 | 9400680 | 9401179 | 9401677 | 9402176 |
| 872 | 9405165 | 9405663 | 9406161 | 9406659 | 9407157 |
| 873 | 9410142 | 9410640 | 9411137 | 9411635 | 9412132 |
| 874 | 9415114 | 9415611 | 9416108 | 9416605 | 9417101 |
| 875 | 9420081 | 9420577 | 9421073 | 9421569 | 9422065 |
| 876 | 9425041 | 9425537 | 9426032 | 9426528 | 9427024 |
| 877 | 9429996 | 9430491 | 9430986 | 9431481 | 9431976 |
| 878 | 9434945 | 9435440 | 9435934 | 9436429 | 9436923 |
| 879 | 9439889 | 9440383 | 9440877 | 9441371 | 9441865 |
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| 882 | 9454686 | 9455178 | 9455671 | 9456163 | 9456655 |

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| 840 | 9245377 | 9245894 | 9246410 | 9246927 | 9247444 |
| 841 | 9250541 | 9251057 | 9251573 | 9252689 | 9252605 |
| 842 | 9255699 | 9256215 | 9256730 | 9257245 | 9257761 |
| 843 | 9260851 | 9261366 | 9261880 | 9262395 | 9262910 |
| 844 | 9265995 | 9266511 | 9267025 | 9267539 | 9268053 |
| 845 | 9271136 | 9271650 | 9272163 | 9272677 | 9273190 |
| 846 | 9276270 | 9276783 | 9277296 | 9277808 | 9278321 |
| 847 | 9281397 | 9281909 | 9282422 | 9282934 | 9283446 |
| 848 | 9286518 | 9287030 | 9287542 | 9288054 | 9288565 |
| 849 | 9291634 | 9292145 | 9292656 | 9293167 | 9293678 |
| 850 | 9296743 | 9297254 | 9297764 | 9298275 | 9298785 |
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| 852 | 9306944 | 9307453 | 9307963 | 9308472 | 9308981 |
| 853 | 9312035 | 9312544 | 9313053 | 9313561 | 9314070 |
| 854 | 9317121 | 9317629 | 9318137 | 9318645 | 9319153 |
| 855 | 9322200 | 9322708 | 9323215 | 9323723 | 9324230 |
| 856 | 9327274 | 9327781 | 9328288 | 9328795 | 9329301 |
| 857 | 9332341 | 9332848 | 9333354 | 9333860 | 9334367 |
| 858 | 9337403 | 9337909 | 9338415 | 9338920 | 9339426 |
| 859 | 9342459 | 9342964 | 9343469 | 9343974 | 9344479 |
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| 862 | 9357591 | 9358095 | 9358598 | 9359101 | 9359605 |
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| 864 | 9367650 | 9368152 | 9368655 | 9369157 | 9369659 |
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| 867 | 9382695 | 9383195 | 9383696 | 9384196 | 9384697 |
| 868 | 9387698 | 9388198 | 9388698 | 9389198 | 9389698 |
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| 872 | 9407654 | 9408152 | 9408650 | 9409147 | 9409645 |
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| 877 | 9432471 | 9432966 | 9433461 | 9433956 | 9434450 |
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| 882 | 9457147 | 9457639 | 9458131 | 9458623 | 9459115 |

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| 885 | 9469433 | 9469923 | 9470414 | 9470905 | 9471395 |
| 886 | 9474337 | 9474827 | 9475317 | 9475807 | 9476297 |
| 887 | 9479236 | 9479726 | 9480215 | 9480705 | 9481194 |
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| 893 | 9508515 | 9509001 | 9509487 | 9509973 | 9510459 |
| 894 | 9513375 | 9513861 | 9514347 | 9514832 | 9515318 |
| 895 | 9518230 | 9518716 | 9519201 | 9519686 | 9520171 |
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| 898 | 9532763 | 9533247 | 9533730 | 9534214 | 9534697 |
| 899 | 9537597 | 9538080 | 9538563 | 9539046 | 9539529 |
| 900 | 9542425 | 9542908 | 9543390 | 9543872 | 9544355 |
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| 903 | 9556877 | 9557358 | 9557839 | 9558320 | 9558801 |
| 904 | 9561684 | 9562165 | 9562645 | 9563125 | 9563605 |
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| 908 | 9580858 | 9581337 | 9581815 | 9582293 | 9582771 |
| 909 | 9585639 | 9586117 | 9586594 | 9587072 | 9587549 |
| 910 | 9590414 | 9590891 | 9591368 | 9591845 | 9592322 |
| 911 | 9595184 | 9595660 | 9596137 | 9596614 | 9597090 |
| 912 | 9599948 | 9600425 | 9600901 | 9601377 | 9601853 |
| 913 | 9604708 | 9605183 | 9605659 | 9606135 | 9606610 |
| 914 | 9609462 | 9609937 | 9610412 | 9610887 | 9611362 |
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| 920 | 9637878 | 9638350 | 9638822 | 9639294 | 9639766 |
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| 922 | 9647309 | 9647780 | 9648251 | 9648722 | 9649193 |
| 923 | 9652017 | 9652488 | 9652958 | 9653428 | 9653899 |
| 924 | 9656720 | 9657190 | 9657660 | 9658130 | 9658599 |
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| 926 | 9666110 | 9667579 | 9667048 | 9667517 | 9667985 |

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| 885 | 9471886 | 9472376 | 9472866 | 9473357 | 9473847 |
| 886 | 9476787 | 9477277 | 9477767 | 9478257 | 9478747 |
| 887 | 9481684 | 9482173 | 9482662 | 9483151 | 9483641 |
| 888 | 9486574 | 9487063 | 9487552 | 9488040 | 9488529 |
| 889 | 9491460 | 9491948 | 9492436 | 9492924 | 9493412 |
| 890 | 9496330 | 9496827 | 9497314 | 9497802 | 9498290 |
| 891 | 9501213 | 9501701 | 9502188 | 9502675 | 9503162 |
| 892 | 9506082 | 9506569 | 9507055 | 9507542 | 9508028 |
| 893 | 9510946 | 9511432 | 9511918 | 9512404 | 9512889 |
| 894 | 9515803 | 9516289 | 9516774 | 9517260 | 9517745 |
| 895 | 9520656 | 9521141 | 9521626 | 9522111 | 9522595 |
| 896 | 9525503 | 9525987 | 9526472 | 9526956 | 9527440 |
| 897 | 9530345 | 9530828 | 9531312 | 9531796 | 9532280 |
| 898 | 9535181 | 9535664 | 9536147 | 9536631 | 9537114 |
| 899 | 9540012 | 9540494 | 9540977 | 9541460 | 9541943 |
| 900 | 9544837 | 9545319 | 9545802 | 9546284 | 9546766 |
| 901 | 9549657 | 9550139 | 9550621 | 9551102 | 9551584 |
| 902 | 9554472 | 9554953 | 9555434 | 9555915 | 9556397 |
| 903 | 9559282 | 9559762 | 9560243 | 9560723 | 9561204 |
| 904 | 9564086 | 9564566 | 9565046 | 9565526 | 9566006 |
| 905 | 9568885 | 9569364 | 9569844 | 9570323 | 9570803 |
| 906 | 9573678 | 9574157 | 9574636 | 9575115 | 9575594 |
| 907 | 9578466 | 9578945 | 9579423 | 9579902 | 9580380 |
| 908 | 9583249 | 9583727 | 9584205 | 9584683 | 9585161 |
| 909 | 9588027 | 9588505 | 9588982 | 9589459 | 9589937 |
| 910 | 9592799 | 9593276 | 9593754 | 9594230 | 9594707 |
| 911 | 9597567 | 9598043 | 9598520 | 9598996 | 9599472 |
| 912 | 9602329 | 9602805 | 9603280 | 9603756 | 9604232 |
| 913 | 9607086 | 9607561 | 9608036 | 9608511 | 9608987 |
| 914 | 9611837 | 9612312 | 9612787 | 9613261 | 9613736 |
| 915 | 9616583 | 9617058 | 9617532 | 9618006 | 9618481 |
| 916 | 9621325 | 9621799 | 9622272 | 9622746 | 9623220 |
| 917 | 9626061 | 9626534 | 9627007 | 9627481 | 9627954 |
| 918 | 9630792 | 9631264 | 9631737 | 9632210 | 9632683 |
| 919 | 9635517 | 9635990 | 9636462 | 9636934 | 9637406 |
| 920 | 9640238 | 9640710 | 9641181 | 9641653 | 9642125 |
| 921 | 9644953 | 9645425 | 9645896 | 9646367 | 9646838 |
| 922 | 9649664 | 9650134 | 9650605 | 9651076 | 9651546 |
| 923 | 9654369 | 9654839 | 9655309 | 9655780 | 9656250 |
| 924 | 9659069 | 9659539 | 9660009 | 9660478 | 9660948 |
| 925 | 9663764 | 9664233 | 9664703 | 9665172 | 9665641 |
| 926 | 9668454 | 9668923 | 9669392 | 9669860 | 9670329 |

| Natural Numbers. | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 927 | 9670797 | 9671266 | 9671734 | 9672203 | 9672671 |
| 928 | 9675480 | 9675948 | 9676416 | 9676883 | 9677351 |
| 929 | 9680157 | 9680625 | 9681092 | 9681559 | 9682027 |
| 930 | 9684829 | 9685296 | 9685763 | 9686230 | 9686697 |
| 931 | 9689497 | 9689963 | 9690430 | 9690896 | 9691362 |
| 932 | 9694159 | 9694625 | 9695091 | 9695557 | 9696023 |
| 933 | 9698816 | 9699282 | 9699747 | 9700213 | 9700678 |
| 934 | 9703469 | 9703934 | 9704399 | 9704863 | 9705328 |
| 935 | 9708116 | 9708581 | 9709045 | 9709509 | 9709974 |
| 936 | 9712758 | 9713222 | 9713686 | 9714150 | 9714614 |
| 937 | 9717396 | 9717859 | 9718323 | 9718786 | 9719249 |
| 938 | 9722028 | 9722491 | 9722954 | 9723417 | 9723880 |
| 939 | 9726656 | 9727118 | 9727581 | 9728043 | 9728506 |
| 940 | 9731278 | 9731741 | 9732202 | 9732664 | 9733126 |
| 941 | 9735896 | 9736358 | 9736819 | 9737281 | 9737742 |
| 942 | 9740509 | 9740970 | 9741431 | 9741892 | 9742353 |
| 943 | 9745117 | 9745577 | 9746038 | 9746498 | 9746959 |
| 944 | 9749720 | 9750180 | 9750640 | 9751100 | 9751560 |
| 945 | 9754318 | 9754778 | 9755237 | 9755695 | 9756156 |
| 946 | 9758911 | 9759370 | 9759829 | 9760288 | 9760747 |
| 947 | 9763500 | 9763958 | 9764417 | 9764875 | 9765334 |
| 948 | 9768083 | 9768541 | 9768999 | 9769457 | 9769915 |
| 949 | 9772662 | 9773120 | 9773577 | 9774035 | 9774492 |
| 950 | 9777236 | 9777693 | 9778150 | 9778607 | 9779064 |
| 951 | 9781805 | 9782262 | 9782718 | 9783175 | 9783631 |
| 952 | 9786369 | 9786826 | 9787282 | 9787738 | 9788194 |
| 953 | 9790929 | 9791385 | 9791840 | 9792296 | 9792751 |
| 954 | 9795484 | 9795939 | 9796394 | 9796849 | 9797304 |
| 955 | 9800034 | 9800488 | 9800943 | 9801398 | 9801852 |
| 956 | 9804579 | 9805033 | 9805487 | 9805942 | 9806396 |
| 957 | 9809119 | 9809573 | 9810027 | 9810481 | 9810934 |
| 958 | 9813655 | 9814108 | 9814562 | 9815015 | 9815468 |
| 959 | 9818186 | 9818639 | 9819092 | 9819544 | 9819997 |
| 960 | 9822712 | 9823165 | 9823617 | 9824069 | 9824522 |
| 961 | 9827234 | 9827686 | 9828138 | 9828589 | 9829041 |
| 962 | 9831751 | 9832202 | 9832654 | 9833105 | 9833556 |
| 963 | 9836263 | 9836714 | 9837165 | 9837616 | 9838066 |
| 964 | 9840770 | 9841221 | 9841671 | 9842122 | 9842572 |
| 965 | 9845273 | 9845723 | 9846173 | 9846623 | 9847073 |
| 966 | 9849771 | 9850221 | 9850670 | 9851120 | 9851569 |
| 967 | 9854265 | 9854714 | 9855163 | 9855612 | 9856061 |
| 968 | 9858754 | 9859202 | 9859651 | 9860099 | 9860548 |
| 969 | 9863238 | 9863686 | 9864134 | 9864582 | 9865030 |
| 970 | 9867717 | 9868165 | 9868613 | 9869060 | 9869508 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 927 | 9673139 | 9673607 | 9674076 | 9674544 | 9675012 |
| 928 | 9677819 | 9678287 | 9678754 | 9679222 | 9679690 |
| 929 | 9682494 | 9682961 | 9683428 | 9683895 | 9684362 |
| 930 | 9687164 | 9687630 | 9688097 | 9688564 | 9689030 |
| 931 | 9691829 | 9692295 | 9692761 | 9693227 | 9693693 |
| 932 | 9696488 | 9696954 | 9697420 | 9697885 | 9698351 |
| 933 | 9701143 | 9701608 | 9702074 | 9702539 | 9703004 |
| 934 | 9705793 | 9706258 | 9706722 | 9707187 | 9707652 |
| 935 | 9710438 | 9710902 | 9711366 | 9711830 | 9712294 |
| 936 | 9715078 | 9715542 | 9716005 | 9716469 | 9716932 |
| 937 | 9719713 | 9720176 | 9720639 | 9721102 | 9721565 |
| 938 | 9724343 | 9724805 | 9725268 | 9725731 | 9726193 |
| 939 | 9728968 | 9729430 | 9729892 | 9730354 | 9730816 |
| 940 | 9733588 | 9734050 | 9734511 | 9734973 | 9735435 |
| 941 | 9738203 | 9738664 | 9739126 | 9739587 | 9740048 |
| 942 | 9742814 | 9743274 | 9743735 | 9744196 | 9744656 |
| 943 | 9747419 | 9747879 | 9748340 | 9748800 | 9749260 |
| 944 | 9752020 | 9752479 | 9752939 | 9753399 | 9753858 |
| 945 | 9756615 | 9757075 | 9757534 | 9757993 | 9758452 |
| 946 | 9761206 | 9761665 | 9762124 | 9762582 | 9763041 |
| 947 | 9765792 | 9766251 | 9766709 | 9767167 | 9767625 |
| 948 | 9770373 | 9770831 | 9771289 | 9771747 | 9772204 |
| 949 | 9774950 | 9775407 | 9775864 | 9776322 | 9776779 |
| 950 | 9779521 | 9779978 | 9780435 | 9780892 | 9781348 |
| 951 | 9784088 | 9784544 | 9785001 | 9785457 | 9785913 |
| 952 | 9788650 | 9789106 | 9789562 | 9790017 | 9790473 |
| 953 | 9793207 | 9793662 | 9794118 | 9794573 | 9795028 |
| 954 | 9797759 | 9798214 | 9798669 | 9799124 | 9799579 |
| 955 | 9802307 | 9802761 | 9803216 | 9803670 | 9804125 |
| 956 | 9806850 | 9807304 | 9807758 | 9808212 | 9808666 |
| 957 | 9811388 | 9811841 | 9812295 | 9812748 | 9813202 |
| 958 | 9815921 | 9816374 | 9816827 | 9817280 | 9817733 |
| 959 | 9820450 | 9820902 | 9821355 | 9821807 | 9822260 |
| 960 | 9824974 | 9825426 | 9825878 | 9826330 | 9826782 |
| 961 | 9829493 | 9829945 | 9830396 | 9830848 | 9831299 |
| 962 | 9834007 | 9834459 | 9834910 | 9835361 | 9835812 |
| 963 | 9838517 | 9838968 | 9839419 | 9839869 | 9840320 |
| 964 | 9843022 | 9843473 | 9843923 | 9844373 | 9844823 |
| 965 | 9847523 | 9847973 | 9848422 | 9848872 | 9849322 |
| 966 | 9852019 | 9852468 | 9852917 | 9853366 | 9853816 |
| 967 | 9856510 | 9856959 | 9857407 | 9857856 | 9858305 |
| 968 | 9860996 | 9861445 | 9861893 | 9862341 | 9862790 |
| 969 | 9865478 | 9865926 | 9866374 | 9866822 | 9867270 |
| 970 | 9869955 | 9870403 | 9870850 | 9871298 | 9871745 |

| Natural Numbers | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 971 | 9872192 | 9872640 | 9873087 | 9873534 | 9873981 |
| 972 | 9876663 | 9877109 | 9877556 | 9878003 | 9878449 |
| 973 | 9881128 | 9881575 | 9882021 | 9882467 | 9882913 |
| 974 | 9885590 | 9886035 | 9886481 | 9886927 | 9887373 |
| 975 | 9890046 | 9890492 | 9890937 | 9891382 | 9891828 |
| 976 | 9894498 | 9894943 | 9895388 | 9895833 | 9896278 |
| 977 | 9898946 | 9899390 | 9899835 | 9900279 | 9900723 |
| 978 | 9903389 | 9903833 | 9904277 | 9904721 | 9905164 |
| 979 | 9907827 | 9908270 | 9908714 | 9909158 | 9909601 |
| 980 | 9912261 | 9912704 | 9913147 | 9913590 | 9914033 |
| 981 | 9916690 | 9917133 | 9917575 | 9918018 | 9918461 |
| 982 | 9921115 | 9921557 | 9921999 | 9922441 | 9922884 |
| 983 | 9925535 | 9925977 | 9926419 | 9926860 | 9927302 |
| 984 | 9929951 | 9930392 | 9930834 | 9931275 | 9931716 |
| 985 | 9934362 | 9934803 | 9935244 | 9935685 | 9936126 |
| 986 | 9938769 | 9939210 | 9939650 | 9940090 | 9940531 |
| 987 | 9943172 | 9943612 | 9944051 | 9944491 | 9944931 |
| 988 | 9947569 | 9948009 | 9948448 | 9948888 | 9949327 |
| 989 | 9951963 | 9952402 | 9952841 | 9953280 | 9953719 |
| 990 | 9956352 | 9956791 | 9957229 | 9957668 | 9958106 |
| 991 | 9960737 | 9961175 | 9961613 | 9962051 | 9962489 |
| 992 | 9965117 | 9965554 | 9965992 | 9966430 | 9966868 |
| 993 | 9969492 | 9969930 | 9970367 | 9970804 | 9971242 |
| 994 | 9973864 | 9974301 | 9974738 | 9975174 | 9975611 |
| 995 | 9978231 | 9978667 | 9979104 | 9979540 | 9979976 |
| 996 | 9982593 | 9983029 | 9983465 | 9983901 | 9984337 |
| 997 | 9986952 | 9987387 | 9987823 | 9988258 | 9988694 |
| 998 | 9991305 | 9991740 | 9992176 | 9992611 | 9993046 |
| 999 | 9995655 | 9996090 | 9996524 | 9996959 | 9997393 |

| Natural Numbers. | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|
| 971 | 9874428 | 9874875 | 9875322 | 9875769 | 9876216 |
| 972 | 9878896 | 9879343 | 9879789 | 9880236 | 9880682 |
| 973 | 9883360 | 9883806 | 9884252 | 9884698 | 9885144 |
| 974 | 9887818 | 9888264 | 9888710 | 9889155 | 9889601 |
| 975 | 9892273 | 9892718 | 9893163 | 9893608 | 9894050 |
| 976 | 9896722 | 9897167 | 9897612 | 9898056 | 9898501 |
| 977 | 9901168 | 9901612 | 9902056 | 9902500 | 9902944 |
| 978 | 9905608 | 9906052 | 9906496 | 9906940 | 9907383 |
| 979 | 9910044 | 9910488 | 9910931 | 9911374 | 9911818 |
| 980 | 9914476 | 9914919 | 9915362 | 9915805 | 9916247 |
| 981 | 9918903 | 9919345 | 9919788 | 9920230 | 9920673 |
| 982 | 9923326 | 9923768 | 9924210 | 9924651 | 9925093 |
| 983 | 9927744 | 9928185 | 9928627 | 9929068 | 9929510 |
| 984 | 9932157 | 9932598 | 9933039 | 9933480 | 9933921 |
| 985 | 9936566 | 9937007 | 9937448 | 9937888 | 9938329 |
| 986 | 9940971 | 9941411 | 9941851 | 9942291 | 9942731 |
| 987 | 9945371 | 9945811 | 9946251 | 9946690 | 9947130 |
| 988 | 9949767 | 9950206 | 9950645 | 9951085 | 9951524 |
| 989 | 9954158 | 9954597 | 9955036 | 9955474 | 9955913 |
| 990 | 9958545 | 9958983 | 9959422 | 9959860 | 9960298 |
| 991 | 9962927 | 9963365 | 9963803 | 9964241 | 9964679 |
| 992 | 9967305 | 9967743 | 9968180 | 9968618 | 9969055 |
| 993 | 9971679 | 9972116 | 9972553 | 9972990 | 9973427 |
| 994 | 9976048 | 9976485 | 9976921 | 9977358 | 9977794 |
| 995 | 9980413 | 9980849 | 9981285 | 9981721 | 9982157 |
| 996 | 9984773 | 9985209 | 9985645 | 9986080 | 9986516 |
| 997 | 9989129 | 9989564 | 9990000 | 9990435 | 9990870 |
| 998 | 9993481 | 9993916 | 9994350 | 9994785 | 9995220 |
| 999 | 9997828 | 9998262 | 9998697 | 9999131 | 9999566 |

10000 it's Log.=4.0000000

*The End of the Table of Logarithms.*

# A TABLE OF THE Proportional Parts OF THE DIFFERENCES OF LOGARITHMS.

By which the foregoing TABLE is made to extend to the Logarithm of 100000, &c.

| Differences of Logarithms. | Tenth Parts of those Differences. (VIZ.) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 435 | 44 | 87 | 130 | 174 | 218 | 261 | 305 | 348 | 392 |
| 436 | 44 | 87 | 131 | 174 | 218 | 262 | 305 | 349 | 392 |
| 438 | 44 | 88 | 131 | 175 | 219 | 263 | 307 | 350 | 394 |
| 440 | 44 | 88 | 132 | 176 | 220 | 264 | 308 | 352 | 396 |
| 442 | 44 | 88 | 133 | 177 | 221 | 265 | 309 | 354 | 398 |
| 444 | 44 | 89 | 133 | 178 | 222 | 266 | 311 | 355 | 400 |
| 446 | 45 | 89 | 134 | 178 | 223 | 268 | 312 | 357 | 401 |
| 448 | 45 | 90 | 134 | 179 | 224 | 269 | 314 | 358 | 403 |
| 450 | 45 | 90 | 135 | 180 | 225 | 270 | 315 | 360 | 405 |
| 452 | 45 | 90 | 136 | 181 | 226 | 271 | 316 | 362 | 407 |
| 454 | 45 | 91 | 136 | 182 | 227 | 272 | 318 | 363 | 409 |
| 456 | 46 | 91 | 137 | 182 | 228 | 274 | 319 | 365 | 410 |
| 458 | 46 | 92 | 137 | 183 | 229 | 275 | 321 | 366 | 412 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 460 | 46 | 92 | 138 | 184 | 230 | 276 | 322 | 368 | 414 |
| 462 | 46 | 92 | 139 | 185 | 231 | 277 | 323 | 370 | 416 |
| 464 | 46 | 93 | 139 | 186 | 232 | 278 | 325 | 371 | 418 |
| 466 | 47 | 93 | 140 | 186 | 233 | 280 | 326 | 373 | 419 |
| 468 | 47 | 94 | 140 | 187 | 234 | 281 | 328 | 374 | 421 |
| 470 | 47 | 94 | 141 | 188 | 235 | 282 | 329 | 376 | 423 |
| 472 | 47 | 94 | 142 | 189 | 236 | 283 | 330 | 378 | 425 |
| 474 | 47 | 95 | 142 | 190 | 237 | 284 | 332 | 379 | 427 |
| 476 | 48 | 95 | 143 | 190 | 238 | 286 | 333 | 381 | 428 |
| 478 | 48 | 96 | 143 | 191 | 239 | 287 | 335 | 382 | 430 |
| 480 | 48 | 96 | 144 | 192 | 240 | 288 | 336 | 384 | 432 |
| 482 | 48 | 96 | 145 | 193 | 241 | 289 | 337 | 386 | 434 |
| 484 | 48 | 97 | 145 | 194 | 242 | 290 | 339 | 387 | 436 |
| 486 | 49 | 97 | 146 | 194 | 243 | 292 | 340 | 389 | 437 |
| 488 | 49 | 98 | 146 | 195 | 244 | 293 | 342 | 390 | 439 |
| 490 | 49 | 98 | 147 | 196 | 245 | 294 | 343 | 392 | 441 |
| 492 | 49 | 98 | 148 | 197 | 246 | 295 | 344 | 394 | 443 |
| 494 | 49 | 99 | 148 | 198 | 247 | 296 | 346 | 395 | 445 |
| 496 | 50 | 99 | 149 | 198 | 248 | 298 | 347 | 397 | 446 |
| 498 | 50 | 100 | 149 | 199 | 249 | 299 | 348 | 398 | 448 |
| 500 | 50 | 100 | 150 | 200 | 250 | 300 | 350 | 400 | 450 |
| 502 | 50 | 100 | 151 | 201 | 251 | 301 | 351 | 402 | 452 |
| 504 | 50 | 101 | 151 | 202 | 252 | 302 | 353 | 403 | 454 |
| 506 | 51 | 101 | 152 | 202 | 253 | 304 | 354 | 405 | 455 |
| 508 | 51 | 102 | 152 | 203 | 254 | 305 | 356 | 406 | 457 |
| 510 | 51 | 102 | 153 | 204 | 255 | 306 | 357 | 408 | 459 |
| 512 | 51 | 102 | 154 | 205 | 256 | 307 | 358 | 410 | 461 |
| 514 | 51 | 103 | 154 | 206 | 257 | 308 | 360 | 411 | 463 |
| 516 | 52 | 103 | 155 | 206 | 258 | 310 | 361 | 413 | 464 |
| 518 | 52 | 104 | 155 | 207 | 259 | 311 | 363 | 414 | 466 |
| 520 | 52 | 104 | 156 | 208 | 260 | 312 | 364 | 416 | 468 |
| 522 | 52 | 104 | 157 | 209 | 261 | 313 | 365 | 418 | 470 |
| 524 | 52 | 105 | 157 | 210 | 262 | 314 | 367 | 419 | 472 |
| 526 | 53 | 105 | 158 | 210 | 263 | 316 | 368 | 421 | 473 |
| 528 | 53 | 106 | 158 | 211 | 264 | 317 | 370 | 422 | 475 |
| 530 | 53 | 106 | 159 | 212 | 265 | 318 | 371 | 424 | 477 |
| 532 | 53 | 106 | 160 | 213 | 266 | 319 | 372 | 426 | 479 |
| 534 | 53 | 107 | 160 | 214 | 267 | 320 | 374 | 427 | 481 |
| 536 | 54 | 107 | 161 | 214 | 268 | 322 | 375 | 429 | 482 |
| 538 | 54 | 108 | 161 | 215 | 269 | 323 | 377 | 430 | 484 |

# A TABLE of Proportional Parts.

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 540 | 54 | 108 | 162 | 216 | 270 | 324 | 378 | 432 | 486 |
| 542 | 54 | 108 | 163 | 217 | 271 | 325 | 379 | 434 | 488 |
| 544 | 54 | 109 | 163 | 218 | 272 | 326 | 381 | 435 | 490 |
| 546 | 55 | 109 | 164 | 218 | 273 | 328 | 382 | 437 | 491 |
| 548 | 55 | 110 | 164 | 219 | 274 | 329 | 384 | 438 | 493 |
| 550 | 55 | 110 | 165 | 220 | 275 | 330 | 385 | 440 | 495 |
| 552 | 55 | 110 | 166 | 221 | 276 | 331 | 386 | 442 | 497 |
| 554 | 55 | 111 | 166 | 222 | 277 | 332 | 388 | 443 | 499 |
| 556 | 56 | 111 | 167 | 222 | 278 | 334 | 389 | 445 | 500 |
| 558 | 56 | 112 | 167 | 223 | 279 | 335 | 391 | 446 | 502 |
| 560 | 56 | 112 | 168 | 224 | 280 | 336 | 392 | 448 | 504 |
| 562 | 56 | 112 | 169 | 225 | 281 | 337 | 393 | 450 | 506 |
| 564 | 56 | 113 | 169 | 226 | 282 | 338 | 395 | 451 | 508 |
| 566 | 57 | 113 | 170 | 226 | 283 | 340 | 396 | 453 | 509 |
| 568 | 57 | 114 | 170 | 227 | 284 | 341 | 398 | 454 | 511 |
| 570 | 57 | 114 | 171 | 228 | 285 | 342 | 399 | 456 | 513 |
| 572 | 57 | 114 | 172 | 229 | 286 | 343 | 400 | 458 | 515 |
| 574 | 57 | 115 | 172 | 230 | 287 | 344 | 401 | 459 | 516 |
| 576 | 58 | 115 | 173 | 230 | 288 | 346 | 403 | 461 | 518 |
| 578 | 58 | 116 | 173 | 231 | 289 | 347 | 405 | 462 | 520 |
| 580 | 58 | 116 | 174 | 232 | 290 | 348 | 406 | 464 | 522 |
| 582 | 58 | 116 | 175 | 233 | 291 | 349 | 407 | 466 | 524 |
| 584 | 58 | 117 | 175 | 234 | 292 | 350 | 409 | 467 | 526 |
| 586 | 59 | 117 | 176 | 234 | 293 | 352 | 410 | 469 | 527 |
| 588 | 59 | 118 | 176 | 235 | 294 | 353 | 412 | 470 | 529 |
| 590 | 59 | 118 | 177 | 236 | 295 | 354 | 413 | 472 | 531 |
| 592 | 59 | 118 | 178 | 237 | 296 | 355 | 414 | 474 | 533 |
| 594 | 59 | 119 | 178 | 238 | 297 | 356 | 416 | 475 | 535 |
| 596 | 60 | 119 | 179 | 238 | 298 | 358 | 417 | 477 | 536 |
| 598 | 60 | 120 | 179 | 239 | 299 | 359 | 419 | 478 | 538 |
| 600 | 60 | 120 | 180 | 240 | 300 | 360 | 420 | 480 | 540 |
| 602 | 60 | 120 | 181 | 241 | 301 | 361 | 421 | 482 | 542 |
| 604 | 60 | 121 | 181 | 242 | 302 | 362 | 423 | 483 | 544 |
| 606 | 61 | 121 | 182 | 242 | 303 | 364 | 424 | 485 | 545 |
| 608 | 61 | 122 | 182 | 243 | 304 | 365 | 426 | 486 | 547 |
| 610 | 61 | 122 | 183 | 244 | 305 | 366 | 427 | 488 | 549 |
| 612 | 61 | 122 | 184 | 245 | 306 | 367 | 428 | 490 | 551 |
| 614 | 61 | 123 | 184 | 246 | 307 | 368 | 430 | 492 | 553 |
| 616 | 62 | 123 | 185 | 246 | 308 | 370 | 431 | 493 | 554 |
| 618 | 62 | 124 | 185 | 247 | 309 | 371 | 433 | 494 | 556 |

# A TABLE of Proportional Parts.

| Diff of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 620 | 62 | 124 | 186 | 248 | 310 | 372 | 434 | 496 | 558 |
| 622 | 62 | 124 | 187 | 249 | 311 | 373 | 435 | 498 | 560 |
| 624 | 62 | 125 | 187 | 250 | 312 | 374 | 437 | 499 | 562 |
| 626 | 63 | 125 | 188 | 250 | 313 | 376 | 438 | 501 | 563 |
| 628 | 63 | 126 | 188 | 251 | 314 | 377 | 440 | 502 | 565 |
| 630 | 63 | 126 | 189 | 252 | 315 | 378 | 441 | 504 | 567 |
| 632 | 63 | 126 | 190 | 253 | 316 | 379 | 442 | 506 | 569 |
| 634 | 63 | 127 | 190 | 254 | 317 | 380 | 444 | 507 | 571 |
| 636 | 64 | 127 | 191 | 254 | 318 | 382 | 445 | 509 | 572 |
| 638 | 64 | 128 | 191 | 255 | 319 | 383 | 447 | 510 | 574 |
| 640 | 64 | 128 | 192 | 256 | 320 | 384 | 448 | 512 | 576 |
| 642 | 64 | 128 | 193 | 257 | 321 | 385 | 449 | 514 | 578 |
| 644 | 64 | 129 | 193 | 258 | 322 | 386 | 451 | 515 | 580 |
| 646 | 65 | 129 | 194 | 258 | 323 | 388 | 452 | 517 | 581 |
| 648 | 65 | 130 | 194 | 259 | 324 | 389 | 454 | 518 | 583 |
| 650 | 65 | 130 | 195 | 260 | 325 | 390 | 455 | 520 | 585 |
| 652 | 65 | 130 | 196 | 261 | 326 | 391 | 456 | 522 | 587 |
| 654 | 65 | 131 | 196 | 262 | 327 | 392 | 458 | 523 | 589 |
| 656 | 66 | 131 | 197 | 262 | 328 | 394 | 459 | 525 | 590 |
| 658 | 66 | 132 | 197 | 263 | 329 | 395 | 461 | 526 | 592 |
| 660 | 66 | 132 | 198 | 264 | 330 | 396 | 462 | 528 | 594 |
| 662 | 66 | 132 | 199 | 265 | 331 | 397 | 463 | 530 | 596 |
| 664 | 66 | 133 | 199 | 266 | 332 | 398 | 465 | 531 | 598 |
| 666 | 67 | 133 | 200 | 266 | 333 | 400 | 466 | 533 | 599 |
| 668 | 67 | 134 | 200 | 267 | 334 | 401 | 468 | 534 | 601 |
| 670 | 67 | 134 | 201 | 268 | 335 | 402 | 469 | 536 | 603 |
| 672 | 67 | 134 | 202 | 269 | 336 | 403 | 470 | 538 | 605 |
| 674 | 67 | 135 | 202 | 270 | 337 | 404 | 472 | 539 | 607 |
| 676 | 68 | 135 | 203 | 270 | 338 | 406 | 473 | 541 | 608 |
| 678 | 68 | 136 | 203 | 271 | 339 | 407 | 475 | 542 | 610 |
| 680 | 68 | 136 | 204 | 272 | 340 | 408 | 476 | 544 | 612 |
| 982 | 68 | 136 | 205 | 273 | 341 | 409 | 477 | 546 | 614 |
| 684 | 68 | 137 | 205 | 274 | 342 | 410 | 479 | 547 | 616 |
| 686 | 69 | 137 | 206 | 274 | 343 | 412 | 480 | 549 | 617 |
| 688 | 69 | 138 | 206 | 275 | 344 | 413 | 482 | 550 | 619 |
| 690 | 69 | 138 | 207 | 276 | 345 | 414 | 483 | 552 | 621 |
| 692 | 69 | 138 | 208 | 277 | 346 | 415 | 484 | 554 | 623 |
| 694 | 69 | 139 | 208 | 278 | 347 | 416 | 486 | 555 | 625 |
| 696 | 70 | 139 | 209 | 278 | 348 | 417 | 487 | 557 | 626 |
| 698 | 70 | 140 | 209 | 279 | 349 | 419 | 489 | 558 | 628 |

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 700 | 70 | 140 | 210 | 280 | 350 | 420 | 490 | 560 | 630 |
| 702 | 70 | 140 | 211 | 281 | 351 | 421 | 491 | 562 | 632 |
| 704 | 70 | 141 | 211 | 282 | 352 | 422 | 492 | 563 | 634 |
| 706 | 71 | 141 | 212 | 282 | 353 | 424 | 494 | 565 | 635 |
| 708 | 71 | 142 | 212 | 283 | 354 | 425 | 496 | 566 | 637 |
| 710 | 71 | 142 | 213 | 284 | 355 | 426 | 497 | 568 | 639 |
| 712 | 71 | 142 | 214 | 285 | 356 | 427 | 498 | 570 | 641 |
| 714 | 71 | 143 | 214 | 286 | 357 | 428 | 500 | 571 | 643 |
| 716 | 72 | 143 | 215 | 286 | 358 | 430 | 501 | 573 | 644 |
| 718 | 72 | 144 | 215 | 287 | 359 | 431 | 503 | 574 | 646 |
| 720 | 72 | 144 | 216 | 288 | 360 | 432 | 504 | 576 | 648 |
| 722 | 72 | 144 | 217 | 289 | 361 | 433 | 505 | 578 | 650 |
| 724 | 72 | 145 | 217 | 290 | 362 | 434 | 507 | 579 | 652 |
| 726 | 73 | 145 | 218 | 290 | 363 | 436 | 508 | 581 | 653 |
| 728 | 73 | 146 | 218 | 291 | 364 | 437 | 510 | 582 | 655 |
| 730 | 73 | 146 | 219 | 292 | 365 | 438 | 511 | 584 | 657 |
| 732 | 73 | 146 | 220 | 293 | 366 | 439 | 512 | 586 | 659 |
| 734 | 73 | 147 | 220 | 294 | 367 | 440 | 514 | 587 | 661 |
| 736 | 74 | 147 | 221 | 294 | 368 | 442 | 515 | 589 | 662 |
| 738 | 74 | 148 | 221 | 295 | 369 | 443 | 517 | 590 | 664 |
| 740 | 74 | 148 | 222 | 296 | 370 | 444 | 518 | 592 | 666 |
| 742 | 74 | 148 | 223 | 297 | 371 | 445 | 519 | 594 | 668 |
| 744 | 74 | 149 | 223 | 298 | 372 | 446 | 521 | 595 | 670 |
| 746 | 75 | 149 | 224 | 298 | 373 | 448 | 522 | 597 | 671 |
| 748 | 75 | 150 | 224 | 299 | 374 | 449 | 524 | 598 | 673 |
| 750 | 75 | 150 | 225 | 300 | 375 | 450 | 525 | 600 | 675 |
| 752 | 75 | 150 | 226 | 301 | 376 | 451 | 526 | 602 | 677 |
| 754 | 75 | 151 | 226 | 302 | 377 | 452 | 528 | 603 | 679 |
| 756 | 76 | 151 | 227 | 302 | 378 | 454 | 529 | 605 | 680 |
| 758 | 76 | 152 | 227 | 303 | 379 | 455 | 531 | 606 | 682 |
| 760 | 76 | 152 | 228 | 304 | 380 | 456 | 532 | 608 | 684 |
| 762 | 76 | 152 | 229 | 305 | 381 | 457 | 533 | 610 | 686 |
| 764 | 76 | 153 | 229 | 306 | 382 | 458 | 535 | 611 | 688 |
| 766 | 77 | 153 | 230 | 306 | 383 | 460 | 536 | 613 | 689 |
| 768 | 77 | 154 | 230 | 307 | 384 | 461 | 538 | 614 | 691 |
| 770 | 77 | 154 | 231 | 308 | 385 | 462 | 539 | 616 | 693 |
| 772 | 77 | 154 | 232 | 309 | 386 | 463 | 540 | 618 | 695 |
| 774 | 77 | 155 | 232 | 310 | 387 | 464 | 542 | 619 | 697 |
| 776 | 78 | 155 | 233 | 310 | 388 | 466 | 543 | 621 | 699 |
| 778 | 78 | 156 | 233 | 311 | 389 | 467 | 545 | 622 | 700 |

# A TABLE of Proportional Parts.

| Diff of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 780 | 78 | 156 | 234 | 312 | 390 | 468 | 546 | 624 | 702 |
| 782 | 78 | 156 | 235 | 313 | 391 | 469 | 547 | 626 | 704 |
| 784 | 78 | 157 | 235 | 314 | 392 | 470 | 549 | 627 | 706 |
| 786 | 79 | 157 | 236 | 314 | 393 | 472 | 550 | 629 | 707 |
| 788 | 79 | 158 | 236 | 315 | 394 | 473 | 552 | 630 | 709 |
| 790 | 79 | 158 | 237 | 316 | 395 | 474 | 553 | 632 | 711 |
| 792 | 79 | 158 | 238 | 317 | 396 | 475 | 554 | 634 | 713 |
| 794 | 79 | 159 | 238 | 318 | 397 | 477 | 556 | 635 | 715 |
| 796 | 80 | 159 | 239 | 318 | 398 | 478 | 557 | 637 | 717 |
| 798 | 80 | 160 | 239 | 319 | 399 | 479 | 559 | 638 | 718 |
| 800 | 80 | 160 | 240 | 320 | 400 | 480 | 560 | 640 | 720 |
| 802 | 80 | 160 | 241 | 321 | 401 | 481 | 561 | 642 | 722 |
| 804 | 80 | 161 | 241 | 322 | 402 | 482 | 563 | 643 | 724 |
| 806 | 81 | 161 | 242 | 322 | 403 | 484 | 564 | 645 | 725 |
| 808 | 81 | 162 | 242 | 323 | 404 | 485 | 566 | 645 | 727 |
| 810 | 81 | 162 | 243 | 324 | 405 | 486 | 567 | 648 | 729 |
| 812 | 81 | 162 | 244 | 325 | 406 | 487 | 568 | 650 | 731 |
| 814 | 81 | 163 | 244 | 326 | 407 | 488 | 570 | 651 | 733 |
| 816 | 82 | 163 | 245 | 326 | 408 | 490 | 571 | 653 | 734 |
| 818 | 82 | 164 | 245 | 327 | 409 | 491 | 572 | 654 | 736 |
| 820 | 82 | 164 | 246 | 328 | 410 | 492 | 574 | 656 | 738 |
| 822 | 82 | 164 | 247 | 329 | 411 | 493 | 575 | 658 | 740 |
| 824 | 82 | 165 | 247 | 330 | 412 | 494 | 577 | 659 | 742 |
| 826 | 83 | 165 | 248 | 330 | 413 | 496 | 578 | 661 | 743 |
| 828 | 83 | 166 | 248 | 331 | 414 | 497 | 580 | 662 | 745 |
| 830 | 83 | 166 | 249 | 332 | 415 | 498 | 581 | 664 | 747 |
| 832 | 83 | 166 | 250 | 333 | 416 | 499 | 582 | 666 | 749 |
| 834 | 83 | 167 | 250 | 334 | 417 | 500 | 584 | 667 | 751 |
| 836 | 84 | 167 | 251 | 334 | 418 | 502 | 585 | 669 | 753 |
| 838 | 84 | 168 | 251 | 335 | 419 | 503 | 587 | 670 | 754 |
| 840 | 84 | 168 | 252 | 336 | 420 | 504 | 588 | 672 | 756 |
| 842 | 84 | 168 | 253 | 337 | 421 | 505 | 589 | 674 | 758 |
| 844 | 84 | 169 | 253 | 338 | 422 | 506 | 591 | 675 | 760 |
| 846 | 85 | 169 | 254 | 338 | 423 | 508 | 592 | 677 | 761 |
| 848 | 85 | 170 | 254 | 339 | 424 | 509 | 594 | 678 | 763 |
| 850 | 85 | 170 | 255 | 340 | 425 | 510 | 595 | 680 | 765 |
| 852 | 85 | 170 | 256 | 341 | 426 | 511 | 596 | 682 | 767 |
| 854 | 85 | 171 | 256 | 342 | 427 | 512 | 598 | 683 | 769 |
| 856 | 86 | 171 | 257 | 342 | 428 | 514 | 599 | 685 | 771 |
| 858 | 86 | 172 | 257 | 343 | 429 | 515 | 601 | 686 | 773 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 860 | 86 | 172 | 258 | 344 | 430 | 516 | 602 | 688 | 774 |
| 862 | 86 | 172 | 259 | 345 | 431 | 517 | 603 | 690 | 776 |
| 864 | 86 | 173 | 259 | 346 | 432 | 518 | 605 | 691 | 778 |
| 866 | 87 | 173 | 260 | 346 | 433 | 520 | 606 | 693 | 779 |
| 868 | 87 | 174 | 260 | 347 | 434 | 521 | 608 | 694 | 781 |
| 870 | 87 | 174 | 261 | 348 | 435 | 522 | 609 | 696 | 783 |
| 872 | 87 | 174 | 262 | 349 | 436 | 523 | 611 | 698 | 785 |
| 874 | 87 | 175 | 262 | 350 | 437 | 524 | 612 | 699 | 787 |
| 876 | 88 | 175 | 263 | 350 | 438 | 526 | 613 | 701 | 789 |
| 878 | 88 | 176 | 263 | 351 | 439 | 527 | 615 | 702 | 791 |
| 880 | 88 | 176 | 264 | 352 | 440 | 528 | 616 | 704 | 793 |
| 882 | 88 | 176 | 265 | 353 | 441 | 529 | 617 | 706 | 795 |
| 884 | 88 | 177 | 265 | 354 | 442 | 530 | 619 | 707 | 796 |
| 886 | 89 | 177 | 266 | 354 | 443 | 532 | 620 | 709 | 797 |
| 888 | 89 | 178 | 266 | 355 | 444 | 533 | 622 | 710 | 799 |
| 890 | 89 | 178 | 267 | 356 | 445 | 534 | 623 | 712 | 801 |
| 892 | 89 | 178 | 268 | 357 | 446 | 535 | 624 | 714 | 803 |
| 894 | 89 | 179 | 268 | 358 | 447 | 536 | 626 | 715 | 805 |
| 896 | 90 | 179 | 269 | 358 | 448 | 538 | 627 | 717 | 806 |
| 898 | 90 | 180 | 269 | 359 | 449 | 539 | 629 | 718 | 808 |
| 900 | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 |
| 903 | 90 | 181 | 271 | 361 | 451 | 542 | 632 | 722 | 813 |
| 906 | 91 | 181 | 272 | 363 | 453 | 544 | 634 | 725 | 815 |
| 909 | 91 | 182 | 273 | 364 | 454 | 545 | 636 | 727 | 818 |
| 912 | 91 | 182 | 274 | 365 | 456 | 547 | 638 | 730 | 821 |
| 915 | 91 | 183 | 274 | 366 | 457 | 549 | 640 | 732 | 823 |
| 918 | 92 | 184 | 275 | 367 | 459 | 551 | 643 | 734 | 826 |
| 921 | 92 | 184 | 276 | 368 | 460 | 553 | 645 | 737 | 829 |
| 924 | 92 | 185 | 277 | 370 | 462 | 554 | 647 | 739 | 832 |
| 927 | 93 | 185 | 278 | 371 | 463 | 556 | 649 | 742 | 834 |
| 930 | 93 | 186 | 279 | 372 | 465 | 558 | 651 | 744 | 837 |
| 933 | 93 | 187 | 280 | 373 | 466 | 560 | 653 | 746 | 839 |
| 936 | 94 | 187 | 281 | 374 | 468 | 562 | 655 | 749 | 842 |
| 939 | 94 | 188 | 282 | 376 | 469 | 563 | 657 | 751 | 845 |
| 942 | 94 | 188 | 283 | 377 | 471 | 565 | 659 | 754 | 848 |
| 945 | 94 | 189 | 283 | 378 | 472 | 567 | 661 | 756 | 850 |
| 948 | 95 | 190 | 284 | 379 | 474 | 569 | 664 | 758 | 853 |
| 951 | 95 | 190 | 285 | 380 | 475 | 571 | 666 | 761 | 856 |
| 954 | 95 | 191 | 286 | 382 | 477 | 572 | 668 | 763 | 859 |
| 957 | 96 | 191 | 287 | 383 | 478 | 574 | 670 | 766 | 861 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of those Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 960 | 96 | 192 | 288 | 384 | 480 | 576 | 672 | 768 | 864 |
| 963 | 96 | 193 | 289 | 385 | 481 | 578 | 674 | 770 | 867 |
| 966 | 97 | 193 | 290 | 386 | 483 | 580 | 676 | 773 | 869 |
| 969 | 97 | 194 | 291 | 388 | 484 | 581 | 678 | 775 | 872 |
| 972 | 97 | 194 | 292 | 389 | 486 | 583 | 680 | 778 | 875 |
| 975 | 98 | 195 | 292 | 390 | 487 | 585 | 682 | 780 | 878 |
| 978 | 98 | 196 | 293 | 391 | 489 | 587 | 685 | 782 | 880 |
| 981 | 98 | 196 | 294 | 392 | 490 | 589 | 687 | 785 | 883 |
| 984 | 98 | 197 | 295 | 394 | 492 | 590 | 689 | 787 | 886 |
| 987 | 99 | 197 | 296 | 395 | 493 | 592 | 691 | 790 | 888 |
| 990 | 99 | 198 | 297 | 396 | 495 | 594 | 693 | 792 | 891 |
| 993 | 99 | 199 | 298 | 397 | 496 | 596 | 695 | 794 | 894 |
| 996 | 100 | 199 | 299 | 398 | 498 | 598 | 697 | 797 | 897 |
| 999 | 100 | 200 | 300 | 400 | 499 | 599 | 699 | 799 | 899 |
| 1005 | 101 | 201 | 301 | 402 | 502 | 603 | 703 | 804 | 904 |
| 1009 | 101 | 202 | 303 | 404 | 504 | 605 | 706 | 807 | 908 |
| 1013 | 101 | 203 | 304 | 405 | 506 | 608 | 709 | 810 | 912 |
| 1017 | 102 | 203 | 305 | 407 | 508 | 610 | 712 | 814 | 915 |
| 1021 | 102 | 204 | 306 | 408 | 510 | 613 | 715 | 817 | 919 |
| 1025 | 103 | 205 | 307 | 410 | 512 | 615 | 717 | 820 | 923 |
| 1029 | 103 | 206 | 309 | 412 | 514 | 617 | 720 | 823 | 926 |
| 1033 | 103 | 207 | 310 | 413 | 516 | 620 | 723 | 826 | 930 |
| 1037 | 104 | 207 | 311 | 415 | 518 | 622 | 726 | 830 | 933 |
| 1041 | 104 | 208 | 312 | 416 | 520 | 625 | 729 | 833 | 937 |
| 1045 | 105 | 209 | 313 | 418 | 522 | 627 | 731 | 836 | 941 |
| 1049 | 105 | 210 | 315 | 420 | 524 | 629 | 734 | 839 | 944 |
| 1053 | 105 | 211 | 316 | 421 | 526 | 632 | 737 | 842 | 948 |
| 1057 | 106 | 211 | 317 | 423 | 528 | 634 | 740 | 846 | 951 |
| 1061 | 106 | 212 | 318 | 424 | 530 | 637 | 743 | 849 | 955 |
| 1065 | 107 | 213 | 319 | 426 | 532 | 639 | 745 | 852 | 959 |
| 1069 | 107 | 214 | 321 | 428 | 534 | 641 | 748 | 855 | 962 |
| 1073 | 107 | 215 | 322 | 429 | 536 | 644 | 751 | 858 | 966 |
| 1077 | 108 | 215 | 323 | 431 | 538 | 646 | 754 | 862 | 969 |
| 1081 | 108 | 216 | 324 | 432 | 540 | 649 | 757 | 865 | 973 |
| 1085 | 109 | 217 | 325 | 434 | 542 | 651 | 759 | 868 | 977 |
| 1089 | 109 | 218 | 327 | 436 | 544 | 653 | 762 | 871 | 980 |
| 1093 | 110 | 219 | 328 | 437 | 546 | 656 | 765 | 874 | 984 |
| 1097 | 110 | 219 | 329 | 439 | 548 | 658 | 768 | 878 | 987 |
| 1101 | 110 | 220 | 330 | 440 | 550 | 661 | 771 | 881 | 991 |
| 1105 | 111 | 221 | 331 | 442 | 552 | 663 | 773 | 884 | 994 |

# A TABLE of Proportional Parts.

| Diff of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1109 | 111 | 222 | 333 | 444 | 554 | 665 | 776 | 887 | 998 |
| 1113 | 111 | 223 | 334 | 445 | 556 | 668 | 779 | 890 | 1002 |
| 1117 | 112 | 223 | 335 | 447 | 558 | 670 | 782 | 894 | 1005 |
| 1121 | 112 | 224 | 336 | 448 | 560 | 673 | 785 | 897 | 1009 |
| 1125 | 113 | 225 | 337 | 450 | 562 | 675 | 787 | 900 | 1013 |
| 1129 | 113 | 226 | 339 | 452 | 564 | 677 | 790 | 903 | 1016 |
| 1133 | 113 | 227 | 340 | 453 | 566 | 680 | 793 | 906 | 1020 |
| 1137 | 114 | 227 | 341 | 455 | 568 | 682 | 796 | 910 | 1023 |
| 1141 | 114 | 228 | 342 | 456 | 570 | 685 | 799 | 913 | 1027 |
| 1145 | 115 | 229 | 343 | 458 | 572 | 687 | 801 | 916 | 1031 |
| 1149 | 115 | 230 | 345 | 460 | 574 | 689 | 804 | 919 | 1034 |
| 1153 | 115 | 231 | 346 | 461 | 576 | 692 | 807 | 922 | 1038 |
| 1157 | 116 | 231 | 347 | 463 | 578 | 694 | 810 | 926 | 1041 |
| 1161 | 116 | 232 | 348 | 464 | 580 | 697 | 813 | 929 | 1045 |
| 1165 | 117 | 233 | 349 | 466 | 582 | 699 | 815 | 932 | 1049 |
| 1169 | 117 | 234 | 351 | 468 | 584 | 701 | 818 | 935 | 1052 |
| 1173 | 117 | 235 | 352 | 469 | 586 | 704 | 821 | 938 | 1056 |
| 1177 | 118 | 235 | 353 | 471 | 588 | 706 | 824 | 942 | 1059 |
| 1181 | 118 | 236 | 354 | 472 | 590 | 709 | 827 | 945 | 1063 |
| 1185 | 118 | 237 | 355 | 474 | 592 | 711 | 829 | 948 | 1067 |
| 1189 | 119 | 238 | 357 | 476 | 594 | 713 | 832 | 951 | 1070 |
| 1193 | 119 | 239 | 358 | 477 | 596 | 716 | 835 | 954 | 1074 |
| 1197 | 120 | 239 | 359 | 479 | 598 | 718 | 838 | 958 | 1077 |
| 1201 | 120 | 240 | 360 | 480 | 600 | 721 | 841 | 961 | 1081 |
| 1205 | 120 | 241 | 361 | 482 | 602 | 723 | 843 | 964 | 1084 |
| 1209 | 121 | 242 | 363 | 484 | 604 | 725 | 846 | 967 | 1088 |
| 1213 | 121 | 243 | 364 | 485 | 606 | 728 | 849 | 970 | 1092 |
| 1217 | 122 | 243 | 365 | 487 | 608 | 730 | 852 | 974 | 1095 |
| 1221 | 122 | 244 | 366 | 488 | 610 | 732 | 855 | 977 | 1099 |
| 1225 | 122 | 245 | 367 | 490 | 612 | 735 | 857 | 980 | 1102 |
| 1229 | 123 | 246 | 369 | 492 | 614 | 737 | 860 | 983 | 1106 |
| 1233 | 123 | 247 | 370 | 493 | 616 | 740 | 863 | 986 | 1110 |
| 1237 | 124 | 247 | 371 | 495 | 618 | 742 | 866 | 990 | 1113 |
| 1241 | 124 | 248 | 372 | 496 | 620 | 745 | 869 | 993 | 1117 |
| 1245 | 124 | 249 | 373 | 498 | 622 | 747 | 871 | 996 | 1120 |
| 1249 | 125 | 250 | 375 | 500 | 624 | 749 | 874 | 999 | 1124 |
| 1253 | 125 | 251 | 376 | 501 | 626 | 752 | 877 | 1002 | 1128 |
| 1257 | 126 | 251 | 377 | 503 | 628 | 754 | 880 | 1006 | 1131 |
| 1260 | 126 | 252 | 378 | 504 | 630 | 756 | 882 | 1008 | 1134 |
| 1263 | 126 | 253 | 379 | 505 | 631 | 758 | 884 | 1010 | 1137 |

# *A TABLE of Proportional Parts.*

| Diff of Logar. | Tenth Parts of thoſe Differences. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1267 | 127 | 253 | 380 | 507 | 633 | 760 | 887 | 1014 | 1140 |
| 1271 | 127 | 254 | 381 | 508 | 635 | 763 | 890 | 1017 | 1144 |
| 1275 | 127 | 255 | 382 | 510 | 637 | 765 | 892 | 1020 | 1147 |
| 1279 | 128 | 256 | 384 | 512 | 639 | 767 | 895 | 1023 | 1151 |
| 1283 | 128 | 257 | 385 | 513 | 641 | 770 | 898 | 1026 | 1155 |
| 1287 | 129 | 257 | 386 | 515 | 643 | 772 | 901 | 1030 | 1158 |
| 1291 | 129 | 258 | 387 | 516 | 645 | 775 | 904 | 1033 | 1162 |
| 1295 | 129 | 259 | 388 | 518 | 647 | 777 | 905 | 1036 | 1165 |
| 1299 | 130 | 260 | 389 | 520 | 649 | 779 | 909 | 1039 | 1169 |
| 1303 | 130 | 261 | 391 | 521 | 651 | 782 | 912 | 1042 | 1173 |
| 1307 | 131 | 261 | 392 | 523 | 653 | 784 | 915 | 1046 | 1176 |
| 1311 | 131 | 262 | 393 | 524 | 655 | 787 | 918 | 1049 | 1180 |
| 1315 | 131 | 263 | 394 | 526 | 657 | 789 | 920 | 1052 | 1183 |
| 1319 | 132 | 264 | 396 | 528 | 659 | 791 | 923 | 1055 | 1187 |
| 1323 | 132 | 265 | 397 | 529 | 661 | 794 | 926 | 1058 | 1191 |
| 1327 | 133 | 265 | 398 | 531 | 663 | 796 | 929 | 1062 | 1194 |
| 1331 | 133 | 266 | 399 | 532 | 665 | 799 | 932 | 1065 | 1198 |
| 1335 | 133 | 267 | 400 | 534 | 667 | 801 | 934 | 1068 | 1201 |
| 1339 | 134 | 268 | 402 | 536 | 669 | 803 | 937 | 1071 | 1205 |
| 1343 | 134 | 269 | 403 | 537 | 671 | 806 | 940 | 1074 | 1209 |
| 1347 | 135 | 269 | 404 | 539 | 673 | 808 | 943 | 1078 | 1212 |
| 1351 | 135 | 270 | 405 | 540 | 675 | 811 | 946 | 1081 | 1216 |
| 1355 | 135 | 271 | 406 | 542 | 677 | 813 | 948 | 1084 | 1219 |
| 1359 | 136 | 272 | 408 | 544 | 679 | 815 | 951 | 1087 | 1223 |
| 1363 | 136 | 273 | 409 | 545 | 681 | 818 | 954 | 1090 | 1227 |
| 1367 | 137 | 273 | 410 | 547 | 683 | 820 | 957 | 1094 | 1230 |
| 1371 | 137 | 274 | 411 | 548 | 685 | 823 | 960 | 1097 | 1234 |
| 1375 | 137 | 275 | 412 | 550 | 687 | 825 | 962 | 1100 | 1237 |
| 1379 | 138 | 276 | 414 | 552 | 689 | 827 | 965 | 1103 | 1241 |
| 1383 | 138 | 277 | 415 | 553 | 691 | 830 | 968 | 1106 | 1245 |
| 1387 | 139 | 277 | 416 | 555 | 693 | 832 | 971 | 1110 | 1248 |
| 1391 | 139 | 278 | 417 | 556 | 695 | 835 | 974 | 1113 | 1252 |
| 1395 | 140 | 279 | 418 | 558 | 697 | 837 | 976 | 1116 | 1255 |
| 1399 | 140 | 280 | 420 | 559 | 699 | 839 | 979 | 1119 | 1259 |
| 1403 | 140 | 281 | 421 | 561 | 701 | 842 | 982 | 1122 | 1263 |
| 1407 | 141 | 281 | 422 | 563 | 703 | 844 | 985 | 1126 | 1266 |
| 1411 | 141 | 282 | 423 | 564 | 705 | 847 | 988 | 1129 | 1270 |
| 1415 | 141 | 283 | 424 | 566 | 707 | 849 | 990 | 1132 | 1273 |
| 1420 | 142 | 284 | 426 | 568 | 710 | 852 | 994 | 1136 | 1278 |
| 1425 | 142 | 285 | 427 | 570 | 712 | 855 | 997 | 1140 | 1282 |

# A TABLE of Proportional Parts.

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1430 | 143 | 286 | 429 | 572 | 715 | 858 | 1001 | 1144 | 1287 |
| 1435 | 143 | 287 | 430 | 574 | 717 | 861 | 1004 | 1148 | 1291 |
| 1440 | 144 | 288 | 432 | 576 | 720 | 864 | 1008 | 1152 | 1296 |
| 1445 | 144 | 289 | 433 | 578 | 722 | 867 | 1011 | 1156 | 1300 |
| 1450 | 145 | 290 | 435 | 580 | 725 | 870 | 1015 | 1160 | 1305 |
| 1455 | 145 | 291 | 436 | 582 | 727 | 873 | 1018 | 1164 | 1309 |
| 1460 | 146 | 292 | 438 | 584 | 730 | 876 | 1022 | 1168 | 1314 |
| 1465 | 146 | 293 | 439 | 586 | 732 | 879 | 1025 | 1172 | 1318 |
| 1470 | 147 | 294 | 441 | 588 | 735 | 882 | 1029 | 1176 | 1323 |
| 1475 | 147 | 295 | 442 | 590 | 737 | 885 | 1032 | 1180 | 1327 |
| 1480 | 148 | 296 | 444 | 592 | 740 | 888 | 1036 | 1184 | 1332 |
| 1485 | 148 | 297 | 445 | 594 | 742 | 891 | 1039 | 1188 | 1336 |
| 1490 | 149 | 298 | 447 | 596 | 745 | 894 | 1043 | 1192 | 1341 |
| 1495 | 149 | 299 | 448 | 598 | 747 | 897 | 1046 | 1196 | 1345 |
| 1500 | 150 | 300 | 450 | 600 | 750 | 900 | 1050 | 1200 | 1350 |
| 1505 | 150 | 301 | 451 | 602 | 752 | 903 | 1053 | 1204 | 1354 |
| 1510 | 151 | 302 | 453 | 604 | 755 | 906 | 1057 | 1208 | 1359 |
| 1515 | 151 | 303 | 454 | 606 | 757 | 909 | 1060 | 1212 | 1363 |
| 1520 | 152 | 304 | 456 | 608 | 760 | 912 | 1064 | 1216 | 1368 |
| 1525 | 152 | 305 | 457 | 610 | 762 | 915 | 1067 | 1220 | 1372 |
| 1530 | 153 | 306 | 459 | 612 | 765 | 918 | 1071 | 1224 | 1377 |
| 1535 | 153 | 307 | 460 | 614 | 767 | 921 | 1074 | 1228 | 1381 |
| 1540 | 154 | 308 | 462 | 616 | 770 | 924 | 1078 | 1232 | 1386 |
| 1545 | 154 | 309 | 463 | 618 | 772 | 927 | 1081 | 1236 | 1390 |
| 1550 | 155 | 310 | 465 | 620 | 775 | 930 | 1085 | 1240 | 1395 |
| 1555 | 155 | 311 | 466 | 622 | 777 | 933 | 1088 | 1244 | 1399 |
| 1560 | 156 | 312 | 468 | 624 | 780 | 936 | 1092 | 1248 | 1404 |
| 1565 | 156 | 313 | 469 | 626 | 782 | 939 | 1095 | 1252 | 1408 |
| 1570 | 157 | 314 | 471 | 628 | 785 | 942 | 1099 | 1256 | 1413 |
| 1575 | 157 | 315 | 472 | 630 | 787 | 945 | 1102 | 1260 | 1417 |
| 1580 | 158 | 316 | 474 | 632 | 790 | 948 | 1106 | 1264 | 1422 |
| 1585 | 158 | 317 | 475 | 634 | 792 | 951 | 1109 | 1268 | 1426 |
| 1590 | 159 | 318 | 477 | 636 | 795 | 954 | 1113 | 1272 | 1431 |
| 1595 | 159 | 319 | 478 | 638 | 797 | 957 | 1116 | 1276 | 1435 |
| 1600 | 160 | 320 | 480 | 640 | 800 | 960 | 1120 | 1280 | 1440 |
| 1605 | 160 | 321 | 481 | 642 | 802 | 963 | 1123 | 1284 | 1444 |
| 1610 | 161 | 322 | 483 | 644 | 805 | 966 | 1127 | 1288 | 1449 |
| 1615 | 161 | 323 | 484 | 646 | 807 | 969 | 1130 | 1292 | 1453 |
| 1620 | 162 | 324 | 486 | 648 | 810 | 972 | 1134 | 1296 | 1458 |
| 1625 | 162 | 325 | 487 | 650 | 812 | 975 | 1137 | 1300 | 1462 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1630 | 163 | 326 | 489 | 652 | 815 | 978 | 1141 | 1304 | 1467 |
| 1635 | 163 | 327 | 490 | 654 | 817 | 981 | 1144 | 1308 | 1471 |
| 1640 | 164 | 328 | 492 | 656 | 820 | 984 | 1148 | 1312 | 1476 |
| 1645 | 164 | 329 | 493 | 658 | 822 | 987 | 1151 | 1316 | 1480 |
| 1650 | 165 | 330 | 495 | 660 | 825 | 990 | 1155 | 1320 | 1485 |
| 1655 | 165 | 331 | 496 | 662 | 827 | 993 | 1158 | 1324 | 1489 |
| 1660 | 166 | 332 | 498 | 664 | 830 | 996 | 1162 | 1328 | 1494 |
| 1665 | 167 | 333 | 499 | 666 | 832 | 999 | 1166 | 1332 | 1498 |
| 1670 | 167 | 334 | 501 | 668 | 835 | 1002 | 1169 | 1336 | 1503 |
| 1675 | 167 | 335 | 502 | 670 | 837 | 1005 | 1172 | 1340 | 1507 |
| 1680 | 168 | 336 | 504 | 672 | 840 | 1008 | 1176 | 1344 | 1512 |
| 1685 | 168 | 337 | 505 | 674 | 842 | 1011 | 1179 | 1348 | 1516 |
| 1690 | 169 | 338 | 507 | 676 | 845 | 1014 | 1183 | 1352 | 1521 |
| 1695 | 169 | 339 | 508 | 678 | 847 | 1017 | 1186 | 1356 | 1525 |
| 1700 | 170 | 340 | 510 | 680 | 850 | 1020 | 1190 | 1360 | 1530 |
| 1705 | 170 | 341 | 511 | 682 | 852 | 1023 | 1193 | 1364 | 1534 |
| 1710 | 171 | 342 | 513 | 684 | 855 | 1026 | 1197 | 1368 | 1539 |
| 1715 | 171 | 343 | 514 | 686 | 857 | 1029 | 1200 | 1372 | 1543 |
| 1720 | 172 | 344 | 516 | 688 | 860 | 1032 | 1204 | 1376 | 1548 |
| 1725 | 172 | 345 | 517 | 690 | 862 | 1035 | 1207 | 1380 | 1552 |
| 1730 | 173 | 346 | 519 | 692 | 865 | 1038 | 1211 | 1384 | 1557 |
| 1735 | 173 | 347 | 520 | 694 | 867 | 1041 | 1214 | 1388 | 1561 |
| 1740 | 174 | 348 | 522 | 696 | 870 | 1044 | 1218 | 1392 | 1566 |
| 1745 | 174 | 349 | 523 | 698 | 872 | 1047 | 1221 | 1396 | 1570 |
| 1750 | 175 | 350 | 525 | 700 | 875 | 1050 | 1225 | 1400 | 1575 |
| 1755 | 175 | 351 | 526 | 702 | 877 | 1053 | 1228 | 1404 | 1579 |
| 1760 | 176 | 352 | 528 | 704 | 880 | 1056 | 1232 | 1408 | 1584 |
| 1765 | 176 | 353 | 529 | 706 | 882 | 1059 | 1235 | 1412 | 1588 |
| 1770 | 177 | 354 | 531 | 708 | 885 | 1062 | 1239 | 1416 | 1593 |
| 1775 | 178 | 355 | 532 | 710 | 887 | 1065 | 1242 | 1420 | 1597 |
| 1780 | 178 | 356 | 534 | 712 | 890 | 1068 | 1246 | 1424 | 1602 |
| 1785 | 179 | 357 | 535 | 714 | 892 | 1071 | 1249 | 1428 | 1606 |
| 1790 | 179 | 358 | 537 | 716 | 895 | 1074 | 1253 | 1432 | 1611 |
| 1795 | 180 | 359 | 538 | 718 | 897 | 1077 | 1256 | 1436 | 1615 |
| 1800 | 180 | 360 | 540 | 720 | 900 | 1080 | 1260 | 1440 | 1620 |
| 1805 | 181 | 361 | 541 | 722 | 902 | 1083 | 1263 | 1444 | 1624 |
| 1810 | 181 | 362 | 543 | 724 | 905 | 1086 | 1267 | 1448 | 1629 |
| 1815 | 182 | 363 | 544 | 726 | 907 | 1089 | 1270 | 1452 | 1633 |
| 1820 | 182 | 364 | 546 | 728 | 910 | 1092 | 1274 | 1456 | 1638 |
| 1825 | 183 | 365 | 547 | 730 | 912 | 1095 | 1277 | 1460 | 1642 |

# A TABLE of Proportional Parts.

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 1830 | 183 | 366 | 549 | 732 | 915 | 1098 | 1281 | 1464 | 1647 |
| 1835 | 184 | 367 | 550 | 734 | 917 | 1101 | 1284 | 1468 | 1651 |
| 1840 | 184 | 368 | 552 | 736 | 920 | 1104 | 1288 | 1472 | 1656 |
| 1845 | 185 | 369 | 553 | 738 | 922 | 1107 | 1291 | 1476 | 1660 |
| 1850 | 185 | 370 | 555 | 740 | 925 | 1110 | 1295 | 1480 | 1665 |
| 1855 | 186 | 371 | 556 | 742 | 927 | 1113 | 1298 | 1484 | 1669 |
| 1860 | 186 | 372 | 558 | 744 | 930 | 1116 | 1302 | 1488 | 1674 |
| 1865 | 187 | 373 | 559 | 746 | 932 | 1119 | 1305 | 1492 | 1678 |
| 1870 | 187 | 374 | 561 | 748 | 935 | 1122 | 1309 | 1496 | 1683 |
| 1875 | 188 | 375 | 562 | 750 | 937 | 1125 | 1312 | 1500 | 1687 |
| 1880 | 188 | 376 | 564 | 752 | 940 | 1128 | 1316 | 1504 | 1692 |
| 1885 | 189 | 377 | 565 | 754 | 942 | 1131 | 1319 | 1508 | 1696 |
| 1890 | 189 | 378 | 567 | 756 | 945 | 1134 | 1323 | 1512 | 1701 |
| 1895 | 190 | 379 | 568 | 758 | 947 | 1137 | 1326 | 1516 | 1705 |
| 1900 | 190 | 380 | 570 | 760 | 950 | 1140 | 1330 | 1520 | 1710 |
| 1905 | 191 | 381 | 571 | 762 | 952 | 1143 | 1333 | 1524 | 1714 |
| 1910 | 191 | 382 | 573 | 764 | 955 | 1146 | 1337 | 1528 | 1719 |
| 1915 | 192 | 383 | 574 | 766 | 957 | 1149 | 1340 | 1532 | 1723 |
| 1920 | 192 | 384 | 576 | 768 | 960 | 1152 | 1344 | 1536 | 1728 |
| 1925 | 193 | 385 | 577 | 770 | 962 | 1155 | 1347 | 1540 | 1732 |
| 1930 | 193 | 386 | 579 | 772 | 965 | 1158 | 1351 | 1544 | 1737 |
| 1935 | 194 | 387 | 580 | 774 | 967 | 1161 | 1354 | 1548 | 1742 |
| 1940 | 194 | 388 | 582 | 776 | 970 | 1164 | 1358 | 1552 | 1746 |
| 1945 | 195 | 389 | 583 | 778 | 972 | 1167 | 1361 | 1556 | 1750 |
| 1950 | 195 | 390 | 585 | 780 | 975 | 1170 | 1365 | 1560 | 1755 |
| 1955 | 196 | 391 | 586 | 782 | 977 | 1173 | 1368 | 1564 | 1759 |
| 1960 | 196 | 392 | 588 | 784 | 980 | 1176 | 1372 | 1568 | 1764 |
| 1965 | 197 | 393 | 589 | 786 | 982 | 1179 | 1375 | 1572 | 1768 |
| 1970 | 197 | 394 | 591 | 788 | 985 | 1182 | 1379 | 1576 | 1773 |
| 1975 | 198 | 395 | 592 | 790 | 987 | 1185 | 1382 | 1580 | 1777 |
| 1980 | 198 | 396 | 594 | 792 | 990 | 1188 | 1386 | 1584 | 1782 |
| 1985 | 199 | 397 | 595 | 794 | 992 | 1191 | 1389 | 1588 | 1786 |
| 1990 | 199 | 398 | 597 | 796 | 995 | 1194 | 1393 | 1592 | 1791 |
| 1995 | 200 | 399 | 598 | 798 | 997 | 1197 | 1396 | 1596 | 1795 |
| 2000 | 200 | 400 | 600 | 800 | 1000 | 1200 | 1400 | 1600 | 1800 |
| 2010 | 201 | 402 | 603 | 804 | 1005 | 1206 | 1407 | 1608 | 1809 |
| 2020 | 202 | 404 | 606 | 808 | 1010 | 1212 | 1414 | 1616 | 1818 |
| 2030 | 203 | 406 | 609 | 812 | 1015 | 1218 | 1421 | 1624 | 1827 |
| 2040 | 204 | 408 | 612 | 816 | 1020 | 1224 | 1428 | 1632 | 1836 |
| 2050 | 205 | 410 | 615 | 820 | 1025 | 1230 | 1435 | 1640 | 1845 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 2060 | 206 | 412 | 618 | 824 | 1030 | 1236 | 1442 | 1648 | 1854 |
| 2070 | 207 | 414 | 621 | 828 | 1035 | 1242 | 1449 | 1656 | 1863 |
| 2080 | 208 | 416 | 625 | 832 | 1040 | 1248 | 1456 | 1664 | 1872 |
| 2090 | 209 | 418 | 627 | 836 | 1045 | 1254 | 1463 | 1672 | 1881 |
| 2100 | 210 | 420 | 630 | 840 | 1050 | 1260 | 1470 | 1680 | 1890 |
| 2110 | 211 | 422 | 633 | 844 | 1055 | 1266 | 1477 | 1688 | 1899 |
| 2120 | 212 | 424 | 636 | 848 | 1060 | 1272 | 1484 | 1696 | 1908 |
| 2130 | 213 | 426 | 639 | 852 | 1065 | 1278 | 1491 | 1704 | 1917 |
| 2140 | 214 | 428 | 642 | 856 | 1070 | 1284 | 1498 | 1712 | 1926 |
| 2150 | 215 | 430 | 645 | 860 | 1075 | 1290 | 1505 | 1720 | 1935 |
| 2160 | 216 | 432 | 648 | 864 | 1080 | 1296 | 1512 | 1728 | 1944 |
| 2170 | 217 | 434 | 651 | 868 | 1085 | 1302 | 1519 | 1736 | 1953 |
| 2180 | 218 | 436 | 654 | 872 | 1090 | 1308 | 1526 | 1744 | 1962 |
| 2190 | 219 | 438 | 657 | 876 | 1095 | 1314 | 1533 | 1752 | 1971 |
| 2200 | 220 | 440 | 660 | 880 | 1100 | 1320 | 1540 | 1760 | 1980 |
| 2210 | 221 | 442 | 663 | 884 | 1105 | 1326 | 1547 | 1768 | 1989 |
| 2220 | 222 | 444 | 666 | 888 | 1110 | 1332 | 1554 | 1776 | 1998 |
| 2230 | 223 | 446 | 669 | 892 | 1115 | 1338 | 1561 | 1784 | 2007 |
| 2240 | 224 | 448 | 672 | 896 | 1120 | 1344 | 1568 | 1792 | 2016 |
| 2250 | 225 | 450 | 675 | 900 | 1125 | 1350 | 1575 | 1800 | 2025 |
| 2260 | 226 | 452 | 678 | 904 | 1130 | 1356 | 1582 | 1808 | 2034 |
| 2270 | 227 | 454 | 681 | 908 | 1135 | 1362 | 1589 | 1816 | 2043 |
| 2280 | 228 | 456 | 684 | 912 | 1140 | 1368 | 1596 | 1824 | 2052 |
| 2290 | 229 | 458 | 687 | 916 | 1145 | 1374 | 1603 | 1832 | 2061 |
| 2300 | 230 | 460 | 690 | 920 | 1150 | 1380 | 1610 | 1840 | 2070 |
| 2310 | 231 | 462 | 693 | 924 | 1155 | 1386 | 1617 | 1848 | 2079 |
| 2320 | 232 | 464 | 696 | 928 | 1160 | 1392 | 1624 | 1856 | 2088 |
| 2330 | 233 | 466 | 699 | 932 | 1165 | 1398 | 1631 | 1864 | 2097 |
| 2340 | 234 | 468 | 702 | 936 | 1170 | 1404 | 1638 | 1872 | 2106 |
| 2350 | 235 | 470 | 705 | 940 | 1175 | 1410 | 1645 | 1880 | 2115 |
| 2360 | 236 | 472 | 708 | 944 | 1180 | 1416 | 1652 | 1888 | 2124 |
| 2370 | 237 | 474 | 711 | 948 | 1185 | 1422 | 1659 | 1896 | 2133 |
| 2380 | 238 | 476 | 714 | 952 | 1190 | 1428 | 1666 | 1904 | 2142 |
| 2390 | 239 | 478 | 717 | 956 | 1195 | 1434 | 1673 | 1912 | 2151 |
| 2400 | 240 | 480 | 720 | 960 | 1200 | 1440 | 1680 | 1920 | 2160 |
| 2410 | 241 | 482 | 723 | 964 | 1205 | 1446 | 1687 | 1928 | 2169 |
| 2420 | 242 | 484 | 726 | 968 | 1210 | 1452 | 1694 | 1936 | 2178 |
| 2430 | 243 | 486 | 729 | 972 | 1215 | 1458 | 1701 | 1944 | 2187 |
| 2440 | 244 | 488 | 732 | 976 | 1220 | 1464 | 1708 | 1952 | 2196 |
| 2450 | 245 | 490 | 735 | 980 | 1225 | 1470 | 1715 | 1960 | 2205 |

# *A TABLE of Proportional Parts.*

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 2460 | 246 | 492 | 738 | 984 | 1230 | 1476 | 1722 | 1968 | 2214 |
| 2470 | 247 | 494 | 741 | 988 | 1235 | 1482 | 1729 | 1976 | 2223 |
| 2480 | 248 | 496 | 744 | 992 | 1240 | 1488 | 1736 | 1984 | 2232 |
| 2490 | 249 | 498 | 747 | 996 | 1245 | 1494 | 1743 | 1992 | 2241 |
| 2500 | 250 | 500 | 750 | 1000 | 1250 | 1500 | 1750 | 2000 | 2250 |
| 2510 | 251 | 502 | 753 | 1004 | 1255 | 1506 | 1757 | 2008 | 2259 |
| 2520 | 252 | 504 | 756 | 1008 | 1260 | 1512 | 1764 | 2016 | 2268 |
| 2530 | 253 | 506 | 759 | 1012 | 1265 | 1518 | 1771 | 2024 | 2277 |
| 2540 | 254 | 508 | 762 | 1016 | 1270 | 1524 | 1778 | 2032 | 2286 |
| 2550 | 255 | 510 | 765 | 1020 | 1275 | 1530 | 1785 | 2040 | 2295 |
| 2560 | 256 | 512 | 768 | 1024 | 1280 | 1536 | 1792 | 2048 | 2304 |
| 2570 | 257 | 514 | 771 | 1028 | 1285 | 1542 | 1799 | 2056 | 2313 |
| 2580 | 258 | 516 | 774 | 1032 | 1290 | 1548 | 1806 | 2064 | 2322 |
| 2590 | 259 | 518 | 777 | 1036 | 1295 | 1554 | 1813 | 2072 | 2331 |
| 2600 | 260 | 520 | 780 | 1040 | 1300 | 1560 | 1820 | 2080 | 2340 |
| 2610 | 261 | 522 | 783 | 1044 | 1305 | 1566 | 1827 | 2088 | 2349 |
| 2620 | 262 | 524 | 786 | 1048 | 1310 | 1572 | 1834 | 2096 | 2358 |
| 2630 | 263 | 526 | 789 | 1052 | 1315 | 1578 | 1841 | 2104 | 2367 |
| 2640 | 264 | 528 | 792 | 1056 | 1320 | 1584 | 1848 | 2112 | 2376 |
| 2650 | 265 | 530 | 795 | 1060 | 1325 | 1590 | 1855 | 2120 | 2385 |
| 2660 | 266 | 532 | 798 | 1064 | 1330 | 1596 | 1862 | 2128 | 2394 |
| 2670 | 267 | 534 | 801 | 1068 | 1335 | 1602 | 1869 | 2136 | 2403 |
| 2680 | 268 | 536 | 804 | 1072 | 1340 | 1608 | 1876 | 2144 | 2412 |
| 2690 | 269 | 538 | 807 | 1076 | 1345 | 1614 | 1883 | 2152 | 2421 |
| 2700 | 270 | 540 | 810 | 1080 | 1350 | 1620 | 1890 | 2160 | 2430 |
| 2710 | 271 | 542 | 813 | 1084 | 1355 | 1626 | 1897 | 2168 | 2430 |
| 2720 | 272 | 544 | 816 | 1088 | 1360 | 1632 | 1904 | 2176 | 2448 |
| 2730 | 273 | 546 | 819 | 1092 | 1365 | 1638 | 1911 | 2184 | 2457 |
| 2740 | 274 | 548 | 822 | 1096 | 1370 | 1644 | 1918 | 2192 | 2466 |
| 2750 | 275 | 550 | 825 | 1100 | 1375 | 1650 | 1925 | 2200 | 2475 |
| 2760 | 276 | 552 | 828 | 1104 | 1380 | 1656 | 1932 | 2208 | 2484 |
| 2770 | 277 | 554 | 831 | 1108 | 1385 | 1662 | 1939 | 2216 | 2493 |
| 2780 | 278 | 556 | 834 | 1112 | 1390 | 1668 | 1946 | 2224 | 2502 |
| 2790 | 279 | 558 | 837 | 1116 | 1395 | 1674 | 1953 | 2232 | 2511 |
| 2800 | 280 | 560 | 840 | 1120 | 1400 | 1680 | 1960 | 2240 | 2520 |
| 2810 | 281 | 562 | 843 | 1124 | 1405 | 1686 | 1967 | 2248 | 2529 |
| 2820 | 282 | 564 | 846 | 1128 | 1410 | 1692 | 1974 | 2256 | 2538 |
| 2830 | 283 | 566 | 849 | 1132 | 1415 | 1698 | 1981 | 2264 | 2547 |
| 2840 | 284 | 568 | 852 | 1136 | 1420 | 1704 | 1988 | 2272 | 2556 |
| 2850 | 285 | 570 | 855 | 1140 | 1425 | 1710 | 1995 | 2280 | 2565 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of thoſe Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 2860 | 286 | 572 | 858 | 1144 | 1430 | 1716 | 2002 | 2288 | 2574 |
| 2870 | 287 | 574 | 861 | 1148 | 1435 | 1722 | 2009 | 2296 | 2583 |
| 2880 | 288 | 576 | 864 | 1152 | 1440 | 1728 | 2016 | 2304 | 2592 |
| 2890 | 289 | 578 | 867 | 1156 | 1445 | 1734 | 2023 | 2312 | 2601 |
| 2900 | 290 | 580 | 870 | 1160 | 1450 | 1740 | 2030 | 2320 | 2510 |
| 2910 | 291 | 582 | 873 | 1164 | 1455 | 1746 | 2037 | 2328 | 2619 |
| 2920 | 292 | 584 | 876 | 1168 | 1460 | 1752 | 2044 | 2336 | 2628 |
| 2930 | 293 | 586 | 879 | 1172 | 1465 | 1758 | 2051 | 2344 | 2637 |
| 2940 | 294 | 588 | 882 | 1176 | 1470 | 1764 | 2058 | 2352 | 2646 |
| 2950 | 295 | 590 | 885 | 1180 | 1475 | 1770 | 2065 | 2360 | 2655 |
| 2960 | 296 | 592 | 888 | 1184 | 1480 | 1776 | 2072 | 2368 | 2664 |
| 2970 | 297 | 594 | 891 | 1188 | 1485 | 1782 | 2079 | 2376 | 2673 |
| 2980 | 298 | 596 | 894 | 1192 | 1490 | 1788 | 2086 | 2384 | 2682 |
| 2990 | 299 | 598 | 897 | 1196 | 1495 | 1794 | 2093 | 2392 | 2691 |
| 3000 | 300 | 600 | 900 | 1200 | 1500 | 1800 | 2100 | 2400 | 2700 |
| 3010 | 301 | 602 | 903 | 1204 | 1505 | 1806 | 2107 | 2408 | 2709 |
| 3020 | 302 | 604 | 906 | 1208 | 1510 | 1812 | 2114 | 2416 | 2718 |
| 3030 | 303 | 606 | 909 | 1212 | 1515 | 1818 | 2121 | 2424 | 2727 |
| 3040 | 304 | 608 | 912 | 1216 | 1520 | 1824 | 2128 | 2432 | 2736 |
| 3050 | 305 | 610 | 915 | 1220 | 1525 | 1830 | 2135 | 2440 | 2745 |
| 3060 | 306 | 612 | 918 | 1224 | 1530 | 1836 | 2142 | 2448 | 2754 |
| 3070 | 307 | 614 | 921 | 1228 | 1535 | 1842 | 2149 | 2456 | 2763 |
| 3080 | 308 | 616 | 924 | 1232 | 1540 | 1848 | 2156 | 2464 | 2772 |
| 3090 | 309 | 618 | 927 | 1236 | 1545 | 1854 | 2163 | 2472 | 2781 |
| 3100 | 310 | 620 | 930 | 1240 | 1550 | 1860 | 2170 | 2480 | 2790 |
| 3110 | 311 | 622 | 933 | 1244 | 1555 | 1866 | 2177 | 2488 | 2799 |
| 3120 | 312 | 624 | 936 | 1248 | 1560 | 1872 | 2184 | 2496 | 2808 |
| 3130 | 313 | 626 | 939 | 1252 | 1565 | 1878 | 2191 | 2504 | 2817 |
| 3140 | 314 | 628 | 942 | 1256 | 1570 | 1884 | 2198 | 2512 | 2826 |
| 3150 | 315 | 630 | 945 | 1260 | 1575 | 1890 | 2205 | 2520 | 2835 |
| 3160 | 316 | 632 | 948 | 1264 | 1580 | 1896 | 2212 | 2528 | 2844 |
| 3170 | 317 | 634 | 951 | 1268 | 1585 | 1902 | 2219 | 2536 | 2853 |
| 3180 | 318 | 636 | 954 | 1272 | 1590 | 1908 | 2226 | 2544 | 2862 |
| 3190 | 319 | 638 | 957 | 1276 | 1595 | 1914 | 2233 | 2552 | 2871 |
| 3200 | 320 | 640 | 960 | 1280 | 1600 | 1920 | 2240 | 2560 | 2880 |
| 3210 | 321 | 642 | 963 | 1284 | 1605 | 1926 | 2247 | 2568 | 2889 |
| 3220 | 322 | 644 | 966 | 1288 | 1610 | 1932 | 2254 | 2576 | 2898 |
| 3230 | 323 | 646 | 969 | 1292 | 1615 | 1938 | 2261 | 2584 | 2907 |
| 3240 | 324 | 648 | 972 | 1296 | 1620 | 1944 | 2268 | 2592 | 2916 |
| 3250 | 325 | 650 | 975 | 1300 | 1625 | 1950 | 2275 | 2600 | 2925 |

# A TABLE of Proportional Parts.

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 3260 | 326 | 652 | 978 | 1304 | 1630 | 1956 | 2282 | 2608 | 2934 |
| 3270 | 327 | 654 | 981 | 1308 | 1635 | 1962 | 2289 | 2616 | 2943 |
| 3280 | 328 | 656 | 984 | 1312 | 1640 | 1968 | 2296 | 2624 | 2952 |
| 3290 | 329 | 658 | 987 | 1316 | 1645 | 1974 | 2303 | 2632 | 2961 |
| 3300 | 330 | 660 | 990 | 1320 | 1650 | 1980 | 2310 | 2640 | 2970 |
| 3310 | 331 | 662 | 993 | 1324 | 1655 | 1986 | 2317 | 2648 | 2979 |
| 3320 | 332 | 664 | 996 | 1328 | 1660 | 1992 | 2324 | 2656 | 2988 |
| 3330 | 333 | 666 | 999 | 1332 | 1665 | 1998 | 2331 | 2664 | 2997 |
| 3340 | 334 | 668 | 1002 | 1336 | 1670 | 2004 | 2338 | 2672 | 3006 |
| 3350 | 335 | 670 | 1005 | 1340 | 1675 | 2010 | 2345 | 2680 | 3015 |
| 3360 | 336 | 672 | 1008 | 1344 | 1680 | 2016 | 2352 | 2688 | 3024 |
| 3370 | 337 | 674 | 1011 | 1348 | 1685 | 2022 | 2359 | 2696 | 3033 |
| 3380 | 338 | 676 | 1014 | 1352 | 1690 | 2028 | 2366 | 2704 | 3042 |
| 3390 | 339 | 678 | 1017 | 1356 | 1695 | 2034 | 2373 | 2712 | 3051 |
| 3400 | 340 | 680 | 1020 | 1360 | 1700 | 2040 | 2380 | 2720 | 3060 |
| 3410 | 341 | 682 | 1023 | 1364 | 1705 | 2046 | 2387 | 2728 | 3069 |
| 3420 | 342 | 684 | 1026 | 1368 | 1710 | 2052 | 2394 | 2736 | 3078 |
| 3430 | 343 | 686 | 1029 | 1372 | 1715 | 2058 | 2401 | 2744 | 3087 |
| 3440 | 344 | 688 | 1032 | 1376 | 1720 | 2064 | 2408 | 2752 | 3096 |
| 3450 | 345 | 690 | 1035 | 1380 | 1725 | 2070 | 2415 | 2760 | 3105 |
| 3460 | 346 | 692 | 1038 | 1384 | 1730 | 2076 | 2422 | 2768 | 3114 |
| 3470 | 347 | 694 | 1041 | 1388 | 1735 | 2082 | 2429 | 2776 | 3123 |
| 3480 | 348 | 696 | 1044 | 1392 | 1740 | 2088 | 2436 | 2784 | 3132 |
| 3490 | 349 | 698 | 1047 | 1396 | 1745 | 2094 | 2443 | 2792 | 3141 |
| 3500 | 350 | 700 | 1050 | 1400 | 1750 | 2100 | 2450 | 2800 | 3150 |
| 3510 | 351 | 702 | 1053 | 1404 | 1755 | 2106 | 2457 | 2808 | 3159 |
| 3520 | 352 | 704 | 1056 | 1408 | 1760 | 2112 | 2464 | 2816 | 3168 |
| 3530 | 353 | 706 | 1059 | 1412 | 1765 | 2118 | 2471 | 2824 | 3177 |
| 3540 | 354 | 708 | 1062 | 1416 | 1770 | 2124 | 2478 | 2832 | 3186 |
| 3550 | 355 | 710 | 1065 | 1420 | 1775 | 2130 | 2485 | 2840 | 3195 |
| 3560 | 356 | 712 | 1068 | 1424 | 1780 | 2136 | 2492 | 2848 | 3204 |
| 3570 | 357 | 714 | 1071 | 1428 | 1785 | 2142 | 2499 | 2856 | 3213 |
| 3580 | 358 | 716 | 1074 | 1432 | 1790 | 2148 | 2506 | 2864 | 3222 |
| 3590 | 359 | 718 | 1077 | 1436 | 1795 | 2154 | 2513 | 2872 | 3231 |
| 3600 | 360 | 720 | 1080 | 1440 | 1800 | 2160 | 2520 | 2880 | 3240 |
| 3610 | 361 | 722 | 1083 | 1444 | 1805 | 2166 | 2527 | 2888 | 3249 |
| 3620 | 362 | 724 | 1086 | 1448 | 1810 | 2172 | 2534 | 2896 | 3258 |
| 3630 | 363 | 726 | 1089 | 1452 | 1815 | 2178 | 2541 | 2904 | 3267 |
| 3640 | 364 | 728 | 1092 | 1456 | 1820 | 2184 | 2548 | 2912 | 3276 |
| 3650 | 365 | 730 | 1095 | 1460 | 1825 | 2190 | 2555 | 2920 | 3285 |

# A TABLE of Proportional Parts.

| Diff. of Logar. | Tenth Parts of those Differences. 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 3660 | 366 | 732 | 1098 | 1464 | 1830 | 2196 | 2562 | 2928 | 3294 |
| 3670 | 367 | 734 | 1101 | 1468 | 1835 | 2202 | 2569 | 2936 | 3303 |
| 3680 | 368 | 736 | 1104 | 1472 | 1840 | 2208 | 2576 | 2944 | 3312 |
| 3690 | 369 | 738 | 1107 | 1476 | 1845 | 2214 | 2583 | 2952 | 3321 |
| 3700 | 370 | 740 | 1110 | 1480 | 1850 | 2220 | 2590 | 2960 | 3330 |
| 3710 | 371 | 742 | 1113 | 1484 | 1855 | 2226 | 2597 | 2968 | 3339 |
| 3720 | 372 | 744 | 1116 | 1488 | 1860 | 2232 | 2604 | 2976 | 3348 |
| 3730 | 373 | 746 | 1119 | 1492 | 1865 | 2238 | 2611 | 2984 | 3357 |
| 3740 | 374 | 748 | 1122 | 1496 | 1870 | 2244 | 2618 | 2992 | 3366 |
| 3750 | 375 | 750 | 1125 | 1500 | 1875 | 2250 | 2625 | 3000 | 3375 |
| 3760 | 376 | 752 | 1128 | 1504 | 1880 | 2256 | 2632 | 3008 | 3384 |
| 3770 | 377 | 754 | 1131 | 1508 | 1885 | 2262 | 2639 | 3016 | 3393 |
| 3780 | 378 | 756 | 1134 | 1512 | 1890 | 2268 | 2646 | 3024 | 3402 |
| 3790 | 379 | 758 | 1137 | 1516 | 1895 | 2274 | 2653 | 3032 | 3411 |
| 3800 | 380 | 760 | 1140 | 1520 | 1900 | 2280 | 2660 | 3040 | 3420 |
| 3810 | 381 | 762 | 1143 | 1524 | 1905 | 2286 | 2667 | 3048 | 3429 |
| 3820 | 382 | 764 | 1146 | 1528 | 1910 | 2292 | 2674 | 3056 | 3438 |
| 3830 | 383 | 766 | 1149 | 1532 | 1915 | 2298 | 2681 | 3064 | 3447 |
| 3840 | 384 | 768 | 1152 | 1536 | 1920 | 2304 | 2688 | 3072 | 3456 |
| 3850 | 385 | 770 | 1155 | 1540 | 1925 | 2310 | 2695 | 3080 | 3465 |
| 3860 | 386 | 772 | 1158 | 1544 | 1930 | 2316 | 2702 | 3088 | 3474 |
| 3870 | 387 | 774 | 1161 | 1548 | 1935 | 2322 | 2709 | 3096 | 3483 |
| 3880 | 388 | 776 | 1164 | 1552 | 1940 | 2328 | 2716 | 3104 | 3492 |
| 3890 | 389 | 778 | 1167 | 1556 | 1945 | 2334 | 2723 | 3112 | 3501 |
| 3900 | 390 | 780 | 1170 | 1560 | 1950 | 2340 | 2730 | 3120 | 3510 |
| 3910 | 391 | 782 | 1173 | 1564 | 1955 | 2346 | 2737 | 3128 | 3519 |
| 3920 | 392 | 784 | 1176 | 1568 | 1960 | 2352 | 2744 | 3136 | 3528 |
| 3930 | 393 | 786 | 1179 | 1572 | 1965 | 2358 | 2751 | 3144 | 3537 |
| 3940 | 394 | 788 | 1182 | 1576 | 1970 | 2364 | 2758 | 3152 | 3546 |
| 3950 | 395 | 790 | 1185 | 1580 | 1975 | 2370 | 2765 | 3160 | 3555 |
| 3960 | 396 | 792 | 1188 | 1584 | 1980 | 2376 | 2772 | 3168 | 3564 |
| 3970 | 397 | 794 | 1191 | 1588 | 1985 | 2382 | 2779 | 3176 | 3573 |
| 3980 | 398 | 796 | 1194 | 1592 | 1990 | 2388 | 2786 | 3184 | 3582 |
| 3990 | 399 | 798 | 1197 | 1596 | 1995 | 2394 | 2793 | 3192 | 3591 |
| 4000 | 400 | 800 | 1200 | 1600 | 2000 | 2400 | 2800 | 3200 | 3600 |
| 4010 | 401 | 802 | 1203 | 1604 | 2005 | 2406 | 2807 | 3208 | 3609 |
| 4020 | 402 | 804 | 1206 | 1608 | 2010 | 2412 | 2814 | 3216 | 3618 |
| 4030 | 403 | 806 | 1209 | 1612 | 2015 | 2418 | 2821 | 3224 | 3627 |
| 4040 | 404 | 808 | 1212 | 1616 | 2020 | 2424 | 2828 | 3232 | 3636 |
| 4050 | 405 | 810 | 1215 | 1620 | 2025 | 2430 | 2835 | 3240 | 3645 |

# *A TABLE of Proportional Parts.*

## Tenth Parts of thoſe Differences.

| Diff. of Logar. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| 4060 | 406 | 812 | 1218 | 1624 | 2030 | 2436 | 2842 | 3248 | 3654 |
| 4070 | 407 | 814 | 1221 | 1628 | 2035 | 2442 | 2849 | 3256 | 3663 |
| 4080 | 408 | 816 | 1224 | 1632 | 2040 | 2448 | 2856 | 3264 | 3672 |
| 4090 | 409 | 818 | 1227 | 1636 | 2045 | 2454 | 2863 | 3272 | 3681 |
| 4100 | 410 | 820 | 1230 | 1640 | 2050 | 2460 | 2870 | 3280 | 3690 |
| 4110 | 411 | 822 | 1233 | 1644 | 2055 | 2466 | 2877 | 3288 | 3699 |
| 4120 | 412 | 824 | 1236 | 1648 | 2060 | 2472 | 2884 | 3296 | 3708 |
| 4130 | 413 | 826 | 1239 | 1652 | 2065 | 2478 | 2891 | 3304 | 3717 |
| 4140 | 414 | 828 | 1242 | 1656 | 2070 | 2484 | 2898 | 3312 | 3726 |
| 4150 | 415 | 830 | 1245 | 1660 | 2075 | 2490 | 2905 | 3320 | 3735 |
| 4160 | 416 | 832 | 1248 | 1664 | 2080 | 2496 | 2912 | 3328 | 3744 |
| 4170 | 417 | 834 | 1251 | 1668 | 2085 | 2502 | 2919 | 3336 | 3753 |
| 4180 | 418 | 836 | 1254 | 1672 | 2090 | 2508 | 2926 | 3344 | 3762 |
| 4190 | 419 | 838 | 1257 | 1676 | 2095 | 2514 | 2933 | 3352 | 3771 |
| 4200 | 420 | 840 | 1260 | 1680 | 2100 | 2520 | 2940 | 3360 | 3780 |
| 4210 | 421 | 842 | 1263 | 1684 | 2105 | 2526 | 2947 | 3368 | 3789 |
| 4220 | 422 | 844 | 1266 | 1688 | 2110 | 2532 | 2954 | 3376 | 3798 |
| 4230 | 423 | 846 | 1269 | 1692 | 2115 | 2538 | 2961 | 3384 | 3807 |
| 4240 | 424 | 848 | 1272 | 1696 | 2120 | 2544 | 2968 | 3392 | 3816 |
| 4250 | 425 | 850 | 1275 | 1700 | 2125 | 2550 | 2975 | 3400 | 3825 |
| 4260 | 426 | 852 | 1278 | 1704 | 2130 | 2556 | 2982 | 3408 | 3834 |
| 4270 | 427 | 854 | 1281 | 1708 | 2135 | 2562 | 2989 | 3416 | 3843 |
| 4280 | 428 | 856 | 1284 | 1712 | 2140 | 2568 | 2996 | 3424 | 3852 |
| 4290 | 429 | 858 | 1287 | 1716 | 2145 | 2574 | 3003 | 3432 | 3861 |
| 4300 | 430 | 860 | 1290 | 1720 | 2150 | 2580 | 3010 | 3440 | 3870 |
| 4310 | 431 | 862 | 1293 | 1724 | 2155 | 2586 | 3017 | 3448 | 3879 |
| 4320 | 432 | 864 | 1296 | 1728 | 2160 | 2592 | 3024 | 3456 | 3888 |

*The End of the Table of Parts Proportional.*

# CHAP. VIII.

## *Lineal Arithmetick.* (See Plate *A.*)

THIS fourth Species of Arithmetick is performed by Right Lines, ſo that Magnitude being here conſidered, it may be called *Geometrical Arithmetick.*

2. A Right Line is properly defined to be the neareſt Diſtance between two Points, as the Line (*a c b*, *Fig.* 1.) is the neareſt Diſtance between the Points *a* and *b*.




3. The Diſpoſition of the two principal Lines, by which this kind of Arithmetick is performed, are either in a Right Angle or an Oblique.

4. A Right Angle is when one Line falls Perpendicular on another, as the Line *a b* (*Fig.* 2.) does on *b c*; but when two Lines meet not ſo, they form an Angle called *Oblique*, as the Angle *d e f* (*Fig.* 3.) by both which, Queſtions are anſwer'd.

5. As in Logarithms thoſe Artificial Numbers are not employed about the moſt eaſy and ſelf-evident Rules of *Addition* and *Subſtraction* of Natural Numbers: So here by Lines I ſhall only ſhew how *Multiplication*, *Diviſion*, the *Rule of Proportion*, with *Extraction of Roots*, are performed thereby.

## SECT. II. *Multiplication by Lines, three Ways.*

1. What is the Product of $17\frac{1}{2}$ by $12\frac{1}{2}$? See *Figure* 4. in the folded Sheet (*A.*)

To anſwer this, draw a Line at pleaſure, and ſet off $12\frac{1}{2}$ equal Parts as from *a* to *b*, from the Line of equal Parts.

Then draw a perpendicular Line at pleaſure as *b c*, and ſet off from *b* to *c* $17\frac{1}{2}$.

Laſtly, Draw Lines between each equal Part, and the $\frac{1}{2}$, Parellel to (*a b*) and to (*b c*), then telling the ſeveral little Squares each of which is a Unit, you'll find 204 intire, and 17 and 12 Halves, or $14\frac{1}{2}$ which together, and the $\frac{1}{4}$ at the Angle (*c*) make the Product $218\frac{3}{4}$.

If you would multiply 175 by 125, that is, if you ſuppoſe every Unit in the Line (*a b*) to be 10, then $12\frac{1}{2}$ will be 125, and by

the ſame Rule the Line *b c* will be 175, and by ſuppoſing each little Square 100 (or 10 times 10, the Lines *a b* and *b c* being 10 times as many as before) you'll find 20400 little Squares or Units, and 17 and 12 Fiftys (or 1450) and 25 the 4th of 100 at the Angle (*c*), which added together, makes 21875 = the Product required.

## *A ſecond Way to multiply by Lines.*

But before I proceed therein, it will be neceſſary to ſhew how to draw a Line Parallel to another, and through any Point aſſigned: So to draw a Line through the Point (*a*) (See *Fig.* 5. *Plate A.*) and Parallel to (*m n*), ſet the Foot of your Compaſſes in the Point, and extend the other to the Line *m n*, ſo as to deſcribe the Ark *c c* juſt to touch the Line *m n.*; and with that Diſtance of the Feet ſet one in the Point (*e*) (or thereabout) and deſcribe the Ark (*d d*); then lay a Ruler from that Ark to the Point *a*, and draw the Line *p q*, which is the Anſwer.

*To multiply by* 2 *Lines, including an Oblike Angle as that* (*e d n*), *Fig.* 6.

What is the Product of 3.5 by 2.2? For anſwer, having made an Angle at pleaſure, not too Acute.

*Rule.*] Take a Line of equal Parts, and ſet off 1 from the Angle *d* to 1. Then ſet the Multiplicand from *d* to 3.5 on the other Leg (*d n*). 3*dly*, Set the Multiplier from 1 to 2.2. 4*thly*, Draw the Line 1 to 3.5, produced to about (*m*), for the better drawing the Line (2.2; 7.7) Parallel to (1, *m*); ſo is the diſtance between 3.5 and 7.7 the Anſwer, which meaſured upon the ſame Line of equal Parts, is found 7.7 = the Product.

2. Or if you ſet the Multiplicand from 1 to *b*; and the Multiplier from *d* to *a*, a Line (*b c*) drawn Parallel to (1, *a*) cutteth, the Product (*a c*) 7.7 as before.

3. Or if you ſuppoſe the Factors 10 times as much, the Anſwer will be by the ſame Method 770.

## *A third Way to multiply by Lines.*

This ſhews the *Uſe of a Line of Numbers*, or as 'tis often called *Gunter*'s Line; being a Line of *Logarithms*, as will appear by the *Uſe*, *Addition performing Multiplication*, *Subſtraction*, the *Work of Diviſion*; *Diviſion*, the *Extraction of Roots*, &c.

This

This uſeful Line was the Invention of Mr. *Edmund Gunter*, Profeſſor of *Aſtronomy* at *Greſham-College* about the Year 1620; and it is a Line divided into 10 unequal Parts, and thoſe 10 Parts repeated are called 20, 30, 40, *&c.* when the former Parts are called 1, 2, 3, *&c.*

If the 1ſt 10 are Tens,
If the 1ſt 10 are Hundreds,
The 2d ten Parts are 100's.
The 2d Radius are ſo many (Thouſands.
If you multiply 1 place by 2 places,
The Product is ſo many 10's (as the Figure denotes.

As your own Experience will eaſily diſcover to you. See the making of this Line near the End of *Sect.* 4th following.

To multiply any 2 Numbers, as thoſe in the laſt Caſe 2.2 by 3.5. See *Fig.* 7. *Plate A.*

*Rule.*] Extend the Foot of a Pair of Compaſſes (or other Meaſure) from 1 to 2.2, and that Extent reaches from 3.5 to 7.7;

2*dly*, Or the Extent from 1 to 3.5, reaches from 2.2 to 7.7.

3*dly*, Or if you multiply 35 by 22: In this caſe, what is called 1, 2, *&c.* in the 1ſt part, muſt be ſuppoſed 10, 20, *&c.* and what are 10th parts muſt be called Units, and if both Factors be 10 or more each, the Product is Hundreds. So the Extent as before from 10 to 22, reaches from 35 to 770; for here 35 being multiplied by a Number which is above 10, is ſo many 100's and 10's as the Meaſure extends to; for your Inſpection ſhews, that 35 by 10 is 350, which is 3 places; and if for any Number of Tens, the like: And if the 2d Extent be beyond 10 on the Line, it is ſtill ſo many Hundreds, as 30 by 50 is 1500, *&c.*

And *Note*, That whatever you call the larger Diviſions, the ſmaller in the ſame Radius are a 10th thereof, as is before hinted, and the contrary.

## Sect. III. *Diviſion by Lines, three Ways.*

As I have ſhewed *Multiplication* by three different Lines, and by each two ways; ſo I intend to ſhew how to divide.

*Caſe* 1.] What is the Quotient of $218\frac{3}{4}$ by $17\frac{1}{2}$? See *Fig.* 4.

The Dividend is repreſented by the ſeveral ſmall Squares and Parts in the Superficies *a c*, which if you divide by the ſide (*a* $17\frac{1}{2}$) the Quotient will be one of the ſmaller ſides *a b* or ($17\frac{1}{2}$ *c*).

For having divided a Line (*a*, $17\frac{1}{2}$) into $17\frac{1}{2}$, parts, your Divisor, and made a little Square upon each part, and half of one for the half; if you add to those Squares at Right Angles so many as will make them up $218\frac{3}{4}$, and as many in one Row of Squares as another; you will then find that the Line including a Right Angle with (*a* $17\frac{1}{2}$) will be (*a b*) which will contain $12\frac{1}{2}$ of those small Squares, and is consequently the Quotient, because that Quote multiplied in the Divisor (*a*, $17\frac{1}{2}$) produces the Dividend or Number of small Squares (*a c*)=$218\frac{3}{4}$. And the same may be said of other like Superficies and their Sides, whatever they be for Numbers.

## *A second Way to divide by Lines.*

What is the Quote of 7.7 divided by 3.5? (*Fig.* 6. for I make use of the same Numbers as in *Multiplication*, to prevent Confusion by a Multitude of prickt Lines.)

1*st*. Take 1 from any Line of equal Parts, and set off (*d* 1).

2*dly*. Set the Divisor (*d*, 3.5) off from the same Line of Parts.

3*dly*. Draw the Line (1, 3.5) and produce it to *m*, &c.

4*thly*. Set your Dividend (7.7) from $3\frac{1}{2}$ towards *n*, which extends to 7.7.

5*thly*. Through that Point draw a Line parallel to (1 *m*) and it will pass through the Point 2.2 in the Line (*d e*).

6*thly*. Take the Distance from 1 to 2.2, and apply it to your equal Parts, and you'll find it 2.2=the Quote.

Or if you were to divide 7.7 by 2.2, the Divisor set from *d* to *a*, and the Dividend from *a* to *c*: If a Line be drawn (1 *a*), and another Parallel thereto through the Point *c*, that Line will pass through *b* in the Line *d e*, and the distance (1 *b*) measured on the same Line of equal Parts, will be 3.5=the Quote.

Or if you divide 770 by 35, the Method is the same as the first of these, and 77 by 22 the same as the second or last, *viz.* 35. And the like of other Numbers.

## *A third Way to divide by Lines.*

Suppose by the Line of Numbers, *Fig.* 7. *Plate A.* you would divide 770 by 22. Extend the Compasses from 10 to 22 (calling those 10, *&c.* in the first Radius, which are 1, 2, *&c.* and those 100's in the second, which are 10, 20, 30, *&c.*) and the same

ſame Extent reaches from 770 in the ſecond Radius, backward to 35.

Or to divide 770 by 35, the Extent from 10 to 35, extends backward from 770 to 22. And the like of other Numbers.

## SECT. IV. *The direct Rule of Proportion by Lines.*

If 10 ℔ of Sugar coſt 22 *s.* what will 35 ℔ coſt?

1*ſt.* By the Lines, *Fig.* 6. ſet 10 from the Angle to 1 (which ſuppoſe 10, as if the 1 was not there;) then ſet the 22 *s.* from 1 to 2.2, that is, from 10 to 22. 3*dly*, Set the third Number from the Angle at *d* to 3.5, or to 35. Then draw a Line from 10 to 35=the third Number given, and a Line parallel to that through the Point 2.2 (here 22 will paſs through, or cut the Line *d n* in 7.7, ſo the Diſtance 7.7 to 3.5 (meaſured upon the Line whereby *d* 1 is 10) is 77 *s.*=the value of the 35 ℔ of Sugar. I have done this (as I ſaid before) by the ſame parallel Lines, to prevent Confuſion by a Multiplicity.

2*dly. By the Line of Numbers*, Fig. 7. If 12 C. of Sugar coſt 25 *l.* 4*s.* what will 1 Tun or 20 C. coſt?

Extend the Compaſſes from 12 C. to 25.2, and the ſame Extent reaches from 20 C. to 42 *l.* the Anſwer.

*Q.* 2. If you would know the Intereſt of 90 *l.* for a Year at 6 *per Cent.* extend the Compaſſes from 100 backward to 6; and the ſame Extent reaches from 90 to 5.4 *l.* or *l.* 5 : 8 : =the Anſwer.

3*dly.* To Extract the Square Root, take half the Diſtance between the Square Number given and 1. And 1 third of the Diſtance from a Cube Number to 1 is the Cube Root, taking the third next to 1.

And now I might end this part of Arithmetick, but before I do I ſhall ſhew the making of a Line of Chords of Numbers, and a Line of half Tangents, with ſomething of their Uſe, and that of equal Parts, which will be a Preparative for thoſe who would know how to perform Trigonometry Geometrically.

*The Line of Chords* is 90 Degrees (or equal Parts in the fourth Part of a Circle) ſet off upon a ſtraight Line, as *Fig.* 8. *Plate A.* the Degrees (*e m*) are ſet upon the Diameter (*e c n*) where (*e* 60) of the Circle, is equal to Radius or the Semi-diameter of that Circle, and *e m* equal to *e n*, &c.

Now to meaſure any Angle by this Line, extend the Compaſſes from *e* to 60 in the Line (*e n*), and with that (ſuppoſe in *Fig.* 9.) ſet

ſet one Foot in *h*, and deſcribe the Ark (*r o*), then take the Diſtance (*r o*) in your Compaſſes, and apply it to your Line of Chords, and you'll find it 33 Degrees: That is to ſay, the Line or Leg (*h f*) is diſtant from that *g h* 33 Degrees or equal Parts of the Circle (*d m e*, &c.) as thoſe are transferred upon the Line (*e d*); ſo the Angle (*h f g*) or at *f*, is found 58 Degrees, and the Sum of thoſe two Angles deducted from 180 Leaves the Angle (*f g h*, or) at *g*. And the Sides of this Triangle are meaſured by applying them between the Compaſſes to *the Line of equal Parts* (*y z*), *Fig.* 10.

## *The way to make a Line of Numbers or Logarithms (commonly called* Gunter's *Line.)*

The Line (1, 10, *Plate A.*) is thus made: *Firſt*, Make a Line of equal Parts according to the Length you intend your Line of Numbers, as here (0, 1000) is divided actually into 100 equal Parts, and each Part ſuppoſed to be divided into 10, makes 1000; and if your Line were deſigned to be much longer, you might divide your equal Parts into 10000.

2*dly*. Lay before you a Table of Logarithms, and from that take the Logarithms (omitting the Index) of 2, 3, 4, *&c.* to 10, the largeſt Diviſions of your Line: As, for 2 I find the Logarithm .3010300, but I take Notice only of the three next places to the Point agreeable to my Line of equal Parts, as 301 taken from that Line, and ſet off upon the Line (1, 10,) from 1 to 2 gives the firſt of the firſt 9 unequal Parts of the Line of Numbers.

| | | | | | |
|---|---|---|---|---|---|
| Then I find the Logarithms of | 3 | to be .477 | Which Logarithms taken from the Line of equal Parts, and ſet on that of Numbers from | 1 to 3 | Gives you the other unequal Parts of the Line of Numbers. |
| | 4 | .602 | | 1 to 4 | |
| | 5 | .698 | | 1 to 5 | |
| | 6 | .778 | | 1 to 6 | |
| | 7 | .845 | | 1 to 7 | |
| | 8 | .903 | | 1 to 8 | |
| | 9 | .954 | | 1 to 9 | |

Then for the Sub-diviſions or ſmaller unequal Parts; I proceed in the very ſame manner 'till I have ſet them all off from 1 (the beginning of the Line of Numbers:) As

The

| | | | |
|---|---|---|---|
| The Logarithm: | .041 | | 1 to 1.1 | |
| | .079 | | 1 to 1.2 | |
| | .113 | | 1 to 1.3 | |
| | .146 | Taken from the | 1 to 1.4 | On the Line of |
| | .176 | Line of equal | 1 to 1.5 | Numbers. |
| | .204 | Parts, extends | 1 to 1.6 | |
| | .230 | from | 1 to 1.7 | |
| | .255 | | 1 to 1.8 | |
| | .278 | | 1 to 1.9 | |
| | .322 | | 1 to 2.1 | |
| | .342, *&c.* | | 1 to 2.2, *&c.* | |

I have been more particular in this Affair than has been ſhewed before, becauſe I find moſt Perſons, and ſome even the Inſtrument-makers ignorant, how this very uſeful Line is made originally to any Radius, which depends on the Magnitude of the equal Parts.

But *Note*, That the Line of Numbers conſiſteth of two ſuch Lines, as (1, 10; on *Plate A. Fig.* 10.) next the *Fig.* 7. (or Numbers) and ſo from 10 is numbered 20, 30, *&c.* to 100 on the 2d Part, which is made 1 Line with the firſt, and muſt ſo be made by the Artiſt.

## *To make a Line of equal Parts.*

To make the Line (0, 1000, *Fig.* 10.) imaginarily divided into 1000 Parts, or actually into 100

1*ſt.* I divide it into 5 with my Compaſſes.

2*dly.* I divide each 5th into 2, which makes the whole into 10.

3*dly.* I divide 1 tenth Part into 10 (as 0,100).

4*thly.* I take (0,100) between my Compaſſes, and ſetting 1 Foot in (*i*), the other extends to (*i*), or a 10th of the next 10th, and ſo forward divides the whole Line into 100, and each 100 muſt be ſuppoſed 10, in taking off the 1000th Parts to make the Line of Numbers by, or in other Uſes.

*The Line of half Tangents*, is the Line (*c m*, *Fig.* 8.) made by laying a Ruler to (*d*), and the ſeveral Degrees in the Quadrant (*e m*), and is uſed in meaſuring ſuch ſtraight Lines as paſs through the Center of a Circle, and do repreſent Circles, though they appear Right Lines, as (*c m*), (*c e*), (*c d*), &c. The Uſe of the Line of equal Parts has been fully ſhewn before in meaſuring Right Lines,

and

and there is nothing more falls within the Compaſs of Lineal or Geometrical Arithmetick; which will be of great Uſe to ſuch as intend to Study Geometry, to whom I recommend the reading of *Euclid*, lib. 2. I ſhall therefore proceed to

# CHAP. IX.

## *Inſtrumental Arithmetick.*

INſtrumental Arithmetick is that Part which is performed by Inſtruments, accommodated with Figures or Numbers for that purpoſe. Of this kind, that of *Neper*'s Bones juſtly claims the firſt Conſideration, as being the moſt plain and eaſy Inſtrument for all Operations, whether in Multiplication, Diviſion, or Extraction of Roots.

*A Deſcription of* Neper's *Bones*, (See *Fig.* 1. *Plate B.*) As to their Matter, they are ſometimes made of Ivory, and then called Bones, ſometimes of Box-wood, and then called Virgulæ, *or little Rods*; for as to *their Magnitude and Form*, they may be made as large as you pleaſe: But the common Sort are each Rod about 2.5 Inches long, and .32 Inch broad, and are in Figure what Geometricians call a Parallelopipedon, which is a long Square ſolid, bounded with 6 Superficies, (or like 7, 8 *e c* Dyes joined exactly one to another.)

*The Number of theſe Rods in a Set are* 10, that is, the 9 Digits and Cypher, beſides the Index-bone which is fixed to the Box, and hath only one Face divided into 9 equal Parts, whereon the 9 Digits are placed as you ſee *Fig.* 1. *Plate B.* But the other 10 have 4 Faces each, whereon are placed the 9 Digits at the top, and an Arithmetical Progreſſion of thoſe Digits added as 2, 4, 6; or 3, 9, 12, *&c.* And whatever the firſt Figure of any Rod is, that of the oppoſite Side is the Complement that makes it up 9. And the 10 times 4 Faces or Sides being 40, contain 4 of each of the 9 Digits, and their Series's Arithmetical, and 4 more Faces are of Cyphers.

### *How to add the Figures on the Rods.* See *Fig.* 12. *Plate A.*

To reduce any Line of Figures on the Rods which ſtand againſt any Digit on the Index-face to a true legible Numbers, you muſt

muſt begin at the Right-hand: Thus to add the Digits ſtanding againſt 6 in the ſet, *Fig.* 1. *Plate B.* You ſee there is nothing to do but to add the two Digits together, which ſtand between the ſame Slant or Diagonal Lines, as 4 is 4, then 5 and 8 is 13; 1 and 4 is 5, and 2 is 7; 4 and 6 is 10; 1 and 3 is 4; 3 and 4 is 7; 2 and 8 is 10; 1 and 1 is 2, and 2 is 4; 1 and 6 is 7: So that ſix times 123456789, is found 740740734.

## SECT. II. *To multiply by* Neper's *Rods or Bones.*

*Queſt.* 1. What is the Product of 34729 by 23795; See *Fig.* 13. *Plate A.*

Firſt tabulate the Multiplicand, (or either Factor, but the larger is moſt uſual;) that is, take ſo many Rods out of your Sett, that the firſt Figures (or thoſe at the upper End of each Rod) will compoſe your Multiplicand in order, as you ſee done, *Fig.* 13. of *Plate A.*

Then looking for the Figures of your Multiplier ſeverally in the Index-rod next the Left-hand, you find againſt each its reſpective Product in the Multiplicand, as *Fig.* 13. aforeſaid.

*Queſt.* 2. What is the Product of 3040999 by 2341?

In this Example there being 3 Nines and 2 Cyphers on the oppoſite Sides, whereof are 2 Nines makes 5 Nines, which is one more than is in a Sett of theſe Rods; therefore, if you have not another Sett, you may eaſily ſupply the Defect by obſerving the Product of each Figure (as you go on) in the Multiplier; and thoſe other Nines, and ſo making one of thoſe extraordinary: Thus in Multiplying by the 4 there (as *Fig.* 14.) and if there were another Rod for the third Nine in the Multiplier, I can eaſily ſee there would be more (as *Fig.* 15. *Plate A.*) which therefore, I ſuppoſe, and ſo put down 6, and then two Nines inſtead of the one Nine in the Rods; ſo when I multiply by the 3, I find in the Tabulet toward the Right-hand againſt 3, (ſee *Fig.* 16. *Plate A.*) But I know if I had another Rod of the

3040999=the Multiplicand.

Multiply'd by 1=3040999
by 4=12163996
by 3=9122997
by 2=6081998

711897865 9=Product.

ſame Digit 9, there would have been another, as *Fig.* 17. *Plate A.* which ſuppoſing there, it would make 2997, that is, a 9 more than I find in the Tabulet, and ſo the reſt.

## SECT. III. *Diviſion by* Neper's *Bones or Rods.*

What is the Quote of 23734.691084 divided by 253.24?

An Example of the whole Proceſs, and of what help you have in dividing from the Rods. See *Figure* 18. *Plate A.*

Hence it appears, that the Rods are only uſeful in exhibiting the Products of the Diviſor by each Figure put in the Quotient, which knowing, you proceed in all other Reſpects as in Diviſion of Integers, *Sect.* 5. *Chap.* 1. *&c*, firſt Tabulating the Diviſor, and then whatſoever the Quote be, you have each Figure multiply'd in the Diviſor, although I have only placed in the Index-rod the Figures that fall in the Quote above, and that in the Order they happen therein, for the more orderly ſhewing the Nature of the Work, and how it agrees with common Diviſion; ſo that in doing this Queſtion above, you finding that 227916 is the next Number on the Rods to, and leſs than 237346 (the firſt Part of the Dividend) looking againſt that in the Index-rod you find 9; therefore put 9 in the Quote, and deduct as in the Example, and the Remainder (with 9 brought down to it) is 94309; which found in the Rods, or what is next leſs, I obſerve 3 to ſtand againſt it, which I put in the Quote, and ſo proceed with the reſt as in the Example, which is ſo plain, that it will ſerve for all. I have put the Figures of the Quote againſt their reſpective Dividuals, which added and placed as above in its proper place, is the general Quotient required 93.7241.

## SECT. IV. *The Uſe of* Neper's *Bones or Rods in Extraction of the Square Root.*

What is the Square Root of 54756? (See *Fig.* 3. *Plate B.*)

*Rule.*] Having pointed the Square Number given, as taught in intire Numbers; by the three Points, it appears, there will be Hundreds, Tens, and Units in the Root.

*2dly.* Having put the Rod of the Square of the 9 Digits next towards the Right-hand of the Index-rod, you'll find that againſt 4, the

| *Square.* | *Sq. Root.* |
|---|---|
| 54756 | (234 |
| 4 | |
| 147 | |
| 129 | |
| 1856 | |
| 1856 | |
| 0 Reſts. | |

next

next Square to 5 (the firſt pointed towards the Left-hand) there ſtands its Root upon the ſaid fixed Rod, *viz.* 2; therefore put 2 in the Root, and deduct the ſaid Square 4 from 5, and there reſts 1.

3*dly*. Having brought as uſual the next Branch down, it makes 147. Then take a Rod, whoſe uppermoſt Number is the double of the Figure laſt put in that Root (here 4;) and place the Rod of Squares to the Right-hand, that (as you ſee *Fig.* 3. *Plate B.* in the middle Part over 30) then look in the two Rods next the Right-hand for 147, and the next leſs is 129, right againſt which on the Index-rod ſtands 3; therefore put 3 in the Root, and deduct 129 from 147, and the Remainder is 18.

4*thly*. Having placed the next two Figures (or laſt Binary of the Square given) toward the Right-hand of 18, they make 1856. Then place two Rods (between the Rods of the Index, and that of Squares) whoſe uppermoſt Digits are the double of the 2 in the Root (as you ſee done in *Fig.* 3. aforeſaid next the Right-hand) and look in the 3 Rods next that Hand for your 1856, which you find juſt, and towards the Left-hand thereof, on the Index-rod ſtands 4; therefore put 4 in the Root, and deducting, you ſee nothing remains, and that 234 is the Root required. *This I have made much more plain than has been done before, by* Fig. 3. *which ſhews the Poſition of the Rods upon each Digit put in the Root, and doubt not but it is ſufficiently Intelligible.*

But *Note*, That if the double of the Figure put in the Root be more than 9, you muſt put two Rods next the Index, which begin with the Digits of that double; and three Rods put when the double of two places is above 99. Thus, if 9 is the firſt in the Root, the double is 18; ſo I put two Rods between the Index and the Rod of Squares, beginning with 1 and 8. So alſo, if 99 were the two firſt in the Root, the double is 198; I put therefore three Rods, beginning with 1, 9, and 8, between the Index-rod and that of Squares, *&c.*

For a farther Exemplification; extract the Square Root by the Bones of 97535376. See the Work in the Margin.

1*ſt*. Here you find 81 the next leſs Square to 97, and againſt it is 9 in the fixed Rod; ſo you put 9 in the Root, and deducting 81, there reſts 16, and the next Branch or Period is 1653.

2*dly*. Doubling 9 is 18, which two Rods placed next that fixed, and that of Squares next the Right-hand; the top Digits will ſtand in this Order, ſee *Fig*. 19. *Plate A*. and in the Progreſſion lower you'll find againſt 1504, the next leſs to 1653, ſtands 8, which put in the Root, and deduct 1504, there reſts 149; which with the next Period makes 14953.

| | *Square.* | *Square* |
|---|---|---|
| Example 2= | 97535376 | (9876=Root |
| | 81 | |
| | 1653 | |
| | 1504 | |
| | 14953 | |
| | 13769 | |
| | 118476 | |
| | 118476 | |
| | 0 | |

3*dly*. Doubling 98 is 196; which three Rods placed next that fixed, and that of Squares towards the Right, the firſt Figures on the tabulated Rods will ſtand thus, ſee *Fig*. 20. *Plate A*. And in the Progreſſion thereof lower you'll find 13769; againſt which is 7, which put in the Root, and the 13769 deducted from 14953, there Reſts 1184; to which bringing down the next Branch, makes 118476.

4*thly*. Doubling 987 in the Root, makes 1974; therefore taking 6 out of the laſt Tablet, and putting the Rods beginning with 7 and 4, the top of the ſix Rods will ſtand thus, ſee *Fig*. 21. And in the Progreſſion lower you'll find on the 5 Rods next the Right-hand 118476, againſt which towards the Left-hand upon the fixed Rod is 6: So there Reſts (0), and the Operation is finiſhed.

## SECT. V. *The Uſe of* Neper's *Bones in the Extraction of the Cube Root.*

The Rods make this (which is reckoned the moſt difficult Rule) very eaſy, ſo ſoon as you know how to place your Virgulæ, which I ſhall therefore illuſtrate by an Example of their Poſition at every Figure put in the Root, as I have done in the preceding Parts of Arithmetick by the Rods.

The

The Cube Number given muſt be prepared for Extraction by pointing over the Units, and every third afterwards, as is taught in extracting Roots of Vulgar intire Numbers, *&c.* for Example: What is the Cube Root of 182284628? See *Figures* 22, 23, 24. *Plate A.*

*1ſt.* The firſt Figure in the Root is found by placing your Cube-rod next the Index-rod; for looking on the former for 182 (the firſt Point next the left) you find the next leſs to be 125.

```
182284628 (567
125
 57
```

And right againſt that 125, ſtands 5 on the fixed Rod, which 5 I therefore put in the Root, and ſubſtracting 125 from 182, there Reſts 57. To which bringing down the next Point or Branch, it makes 57284.

```
45216
  540
50616
 6668628
```

Then for a Diviſor to divide the 57284: Take treble the Square of the 5 (in the Root) which is 75, ſee *Fig.* 23. *Plate A.* And having tabulated that, and the Rod of Cubes, as you ſee, I find on thoſe three Rods 45216 the next leſs to be taken, againſt which in the Index is 6, which I put in the Root: To which 45216 I add 540 (which is the Square of 6 in the Root multiplied in three times the 5 there) and the Sum is 50616; which ſubſtracted from the Dividend 57284, there Reſts 6668, to which I bring down 628=the third Branch.

```
9408)  6668628
       6585943
          8232
       6668263
           365
```

And it makes the laſt Dividend 6668628; and treble the Square of the Root (56) make 9408 (a Diviſor) which Tabulated with the Rod of Cubes, as you ſee *Fig.* 24. *Plate A.* I look upon the Rods, and find 6585943 to be the next leſs Number than the 6668628;

6668628; and againſt it is 7 in the Index, which 7 I put in the Root: Then for the 8232 to be added to the Number next above it, I multiply 3 times 56 (in the Root) by the Square of 7 there; and the Sum is 6668263, which I take from the laſt Dividend 6668628, and the Remainder is 365; to which, if you deſire Decimal Places, you may put three Cyphers towards the Right, and ſo proceed as before. I have made this Example ſo demonſtrative, that I ſhall need to give no more, for by the very ſame Method you may do any other.

*Note*, That in the Diviſion (by 75) the next of all on the Rods, and leſs than the Dividend 57284 is 52843, againſt which ſtands 7 on the Index, but then had you put 7 in the Root inſtead of 6, you would have found a Number too great to be deducted from the ſaid Dividend or Reſolvend 57284; and therefore I take ſtill the next leſs Number which ſtands againſt 6.

2*d Note*, That the 540 and the 8232 are placed 1 Degree towards the Left-hand, becauſe they would fall ſo if put under the Tens Places of the Cube of the laſt Figure put in the Root; as is done in the common way of Extraction.

*Note alſo*, That if you have not the Rods of Squares and Cubes, you may do without, either by placing in the Room a Rod, whoſe firſt Number is the ſame with that which falls in the Root (which you may know by comparing the Figures on the Rods toward the Left-hand with thoſe of the Dividend) or by making them (for preſent Uſe) of Paper, according to the many Examples above given of them, till you get the Rods of Squares and Cubes.

## SECT. VI. *Notation and Numeration of the Diagonal Circular Inſtrument*, (See *Plate* B.)

This Inſtrument was by me contrived chiefly for Reduction of Money, Weight, or Meaſure into Decimals; or for finding with Eaſe and Speed the Value of thoſe Fractions, and for Multiplication, Diviſion, and Evolution. It is marked on *Plate* B. with *Fig.* 5. and conſiſteth of ſeven Concentrick Circles (beſides the two outermoſt, which are Lines of Numbers) as

1. The outward or largeſt Circle (ſave two) is of equal Parts, to which all the reſt that are leſſer Circles refer in their Uſe; it is firſt divided into 10 Parts, each of which being ſubdivided into 10, makes 100 Parts; and each 100 Part again divided into 10 by the

the Diagonal Line (1000, *a*) &c. divides the whole Circle into 1000 equal Parts. So that for Example: To find in that Circle 767 thouſandth Parts, you have 760 in the Arch of that Circle, and 7 in the Diagonal Line, telling the Parts upwards from 760.

2. The ſecond Circle is of Engliſh Coin, the whole repreſenting 1 *l.* divided into 20 *s.* each Shilling in 12 Pence, and each Penny by the Diagonal into 4 Farthings. So that to ſet off 13 *s.* 7 *d.* $\frac{1}{4}$ *q.* look for 13 *s.* 7 *d.* in the Ambit of the Circle, and then from 7 *d.* telling upwards, you ſee 1 Farthing in the Diagonal.

3. The next inner Circle (which is the third from thoſe of Numbers) is of Beer Meaſure, being 1 Barrel divided into 4 Firkins, each Firkin into 9 Gallons, and each Gallon into Quarts and Pints as you ſee.

4. The next inward is a Barrel of Ale divided into 4 Firkins, thoſe into each 8 Gallons, and thoſe again into Quarts and Pints.

5. The fifth Circle inward from thoſe of Logarithms is the Pound Averdupois divided into 16 Ounces, and thoſe each Ounce into 16 Drams.

6. The next inward Circle is the Hundred Averdupois divided into four Quarters, and each Quarter into 28 ℔. And

7. The innermoſt Circle is Foot Meaſure, being 1 Foot divided into 12 Inches, and each Inch into 4 Quarters.

And all theſe Diviſions and Subdiviſions are actually done, and not imagined only as ſome Scales are, which are of little or no Uſe. And any Quantity in the ſeven inner Circles is eaſily pointed to by the ſame Rule as given for the two firſt; from thoſe of Numbers, only in theſe there are no Diagonals, there being no need of them, the Parts being ſufficiently expreſſed without.

8. The two outermoſt Circles are Lines of Numbers, whoſe Uſes follow after 10th.

## *Reduction of Decimals by the Circular Inſtrument.*

*Queſt.* 1.] What Decimal of a Pound is 13 *s.* 6 *d.* $\frac{1}{4}$ *q*?

*Rule.*] Lay a Ruler from the Center to 13 *s.* 6 *d.* $\frac{1}{4}$ *q.* in the ſecond Circle, and that will cut in the Diagonal Circle .676, which is the Anſwer.

*Queſt.* 2.] What is the Value of .876543 of a Pound Sterling?

*Rule.*] Lay a Ruler to the Center, and to 876, and half in the Diagonal Circle (or equal Parts) and you'll find it to cut in the Circle of Money 17 *s.* 6 *d.* $\frac{1}{2}$ *q.*

*Queſt.* 3.]

*Queſt.* 3.] What Decimal of a Barrel of Beer is 2 Firkins, 7 Gallons, 3 Quarts, and a Pint?

*Rule.*] Lay the Ruler to the Center, and the Quantity given in the third Circle from the Line of Numbers, and it will ſhew in the firſt Circle the Decimal required to be .7187, or apparently more than .718 and $\frac{1}{2}$.

*Queſt.* 4.] What is the Value of .876543 of a Barrèl of Ale?

*Rule.*] Lay a Ruler to the Center, and the Decimal given in the Diagonal Circle (I mean the four firſt Figures next the Point) and it will cut in the fourth Circle 3 Firkins, 4 Gallons, and about half a Pint.

*Queſt.* 5.] In 3 Firkins, 5 Gallons, 3 Quarts of Ale, how much Beer?

*Rule.*] Lay a Ruler from the Center to the Quantity given in the Circle of Ale Meaſure, and it will give in the Circle of Beer Meaſure 3 Firkins, 6 Gallons, 2 Quarts Beer. And in like manner Beer is reduced to Ale Meaſure.

*Queſt.* 6.] What Decimal of a Pound is 13 Ounces 11 Drams?

*Rule.*] Lay a Ruler from the Center to 13 Ounces and 11 Drams, and it will cut in the Diagonal Circle .855.

*Queſt.* 7.] What is the Value of .876543 of a Pound Averdupois?

*Rule.*] By laying a Ruler to the Center, and the Decimal given, it will ſhew the Value in the fifth Circle from thoſe of Numbers to be 14 Ounces and near half a Dram.

*Queſt.* 8.] What Decimal of a hundred Weight is 2 Quarters 21 Pound?

*Rule.*] Lay a Ruler to the Center, and the given Quantity in the ſixth Circle, and it will cut in the Diagonal Circle .687, or .6875.

*Queſt.* 9.] What is the Value of .876543 of an hundred Weight?

*Rule.*] Laying a Ruler as in the 7th Queſtion, it will cut in the 6th Circle 3 Quarters 14 Pound.

*Queſt.* 10.] What Decimal of a Foot is 10 Inches 3 Quarters?

*Rule.*] Laying a Ruler to the Center, and the Quantity given in the 7th Circle, it will cut in the equal Parts .8958, or viſibly .895 and more.

And a Line of Numbers being delineated without the largeſt or diagonal Circle (as by Rules given for drawing that Line, ſetting the Parts off from the Center) and another to turn round within it; you may by them do moſt Queſtions in Multiplication, Diviſion, and Extraction of Roots very accurately, as by Rules in *Chap.* 8. Or thus; Suppoſe I would do the ſecond Queſtion in

*Sect.* 4.

*Sect.* 4. of *Chap.* 8. i. e. If 12 *C.* of Sugar coſt *l* 25 : 4 :— what will 1 Tun coſt?

Turn 12 in the inner Circle, to 20 in the outer Circle; and then againſt 25.2 in the inner, will ſtand 42 in the outer.

Or in Caſe you have only a Cut of this Inſtrument paſted on a ſtrong Board that will not warp; you may do theſe Queſtions by the Compaſſes, and one of the Lines (or the outmoſt Circle) as if 20 require 30; what will 50 require? In the ſecond half Circle, the Extent of a Pair of Compaſſes from 20 to 30, will extend from 50 to 75, the fourth Proportional or Anſwer.

The compound Intereſt of any Sum for Years, is found by the two Lines of Numbers: As that of *l.*650 for five Years at 5 *per Cent.* the fifth Power of 1.05 is the Amount of *l.* 1 for five Years, which multiplied by *l.*650 gives *l.* 829 : 11 :—the Anſwer: For

Turn 1 in the inner Circle, to 1.05 in the outer Circle, then againſt { 1.05, 1.1, 1.16, 1.22 } In the inner Circle ſtands { 1.1, 1.16, 1.22, 1.28 } In the outer Circles. = the Amount of 1 *l.* for 5 Years.

And turning 1 in the inner Circle, to 1.28 in the innermoſt Circle; againſt 650 (the Principal) in the inner Circle, you will find 829½ in the outer Circle, the Amount required.

Many other Queſtions might be propoſed, and Uſes ſhewn of this Inſtrument; but I leave it to the Reader to find out, of his own accord, to exerciſe his Parts.

*The End of Inſtrumental Arithmetick.*

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# CHAP. X.

## *Algebraical Arithmetick.*

I. ALGEBRA being a Word derived from the *Arabick Algiabr*, makes it probable, that the *Arabs* were the moſt ancient Profeſſors, and great Proficients in this Art. Although ſome have aſcribed the firſt Invention to the *Chineſe*, *Indians*, or *Perſians*; ſome to the *Græcians*, and that it was firſt propagated by *Plato*, though it was kept a Secret from the Principle, that

the Divulging a Truth was profaning of it. The firſt Treatiſe of this Art is ſaid to be wrote by *Diophantus*, extant in *Latin* and *Greek*; but the firſt *European* Author, was moſt probably, *Lucas de Burgo*, a *Fryar*, who publiſh'd a Book of this Subject in *Italian*, printed at *Venice*, *Anno* 1494. yet *Scipio*, *Cardan*, and others about their time firſt ſhewed the Solution of Cubical Equations. The famous *Vieta* was the Inventor of *Specious Algebra* about 1590. which was improved by *Oughtred*; and the Methods of ſolving Quadraticks, by Changing the ſecond Term, and by Compleating of the Square, were with many other excellent Improvements, introduced by the celebrated *Harriot*, who was contemporary with *Oughtred*, and flouriſhed about the Year 1630. And ſo much may ſuffice for the Invention and Improvement of this great Art.

II. Some define *Algebra* to be, *The Art of Reſtitution and Compariſon.* It is ſometimes called, *The Analitick, or the Art of Enquiring into the fundamental Nature and Reaſon of Propoſitions, and of compoſing and ſolving Equations, and diſcovering Canons and Theorems accordingly.*

III. *Algebra* is either Numeral or Literal; the former, when a Queſtion is ſolved by the Numbers themſelves; and Literal, when done by Symbols, Letters, or Species, repreſenting the Numbers in a Queſtion.

What I have propoſed for this firſt Section, I ſhall give the Reader in this Method.

1. Shew him the Characters and Signs uſed in *Algebra*.
2. Say what is neceſſary of the Symbols or Letters, repreſenting Numbers in this Caſe.
3. Of the Notation, Numeration, or way of reading Compound *&c* Quantity.
4. Of the Definitions and Explanations of Terms peculiar to, and uſed in *Algebra*.

IV. The Learned *Algebriſts*, for the more commodious and expeditious Way of Proceſs in ſolving Problems, have very wiſely agreed upon *certain Characters*, whereby Quantities, or the Powers of them are expreſſed and connected, and the Mind of the Artiſt is briefly and methodically explained, which would appear inconveniently confuſed, ſhould Words at Length be uſed: Thus,

| Signs or Characters. | Their Signification. | Literal Examples; as, | Numeral Examples: or, |
|---|---|---|---|
| + | More. | $b+a$. | $3+2=5$. |
| − | Lefs. | $b-a$. | $3-2=1$. |
| = | Equal to | $b+a=c$. | $3+2=5$. |
| ÷ | Tranſpoſed to the Right. | $a+b=c$, is $a=c-b$. | $2+3=5$, is $2=5-3$. |
| × | Multiply'd by | $b\times a=ba$. | $3\times 2=6$. |
| ÷ | Tranſpoſed to the Left. | $b=c-a$, is $a+b=c$. | $3=5-2$; is $2+3=5$. |
| ÷ | Divided by | $b\div a=\frac{b}{a}$ | $3\div 2=\frac{3}{2}=1.5$. |
| ∽ | The Difference between | $b \backsim a$ | $3 \backsim 2=1$. |
| ∠ (or ⊐) | Leſſer than | $a \angle b$. | $2 \sqsupset 3$. |
| ∀ (or ⊏) | Greater than | $b \forall a$. | $3 \sqsubset 2$. |
| ∴ ∴ | *So is* (when in the middle of 4 Geometrical Proportionals.) | $b.\ a::c.\ \frac{ac}{b}$ | $3.\ 2::5.\ \frac{10}{3}$. |
| ÷∴ | Continual Geometrical Proportionals. | $a.\ b.\ d.\ h.\ r.$ ÷∴ | 1. 3. 9. 27. 81 ÷∴ |
| √ | The Square Root of | $\sqrt{bb}=b$. | $\sqrt{9}=3$. |
| $\sqrt[3]{}$ | The Cube Root of | $\sqrt[3]{aaa}=a$. | $\sqrt[3]{8}=2$. |
| $\sqrt[4]{}$ | The Biquadrat Root of | $\sqrt[4]{aaaa}=a$. | $\sqrt[4]{16}=2$. |
| $\sqrt[5]{}$ | The Root of the 5th Power, or ſurſolid Root. | $\sqrt[5]{bbbbb}=b$. | $\sqrt[5]{243}=3$. for |
| $aa$ (or $a^2$) | The Square of a | $a^2=aa$. | $2^2=4$. |
| $aaa$ (or $a^3$) | The Cube of a | $a^3=aaa$. | $2^3=8$. |
| $aaaa$ (or $a^4$) | The Biquadrat of a | $a^4=aaaa$. | $2^4=16$. |
| $aaaaa$ (or $a^5$) | The 5th Power, or ſurſolid of a | $a^5=aaaaa$. | $2^5=32$. |

*Other Characters leſs uſed are*; $a\sqrt{b}$ uſed in Surds, ſignifies $a$ is Multiplied in the Square Root of $b$.

(∟) Right Angle. (▭) Square. (∠) Angle. (△) Triangle. (⏥) Equilateral. (⊙) Involution, obviating that ſuch a Quantity is ſquared; and ($\omega$) Evolution (or that the Root is Extracted.)

V. *The Species or Symbols* uſed in *Algebra*, are Letters, which are placed in an Operation to repreſent the Numbers given in any Queſtion; and theſe are commonly Conſonants, as *b*, *c*, *d*, &c. But the Letters put in an Operation by *Algebra* to ſtand for the unknown Number (or thoſe ſought for) are uſually Vowels. And I ſhall obſerve this way in the following Work, but with thoſe Limitations which I think are very neceſſary.

1. I ſhall put ſuch Conſonants to ſtand for the known Numbers as are moſt expreſſive of the Things to which thoſe Numbers relate; and the like for unknown, or if there is but one unknown or Number ſought in a Queſtion, I ſhall always put (*u*) for it.

2. I ſhall not as has been uſual put the Letters in a Product promiſcuouſly, and without Order, but ſhall place them as they have Order or Priority in the Alphabet; whereby one and the ſame Quantity is eaſier ſeen in contracting manifold compound Quantities, and ſeveral other good Uſes may be made thereof.

VI. *Of the Notation, Numeration*, or way of reading Algebraical Quantities, it may be obſerved from the foregoing Signs.

1. Such as are connected with the Affirmative Sign +, as + $a+b+c+d=p$, are to be Read, a more *b*, more *c*, more *d*, equal to *p*.

*Note*, Which *p*, though it hath no Sign before, is (and the like is always to be obſerved) ſuppoſed to have +, as being always Affirmative.

2. Such Quantities as are connected with the Negative Sign (−) as $a-b-c$, are to be Read thus; *a*, leſs *b*, leſs *c*. And as in the firſt Example, the Numbers ſignified by *b*, *c*, and *d*, are to − be added together to make the Number repreſented by *p*: So in this ſecond Example, either the Sum of the Numbers repreſented by *b* and *c*, are to be deducted from *a*, or elſe *b* and *c* taken from *a* ſinglely, *b* from *a*, and then *c* from the Remainder.

Or ſuppoſe $b+c-d=$ ∽ *p*; that is, the Number repreſented by *d*, being deducted from the Sum of thoſe repreſented by *b* and *c*; the Remainder is equal to the Difference between the Numbers S repreſented by *p*, and that unknown Number by *a*.

*Note*, That ſuch Quantities as have no Numbers ſtanding to the Left-hand, are ſuppoſed to have a Unit or 1; thus *b* is 1*b*, *c* is 1*c*, &c.

In $c-\overline{b+d}-p$ ∽ *a*: Here the Difference between *p* and *a*, is to be deducted from the Remainder, when *b* more *d* is taken from *c*; for 'tis Read, *c* leſs *b* more (*d*) leſs the Difference between *p* and *a*; for

for (*a*) being unknown, it is ſo likewiſe whether the Exceſs lieth in *a* or *p*, therefore the (ᔕ) to repreſent the Difference is neceſſary.

3. Such Quantities as have the Sign × between them, are commonly expreſſed without it, and the Letters put as in a Word; thus $a \times b$, is, *a* multiplied in *b*, or *ab*; $c \times b \times p$, is *c* multiplied in *b*, and that Product in *p*, but is moſtly put thus (*bcp*); and $b\times$ in *b* is *bb*, $c \times c \times d \times d$ is *ccdd*, or *c* ſquared in *d* ſquared. ×

Or $a$ ᔕ $c + b \times p \times r$ is $a$ ᔕ $c + bpr$, that is the Product of *b*, *p*, and *r* added to the Difference between *a* and *c*.

4. So all Quantities having the Sign (÷) may be, and are moſt commonly expreſſed by way of Fraction, the Quantity to be divided being placed over the other thus, $a - b \div c + p$ will ſtand $\frac{a-b}{c+p}$; *i. e. a* leſs (*b*,) divided by *c* more *p*. ÷

5. Surd Quantities are expreſſed with their proper Marks before them, as $\sqrt{g-a}$ is the Square Root of the Remainder, when *a* is deducted from *g*. So alſo $\sqrt[3]{p+q-r}$ is the Cube Root of the Remainder, when *r* is deducted from the Sum of *p* and *q*. Laſtly, $\sqrt[4]{b - c + \frac{d}{q}}$, which is the Biquadrat Root of the Sum when (*c*) is taken from *b*, and that Remainder added to the Quote of *d*, divided by *q*. √ $\sqrt[3]{}$ $\sqrt[4]{}$

*But ſome Examples admit of a double Entendre.*

As under the ſecond of this 6th Head, wherein $c - \overline{b+d}$: If it were not for the Line over *b* and *d*, inſtead of deducting $b+d$ from *c*, *b* would only be ſhewed to be deducted from the Sum of *c* and *d*.

So alſo in the 4th above, if there were a Line over $b \div c + p$, it would ſtand thus $a - \overline{b \div c + p}$, which without the Line (ſuppoſing $a = 40$, $b = 20$, $c = 6$, and $p = 4$) would be but 2; whereas with the Line 'tis = 38, for

| | | |
|---|---|---|
| $a - b = 20$ | is = 2 | But (with the Line) $a = 40$ |
| And that divided by $c + p = 10$ | | Leſs (*b*) = 20<br>Divided by $c + p$ = 2<br>Which deducted is = 38. |

ſo there is 36 difference. According to the 5th Example above, there is nothing more frequent in ſolving Quadratick Equations, than to have Occaſion in the Canon or Anſwer to uſe Lines to diſtinguiſh

ſtinguiſh what Quantities are to have their Roots extracted, and which are not, as $u=\sqrt{bc+p}-d$; that is, ($u$) is equal to the Remainder, when ($d$) is deducted from the Square Root of the Sum of ($p$), and ($b$) multiplied in ($c$), as if $b$ be 16.

The Numbers to which Equation diſcover, from the Letters, the Value of ($u$), and ſo you muſt unfold any Canon of Species.

$c=3$
Product $=48$
More $p=16$
Sum $=64$
Square Root of which is 8
Leſs $d=3$, ſo ($u=5$.)

But had not the Line been drawn over $bc+p$, it would have ſhewed that the Root is to be extracted of $bc+p-d$, which would give too much.

7. In the new Notation $\frac{1}{a}$ is often expreſſed $a^{-1}$; $\frac{1}{a^2}$ is $a^{-2}$; $\frac{1}{a^3}$ is $a^{-3}$; $\frac{1}{a^4}=a^{-4}$, &c. And is Read one divided by $a$; 1 divided by $a$ Squared; 1 by $a$ Cubed, &c.

Alſo $a^7+b^8$, &c. is the 7th Power of $a$, plus, the 8th Power of $b$. And $\frac{1}{a+b}$ is wrote $\overline{a+b}^{-1}$, &c. $a^7+b^7\times\frac{1}{a+b}$ is $a^7+b^7\times\overline{a+b}^{-1}$ that is, 1 divided by $a$ more $b$; and the 7th Power of $a$ more the 7th Power of $b$ multiplied in 1 divided by $a$ more $b$.

Likewiſe $\frac{1}{a^2}b$, $\frac{1}{a^3}b^2$, &c. is in the new Notation $\overline{a}^{-2}b$; $\overline{a}^{-3}b^2$, &c. that is, 1 divided by the Square of $a$ multiplied in $b$; one divided by the Cube of $a$ multiplied in $b$ ſquared.

And $\frac{a^5+b^5}{a+b}=a^5+b^5\times\frac{1}{a+b}=a^5+b^5\times\overline{a+b}^{-1}$; *i. e.* the ſurſolid of $a$, more that of $b$, divided by $a$ more $b$; is expreſſed alſo $a^5+b^5\times\frac{1}{a+b}$. Or rather $a^5+b^5\times\overline{a+b}^{-1}$.

VII. An Alphabetical Explanation and Definition of moſt Terms uſed in *Algebra*, which the Reader may have Recourſe to as he finds Occaſion; for having well underſtood what is above in this Section, he may paſs this Head, and proceed to *Addition*, &c.

*Abſolute Quantity*, or *Abſolute Number*.] Is ſuch a known one as poſſeſſeth one Side of an Equation, as $u+cu=b$, here $b$ is the Abſolute Quantity, whoſe Value is known, and is free from any Dependance or Power of any other. Or it is one marked with the Sign +, contrary to Negative marked −.

*Affirmative*

*Affirmative Quantities.*] Such as have either no Sign before them, or are marked with +; ſo $a+b$ are both Affirmative Quantities, as by the laſt.

*Algoriſm.*] Is the Operations in the ſeveral Parts of *Algebra* practically handled.

*Algorithm.*] The Parts of ſingle Arithmetick, as *Numeration*, *Addition*, *Subſtraction*, &c.

*Arithmetick of Infinites.*] The ſame as infinite Series, or converging Series.

*Adfected Equations.*] When in a Quadratick Equation, (which ſee *Sect.* 10. in this Chap.) the unknown Quantity is in 1 Term, and its Square in another.

*Analogies.*] The ſame as Proportions, as the Analogy of theſe four Quantities, *a. b* : : *c. d*, &c. and becauſe the Rectangle of the two Extreams is equal to that of the means; therefore $ad=bc$, the Analogy being converted to an Equation.

*Approximation.*] See *Converging Series.*

*Arithmetick* of *Negatives.*] See *Negative* under *N.*

*Binomial Quantities* or *Roots.*] Thoſe conſiſting of two Names, Terms, or Members, connected together by the Sign +, as $u+d$, &c.

*Coefficients.*] Thoſe Numbers or known Quantities in a Quadratick, *&c.* Equation, which are (in the ſecond Term commonly) multiplied in the Root of the firſt or unknown Quantity, as $uu+2bu$, here $2b$ is the Coefficient: And *Note*, That where there is no Number nor Quantity multiplied in the Root in the ſecond Term, then 1 or a Unit is the Coefficient, as $uu+u$ is $=uu+1u$. See *Compleating the Square.*

*Compound Quantities.*] Are ſuch as are either the Product, Sum, or Difference of two or more ſingle Quantities: Thus $(up)$ is a Rectangle compounded of $u$ and $p$, $u+d$, or $u-d$, the Sum or Difference of thoſe two Quantities $u$ and $d$.

*Compleating a Square.*] This is to Square half the Coefficient: As ſuppoſe two Members of a Square $uu+bu$ be given; to find what Quantity muſt be put to compleat the Square, is to ſquare half the Coefficient or $\frac{b}{2}$, which Square is $\frac{bb}{4}$, ſo that $uu+bu+\frac{bb}{4}$ is a compleat Square: For Proof of which you may multiply $u+\frac{b}{2}$ (which is the Root of that Square) in itſelf. And if this were a Quadratick Equation, this Square of half the Coefficient muſt

muſt be added to both Parts of the Equation, as admit $uu+bu=c$; half the Coefficient being ſquared and added, will make it $uu+bu+\frac{bb}{4}=c+\frac{bb}{4}$.

*Compoſing an Equation.*] Is the way of proceeding (to reſolve Problems) 'till we make ſome Quantities in the Queſtion equal to ſome others; (ſee *Reſolution of Equations.*) As if I were asked, what Number that is which being multiplied by 7, and that Product divided by 3, the Quotient will make 1000, abating half the Number ſought? Here, to compoſe this Equation, I put $u$ for the unknown Number, or that ſought; then 7 times that is $7u$; that divided by 3, makes $\frac{7u}{3}$. Then putting $b$ repreſent 1000, this Equation is compoſed from the Nature of the Queſtion, *viz.* $\frac{7u}{3}=b-\frac{u}{2}$. (See this Equation reſolved under the word *Reſolution.*) And where the Quantities in the Propoſition or Problem can be reduced naturally into Analogy, then that Analogy is reduced, or an Equation compoſed from it: See *Analogy*.

*Conſtruction of an Equation.*] Is to demonſtrate Geometrically the Truth of the Equation, and Canon, from Lines or Geometrical Figures laid down for that purpoſe, as Simple and Quadratick from the Sides of Triangles and Arks of Circles; and Cubical from the Parabola and Circle, *&c.*

*Converſion of Eqnations.*] Is properly the converting of an Equation into that which is not ſo, *i. e.* into Analogy, as if $ad=bc$; then it follows, that as the Quantity $a$ is to $c$ or $b$, ſo is the other of them to $d$, *viz.* $a.b::c.d$. See *Analogy*, of which Example this is the Converſe, and they ought both to be remembred. But ſome call this following, and the like a Converſion, (but I call it Reduction by Multiplication, of which more under *Reduction of Equations*,) viz. $\frac{u}{r}+cd=p$, this by multiplying each Member by the Denominator ($r$), gives $u+cdr=pr$, &c. of any other.

*Converging Series.*] Lines converge, when two right ones proceed nearer and nearer to each other, 'till they meet in a Point where they make an Angle: So in Numbers, it is the orderly approaching nearer and nearer to the Truth (as to the Root of a ſurd Number or Quantity) by the common Rules. Examples hereof, ſee *Section* the 13th following: *Note*, Sometimes this Method of Proceſs is called alſo *Arithmetick of Infinites, Infinite Series, and Approximiation.*

*Contraction.*]

*Contraction.*] This in *Algebra* is to leave out all repeated or ſuperfluous Terms, or ſingle Quantities; as $4ab-3c+5c-3ab$, this contracted is $ab+2c$; for $4ab$, leſs $3ab$, leaves $ab$; and $5c$, leſs $3c$, leaveth $2c$. So alſo $7uu+3pu-du-3pu$ is $7u-d$; for firſt, $(u)$ being in each Member $1u$, may be left quite out, and then 'tis $7u+3p-d-3p$; ſecondly, $+3p$ and $-3p$ deſtroy each other: So there reſts but $7u-d$ of the Total, and the like may be obſerved in Contraction of other Quantities.

*Calculus.*] A Term in Fluxions: It is either *Calculus Integralis*, which is the Method of finding a flowing Quantity to any Fluxion given; or elſe *Calculus Differentialis*; which is the way of finding a Fluxion of any flowing Quantity.

*Canon.*] As it relates to *Trigonometry*, is the Rule or Proportion by which an Angle or Side of a $\triangle$ is found; the Tables of Artificial Numbers (Logarithms, Signs, and Tangents) are ſometimes called *Canons*; but in *Algebra* it is a reſolved Equation, being the Reſult of an Operation, and is the Value of ſome unknown Quantity in known Species, or Numbers, which being wrote down in Words, contains a plain General Rule for the Solution of any Queſtion of like Nature; which is one of the great Excellencies of *Algebra*.

*Coſſick Powers.*] Algebraick Powers of Number or Quantity; for the firſt, ſee *Chap.* 1. *Sect.* 1. of this Treatiſe; whence it follows, that the Powers of any Quantity will ſtand thus, $u=$ a Root; $u^2=$ the Square or ſecond Power; $u^3=$ the Cube or third Power; $u^4=$ the Biquadrat or fourth Power of $u$, &c. See Extraction of Roots in this Chapter, *Sect.* 6.

*Cubick Equations.*] Vide *Sect.* 12. of this Chapter.

*Depreſſion.*] This Word is uſed in the Solution of Cubick *e c* Equations, where the Equation is depreſſed or reduced from one of the third Dimenſion to a Quadratick; that being one of the ways of reſolving Cubical Equations: Or Depreſſion is the Reducing of an Equation by Diviſion; as $puu+duu=cu-du$ by dividing by $p+d$, that Equation is reduced to $u=\frac{c-d}{p+d}$, firſt expunging or caſting out $(u)$, becauſe found in each Member or Term of the Equation: See this Chapter, *Sect.* 5.

*Diſcrete Proportionals.*] Are Numbers or Quantities, where there is the ſame Proportion between the third and fourth, as between the firſt and ſecond; but not the ſame between the ſecond and third, as in $u.\ b::h.\ k$, there is the ſame Analogy between $b$ and $k$,

as between *u* and *b*, but not the like between *b* and *h*, ſo that 'tis alſo called *Disjunct Proportion*; in Number 3. 7 : : 12, 28.

*Dimenſions.*] In Equations, where the firſt Term is *uuuu*, that is one of 4 Dimenſions; *uuu* is an Equation of 3 Dimenſions; *uu* of 2, *&c.* *u* being unknown.

*Equation*] An Equation in *Algebra* is, when one or ſeveral known or unknown Quantities are made equal to other Quantities upon a due and regular Proceſs of *Compoſing*, (ſee that Word,) as if $u+c-d=rs+tu$, that is, *u* more *c* leſs *d* is equal to *r* multiplied in $s+tu$, which Equation is reſolved as under *Reſolution*; ſee alſo Simple, Quadratick, Cubical, and Lateral Equations in this part.

*Evolution.*] Is marked $\omega$, (or the Extraction of a Root,) and tells us, that the Number or Quantity before which it ſtands, is the Square Root of ſome other Step referred to in an Algebraical Operation.

*Exponents.*] Are thoſe ſmall Figures placed almoſt over any Quantity, to ſhew (or expound) what Power of the ſaid Quantity is expreſſed. See *Coſſick Powers* above. And

*Exponential Quantities.*] Is a Term in Fluxions, theſe being ſuch Quantities, whoſe Exponents are variable or flowing Quantities.

*Fluxions.*] Are defined to be the Doctrine, or Algorithm of the infinitely ſmall Increment or Decrement of variable and indeterminate Quantities: For the beſt Notion we have of Generating a Line, is by the Motion of a Point; of a Surface by the Motion of a Line; and of a Solid by the Motion of a Superficies: Now as this Increaſe of a Figure by Local Motion, is termed a *flowing Quantity*, ſo the Velocity of that Increaſe of the flowing Quantity is called *a Fluxion*; and is diſtinguiſh'd from the Character or Symbol of the flowing Quantity by a Point over it, *&c.* which is a kind of Artificial *Algebra*, but is wrought by Rules peculiar to it ſelf, being a late Invention and a great Improvement.

*Flowing Quantities.*] See *Fluxions* above.

*Generating.*] In Geometry Points Lines and Superficies generate as under the word *Fluxion*; or in Arithmetick, what is produced by Multiplication or Involution is a *generated Number or Quantity* in *Algebra.*

*Homogeneal.*] Is *of the ſame kind*: So ſuch ſurd Roots as have the ſame radical Sign are *Homogeneal*, as $\surd ab$, $\surd ac$ *ec.*; $\sqrt[3]{cd}$, $\sqrt[3]{brke}$; $\sqrt[4]{lp}$, $\sqrt[4]{g}$, &c. See this Word in the *Introduction*, Sect. 1.

*Homologous.*]

*Homologous.*] In a Proportion of four Quantities. If *a. b* : : *c. d.* here the two Antecedents *a* and *c* are *Homologous*, as are the two Conſequents *b* and *d*, &c. and *a* is *Homologous* to *c*, as *b* to *d*.

*Heterogeneal.*] (*Of different kind*;) So Surds are *Heterogeneal* when their radical Signs are not the ſame, as $\sqrt[3]{bc}$; $\sqrt[4]{tu}$, $\sqrt[5]{rst}$, &c.

*Infinite Series.*] See *Converging Series* above, and *Chap.* 10. following, *Sect.* 13.

*Involution.*] Is marked ⊙, and ſhews that the Quantity before which it ſtands is the Square of ſome other which is referred to; the Contrary to *Evolution.*

*Ineffable Quantities.*] Surds, whoſe Roots can only be expreſſed by the proper radical Signs.

*Irrational Quantities.*] The ſame as *Ineffable*, laſt above.

*Lateral Equations.*] Simple Equations, which have but one Root.

*Like Signs.*] Thoſe Quantities in *Algebra* which have the ſame Sign, as $+b+c+d$, or $-b-c-d$, &c.

*Like Quantities.*] Such as have the ſame Symbols or Species, as $23bcd$, $14bcd$, $2bcd$, &c. there being the ſame Number of each Letter.

*Maximis and Minimis.*] Is a Term uſed in *Fluxions.* It is a Method whereby Problems are reſolved, which require the greateſt or leaſt Quantity attainable in that Caſe; it is a ſtable Quantity naturally; and therefore to determine to a *Maximum* or *Minimum* in any flowing Quantity, is to make it a permanent Quantity.

*Members of an Equation.*] Thoſe Quantities contained between every Sign, as $ab+du-cp=r$; here $ab$, $du$, $cp$, and $r$, are the four Members or Terms of this Equation.

*Moments.*] A Term in Fluxions according to ſome; being ſuch Parts of Quantity as are in a continual Flux, either decreaſing or increaſing: Or rather, Moments are the beginning (of the generating Principle) of Magnitude, and are themſelves no Magnitude conceiveable, becauſe infinitely ſmall.

*Multinomial.*] See *Polynomial.*

*Negative Signs.*] See under the laſt Article (or Head) among the Characters uſed in *Algebra* for (—).

*Negative, or the Arithmetick of Negatives.*] Is that kind wherein Negative Numbers or Quantities are employed, which are known by this mark (—) before them.

The Numeration is no more than only adding the Words [leſs than nothing] to what is wrote down; as $-a$, or $-7$ is read, *a* leſs than nothing, or 7 leſs than nothing, and $-.75$ *ec* is that Decimal leſs than nothing.

*Addition.*] Shews the adding of Negative, to Negative or Affirmative, as by the Rule of adding the Quantities of like Signs together, and taking the Difference for the Sum; as the Sum of $-7+-3$ is $-10$; $-7+3$ is $-4$; and $12+-13=-1$.

*Subſtraction.*] Is done by changing the Sign of the Subtrahend, and then adding the Quantities; as from $-7$ take 3, reſts $-4$; from $--$ take 3, reſts $-10$; and if from 12 you take $-13$, there reſts 25, *&c.*

*Multiplication.*] Is nothing difficult, for you only obſerve, that if Quantities of like Signs be multiplied, the Sign of the Product is Affirmative or $+$; But if the Factors have unlike Signs, the Sign of the Product is Negative or $-$: Thus $-7$ by $-3$ is 21, and ſo $+7$ by $+3$; but 7 by $-3$ is $-21$, and $-12$ by 13 is $-156$, *&c.* And the ſame Rule ſerveth for

*Diviſion of Negatives.*] As to prove thoſe in Multiplication, $21\div-7$ quoteth $-3$; $-21\div-3$ quoteth 7; and $-156\div13=-12$. All which you'll find farther Illuſtrated, with Variety of Examples, *Sect.* 3. of *Chap.* 7. alſo *Sections* 2, 3, 4, and 5 of this *Chap.* 10. and the laſt of my Examples in *Converging Series* near the end of the Book.

*Nome.*] Whence Binomial, Trinomial, and Multi- (or Polynomial,) *i. e.* Roots of 2, 3, or more Names; ſee thoſe Words.

*Numeral Algebra.*] When the Work is performed Algebraically with Figures or Numbers, and not Letters.

*Periodical Degrees, or Parodical.*] Of the Terms of a Cubical *ec* Equation, are when the Exponents of the Powers therein riſe or fall gradually in an Arithmetical Ratio, as $u^4-59u^3+1154u^2-8080u=-12000$, &c. are regular Biquadraticks, having all their Terms in a gradual Order.

*Permanent Quantities.*] In Fluxions, ſuch Quantities as are invariable without Motion, or at Reſt, and conſequently have no Fluxions to them.

*Powers of Quantities.*] See that of Numbers, *Chap.* 1. *Sect.* 1. or Exponents here below; as $u$ or $c$, &c. are Roots or firſt Powers, $uu$ or $cc$ are Squares or ſecond Powers, $uuu$ or $ccc$ the Cubes or third Powers.

*Power*;

*Power; to raiſe the Powers of Numbers to that of any other.*] As ſuppoſe I raiſe the Root (*b*) to the ſame Power as *cccc* or $c^4$, this is to multiply *b* in it ſelf *ec*, to bring it to the fourth Power or Biquadrat: Or ſuppoſe $\sqrt[3]{82}$ were to be multiplied by 2, here 2 muſt be raiſed up to the ſame Power or Cube with the 82, and it will ſtand either $2\sqrt[3]{82}$, or $\sqrt[3]{8} \times \sqrt[3]{82} = \sqrt[3]{656}$; this will be of Uſe to obſerve, when you come to the Doctrine of ſurd Quantities, *Sect.* 7. of this Chapter.

*Polynomial Roots or Quantities.*] Thoſe which conſiſt of above three Names or Numbers; as $u+s+t+n$, &c.

*Poſitive Quantities.*] The ſame with Affirmative, which are thoſe that are marked with or follow the Sign +, or elſe ſuch as have no Sign before them.

*Quantity.*] In *Algebra*, either that ſought for, or thoſe given in a Queſtion, or they are all thoſe Symbols which repreſent any Number. See *Simple*, *Compound*, *Surd*, and *Variable Quantity*.

*Quadratick Equations.*] Are ſuch (or reduceable to ſuch) as have the Square of the unknown Quantity in the firſt Term, and the Root (or that multiplied in ſome known Quantity) in the ſecond Term, as $nn+bu+c=rs$. See two Sorts, *Section* the 10th of this Chapter.

*Rational Quantities.*] Either ſuch whoſe Roots can be extracted accurately; or ſuch as are already the Root of ſome Quantities, or which want no Extraction.

*Raiſe the Powers of Number or Quantity.*] See *Powers*.

*Radical Sign.*] Are $\surd$ for the Square Root; $\sqrt[3]{}$ for the Cube; $\sqrt[4]{}$ for the Biquadrat Root, *&c.* ſee *Roots* and the *Powers* in the Article preceding, of Algebraical Characters: And alſo

*Roots of Quantities.*] Are either (in *Algebra*) put actually down (if of Rational Quantities) or if of Surd Quantities the Root is expreſſed by putting the proper Radical Sign before the Quantity; thus in the firſt Caſe, the Square Root of *bb* is *b*, of *cc* is *c*, &c. the Cube Root of *uuu* is *u*, &c. and in the ſecond Caſe, the Square Root of $u+s$ is $\sqrt{u+s}$; the Cube Root of $u+s$ is $\sqrt[3]{u+s}$; the Biquadrat Root of it is $\sqrt[4]{u+s}$, and the like of any other Surd Quantity. See *Binomial*, *Trinomial*, *Polynomial*, and *Reſidual*. *Note*, That the Surd $c\sqrt{d}$, *ec*, is *c* multiplied in the Square Root of *d*, *ec*.

*Reſolution*

*Reſolution of (or to ſolve) Equations.*] After an *Algebriſt* has done with the compoſing an Equation as under that Word, he reſolves it by ſeveral Rules requiſite thereto, as by *Multiplication*, *Diviſion*, *Addition*, *Subſtraction*, *Evolution*, &c. ſo the Equation compoſed under the word *Compoſing* as above, which is $\frac{7u}{3}=b-\frac{u}{2}$; being reſolved by multiplying by the Denominators of the Fraction, and adding $3u$ to each Part of the Equation, dividing by 14 and 3 (or 17) gives the Solution of the Queſtion, or the Cannon $u=\frac{6b}{17}$. Now $(b)$ we find is = 1000 in the Queſtion, ſo $(u)$ the unknown Quantity is eaſily found by dividing 6000 by 17. And the Equation under that Word, *viz.* $x+c-d=rs+tu$ being reſolved, is $x=\frac{d-c+rs}{t-1}$. Particular Rules for reſolving Equations, ſee *Section* the 9th of this Chapter; where it appears, that an Equation is reſolved when the unknown Quantity alone poſſeſſeth one Side of the Equation *a*, here *u* doth.

*Reſidual Roots or Quantities.*] Such as have a Negative Sign between them, as $bc-d$, &c. or $40-10=30$ the Reſidue.

*Simple Quantities.*] In a ſtrict Senſe are thoſe ſingle Letters, which repreſent the ſeveral Numbers in a Problem, given or required: Or ſome take them in a larger Senſe, for ſuch Quantities as are not connected by + or − to any other, though they be multiplied in other; of the firſt ſort are $b$, or $c$, or $d$, &c. of the ſecond are $4b$, $bc$, $drs$, &c.

*Signs.*] See the *Table of Characters*, Article IV. of this Section; alſo *Affirmative*, *Negative*, and *Radical*.

*Simple Equations.*] Such as are free from the Involution or Powers of the unknown Number or Quantity.

*Simple Quadratick Equations.*] Such as have (or are reduceable to have) the Square of the unknown Number or Quantity in only one of the Members or Terms, as $uu=pq$, that is, $u=\sqrt{pq}$, or $xxu+ub+udc$, which by the Seclufion of $u$ in each., is $u=\sqrt{b+dc}$.

*Series.*] A Rank, Row, or Column of Numbers or Quantities orderly placed. See *Progreſſion and Converging Series*.

*Species.*] In *Algebra*, are the Letters which repreſent the Numbers in a Queſtion, they are alſo called *Symbols*; ſo ſpecious Arithmetick is not Numeral, but Literal *Algebra*.

*Subſtitution.*] Is the putting one Symbol or Quantity into the Proceſs of Algebraical Operations, in the room of ſome other: As in the Solution of a Quadratick Equation, by changing the ſecond Term, and as in Fluxions.

*Surd Quantities.*] Such as cannot have the Root expreſſed, and are therefore marked with their proper *Radical Sign*, which Term ſee above, and alſo *Roots of Quantities.*

*Terms.*] *Of an Equation*, are commonly ſaid to be the ſeveral Members thereof; (ſee that Word above;) but if in a Quadratick, as $uu+ub+ud=k+c$, the Terms by ſome are called but three Terms, as $uu$ the firſt; $ub+ud$ the ſecond; and $k+c$ the third Term.

*Tranſpoſition.*] Is the changing the Places of the Members of an Equation, in order to make the unknown Quantities to poſſeſs one Side, and the known the other Side of the Equation; whereby the Knowledge is gained of what Value the Number ſought in a Queſtion, really is, by finding what it is equal to. See *Sect.* 9. of this Chapter, for Rules to perform the ſame.

*Trinomial Roots or Quantities.*] Are ſuch as have three Nomes or Names, as $u+rs+tu$, &c.

*Vaniſh, or be Wanting.*] A Quantity is ſo, when by Reduction or Contraction it is expunged or thrown out of an Equation.

*Variable Quantities.*] Are ſuch as are ſuppoſed in continual Motion, whereby they generate Lines, Superficies, and Solids; the ſame as *Flowing Quantities.*

*Unciæ.*] Are the Numbers prefixed to the Quantities, which are the Powers of ſome Binomial Roots; as in the Square of $a+b=aa+2ab+bb$, the *Unciæ* is 1, 2, 1, for $aa$ and $bb$ are $1aa$ and $1bb$; ſo in the Cube of $a+b$, viz. $a^3+3aab+3abb+bbb$, the *Unciæ* is 1, 3, 3, 1, *&c.* for according to Sir *Iſaac Newton*'s Rule, the *Unciæ* of any Powers are found (as ſuppoſe the third Power or Cube) by this Rule; $1\times\frac{3-0}{1}$ $(3\times\frac{3-1}{2}$ $(3\times\frac{3-2}{3}$ (1, which in Words is read thus: One multiplied in 3 the Power, leſs 0, divided by 1 is 3= the ſecond of the *Unciæ*; 3, the ſecond multiplied in 3 the Power, leſs 1, divided by 2, gives 3= the third of the *Unciæ*; and 3, the third multiplied in the Power 3, leſs 2, divided by 3, gives 1= the forth of the *Unciæ.*

And by the ſame Rule, the *Unciæ* of the ſurſolid is 1, 5, 10, 10, 5, 1, for every Quote and the firſt 1, are the *Unciæ* ar ſung.

Thus

Thus $1x^{\frac{5-0}{1}}$ ($5x^{\frac{5-1}{2}}$ ($10x^{\frac{5-2}{3}}$ ($10x^{\frac{5-3}{4}}$ ($5x^{\frac{5-4}{5}}$(1 ; see also *Sect.* 6.

*Vinculum.*] Is a Compound Surd Quantity multiplied by a Fluxion ; in which this is a Term.

*Unfold.*] To unfold an Aigebraick Canon, expressed Literally ; is to compare the known Quantities with the Numbers which they represent, and working with them as the Canon directs, we discover the Value of the unknown Number, or that sought.

*Wanting.*] See *Vanish.*

Having in this Section laid a plain and copious Foundation ; I shall be something briefer in the subsequent Rules, but no farther than is consistent with evident Demonstration.

## SECT. II. *Addition of Algebra.*

| Note $b=70$ | | | Note $c=35$ | | |
|---|---|---|---|---|---|
| | Example 1. | | | Example 2. | |
| Literal $9b=630$ | Numeral | } Add | Literal $-12c=-420$ | Numeral | |
| $10b=700$ | | | $-7c=-245$ | | |
| $b=70$ | | | $20c=700$ | | |
| Sum $20b=1400$ | Sum or Proof. | | Sum $=c=35$ | Sum or Proof. | |

Compound Quantities. *Note*, $d=4$ ; $g=16$ ; $h=6$.

| | | | |
|---|---|---|---|
| $3ddh+hhg$ | $=$ | $288+576$ | Example 3. |
| $ddh-2hhg$ | $=$ | $96-1152$ | |
| $-5ddh+hhg$ | $=$ | $-480+576$ | |
| Sum $-ddh$ | $0=$ | $-96+0$ | Sum for Proof. |

The General Rule for *Addition.*

*That the Quantities of like Signs are to be added together, and where there are different Signs, as in the second and third Examples, you must subtract the Sum under one Sign from that under another, and put the Difference down marked with the same Sign as that Sum, wherein was the Excess :* As I deduct $19c$ from $20c$ (in the second Example) and the Difference is $c$ for the Sum required.

And in the third Example, (*Colum.* 1.) the Excess lies in the Negative Sign, for taking $4ddh$ from $-5ddh$, the Remainder is $-ddh$ (or $-1ddh$.)

And

And in the ſecond Series of Quantities in that third Example, the Signs making the Quantities to Balance; I therefore put (o) down, all which is proved by the reſpective Numbers.

And whereas it may be thought ſtrange, that a Sum Total ſhould be *ddh*, or 96 leſs than nothing; it is no more than what falls out in many Caſes: For if a Merchant draws on his Factor 480 *l.* (as the Number is in *Example* 3.) and that Factor has but 288 *l.* and 96 *l.*=*viz.* 384 *l.* to pay the Bills in his Hands; 'tis plain, when the Factor has paid what is drawn on him, the Merchant will have 96 *l.* leſs than o in the Factor's Hands, *&c.* And this is alſo plain from the Reaſon given for adding Affirmative and Negative Indices of Logarithms, *Chap.* 4. foregoing; which they who have conſidered, the Addition and Subſtraction of *Algebra* will to them appear evident and reaſonable.

And when Numbers are to be added that have different Quantities annexed, there is nothing to do but to put them down in one Line with their Signs between them; as to add $5ab$ to $3ad$, is $=5ab+3ad$; and $ubc$ to $rst$ is $-rst+ubc$, &c. And the Quantities in the foregoing Examples are truly added by the ſame way, being afterward Contracted as under the word *Contraction* in the laſt Section foregoing.

## Sect. III. *Subſtraction of* Algebra.

*The General Rule*, &c. *ſee after the* Examples.

Example 1.

*Note*, $b=70$

| | | |
|---|---|---|
| From | $20b$ | $=1400$ |
| Take | $11b$ | $=770$ |
| Reſts | $9b$ | $=630$ |

Literal = Numeral

Example 2.

| | | |
|---|---|---|
| From | $c$ | $=35$ |
| Take | $-19c$ | $=-665$ |
| Reſts | $20c$ | $=700$ |

*Note* (as in Addition of Compound Quantities) $d=4$; $g=16$, and $h=6$.

Example 3.

| | | |
|---|---|---|
| From | $-ddh$ | $=-96$ |
| Take | $-5ddh-2ghh$ | $=-480-1152$ |
| Reſts | $4ddh+2ghh$ | $=384+1152$ |

And as a fourth Example; if from $4ddh+ghh+bb$, I take $bh+gg-hh-d$. The Remainder will ſtand thus:

$$4ddh+ghh+bb;\ -bh-gg+hh+d.$$

***For Proof of which.*** By the laſt Section it appears that $4ddh = 384$
From this Sum 5860 taking the $ghh = 576$
Sum of the four Quantities in the $bb = 4900$
Subtrahend (added according to the Sum 5860
Rules of Addition,) which is 636, *viz.* $+bb = 420$, $+gg = 256$ } 676; $-bb = 36$, $-d = 4$ } $-40$ } $= 636$
the Remainder is 5224 as by the Margin
And ſo it will be by this Rule. 5224

*The General Rule for* Subſtraction, *i. e.*

*Change the Sign of the Quantities to be ſubſtracted, and then proceed as in Addition.*

*So in the firſt Example*, $20b - 11b = 9b$, for $1400 - 770 = 630$; which is $9b$, or 9 times 70.

*And in the ſecond Example*, $c + 19c = 20c$, or $35 + 665 = 700$; which is 20 times 35.

*In the third Example*; I change the Sign of $2ghh$, (or 1152), and add the ſame to (0), makes $+2ghh = +1152$; and $-5ddh$, or $-480$ having the Sign changed and added to $-ddh$, and $-96$ gives $4ddh$, or $384 =$ the Remainder as you ſee. And

*In the fourth Example*; If the four Signs of the Subtrahend be changed and made as in the Remainder above, that Remainder will prove true and agreeable to the Deduction foregoing, altho' it be done differently and according to the General Rule for Subſtraction of Algebraical Integers, as by the Margin appears, where $5860 - 636 = 5224$.

*Note*, Theſe Examples prove the Truth of thoſe in Addition.

$4ddh = 384$
$ghh = 576$
$bb = 4900$
Sum $= 5860$
And $-bb = -420$, $-gg = -256$ } $-676$; $+bb = +36$, $+d = +4$ } $+40$ } $= -636$
Reſts $= 5224$

*The Reaſon of the General Rule for* Subſtraction.

1*ſt*. Either the Sign of the Subtrahend is $+$ or $-$, if $+$ as to take 2 from 6, or $\left.\begin{matrix}+6\\+2\end{matrix}\right\}$ to take 2 from 6 is (ſpeaking Algebraically) to ſay $6-2$ (or 6 leſs 2) whereby the Sign of the 2 is of Courſe changed.

2*dly*. And

2*dly.* And if the Sign of 2 (*ec*) as given to be ſubſtracted were (−), then by what is ſaid of Subſtraction of Indices of Logarithms, as well as from the Reaſon of the Thing it ſelf; if I take (0) from 6, there will remain 6; but if −2 (2 leſs than 0) from 6, 'tis the ſame as to add 2 to 6, ſo there muſt remain 8; for by how much the leſs any Number is that is deducted, by ſo much the greater is the Remainder; and by how much ſoever the Number ſubſtracted is leſs than 0, by ſo much does it make the Remainder greater than the Sum ſubſtracted from.

## Sect. IV. *Multiplication in* Algebra.

Note, $u=25$; $b=12$; $c=20$; $d=4$; $g=5$; $h=10$.

*A General Rule, ſee after the* Examples.

*Prop.* 1. To multiply ſimple Quantities by Simple.

Caſe 1. Multiply $-u=25$ by $-b=12$ — Product $bu=300$

Caſe 2. Multiply $d=4$ by $g=5$ — Product $dg=20$

Caſe 3. Multiply $-c=-20$ by $h=10$ — Product $-ch=-200$

Caſe 4. Multiply $u=25$ by $-c=-20$ — Product $-cu=-500$

*Prop.* 2. To multiply compound Quantities by Simple.

Caſe 1. Multiply $2hu+d=500+4$ by $c=20$ — Product $=2chu+cd=10000+80=10080$

Caſe 2. Multiply $-cd-h=-80-10$ by $-c=-20$ — Product $+ccd+ch=+1600+200=1800$

Caſe 3. Multiply $2hu+d=500+4$ by $-c=-20$ — Product $-2chu-cd=-10000-80=-10080$

*Prop.* 3. **To multiply compound Quantities by Compound.**

Case 1. Multiply $bb+g = 100+5 = 105$
by $gg+b = 25+10 = 35$

$gg^{bb}+ggg = 525$
$+bbb+gb = 315$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

---

Case 2. Multiply $-bb-g = -100-5 = -105$
by $-gg-b = -25-10 = -35$

$gg^{bb}+ggg = 525$
$+bbb+gb = 315$

Product $ggbb+ggg+bbb+gb = 3675$ Product.

---

Case 3. Multiply $bb+g = 100+5 = 105$
by $-gg-b = -25-10 = -35$

$-ggbb-ggg = -525$
$-bbb-gb = -315$

Product $= -ggbb-ggg-bbb-gb = -3675$ Product.

---

To multiply *Cossick Powers*, see at the end of *Division* of *Algebra*.

*The General Rule for* Multiplication.

*If the Signs of both the Factors are like, then the Sign of the Product is Affirmative: But if the Signs of the Factors are different, the Sign of the Product is Negative.*

This Rule will appear to be observed in the nine Cases of *Multiplication* as above, which I have demonstrated by Numbers, which agree to the Quantities of the Factors and Products: For Instance, the Product of the first Case of *Prop.* 3. is as in the Margin, where the Value of the Product is equal to the Rectangle of those Numbers 105 and 35 (the Values of the Factors) being multiplied together.

| Quantities. | Values. |
|---|---|
| $ggbb$ | = 2500 |
| $ggg$ | = 125 |
| $bbb$ | = 1000 |
| $gb$ | = 50 |
| Sum | = 3675 |

*The*

*The Reason of the General Rule for* Multiplication.

1*st. Multiplication* being nothing but the Work of many Additions; it follows, that $-ab$ being added to $-ab$, is $-2ab$, which is the same as multiplying $-ab$ by 2; and so $-gh-2gh-3gh$ added, is $=-6gh$, which is the same as $-gh\times 6$; which shews, that the Negative Quantity $-gh$ by the Affirmative 6, must give the Negative $-6gh$, and the like Reason holds for any other Negative by a positive Quantity.

2*dly*. And as in *Logick*, two Negatives make an Affirmative; so in *Algebra*, one Negative Number or Quantity multiplied by another Negative, produceth an Affirmative: For the Sign — being directly contrary to +, must make a Product of a direct contrary Nature when the Multiplier is —, than (as above) when it is +; as substracting a Negative, is adding really so much as we seem to substract, as $-2$ from 4 is 6.

## Sect. V. *Division of* Algebra.

I shall make use of the same Letters and their Values as in the last Section, and the Examples shall be such as to prove both themselves and those in the last Section.

*Prop.* 1. To divide a simple Quantity by a Simple.

Case 1. Divide $bu=300$ by $-b=-12$

$-b)\ bu\ (-u=$ Quote $=25$ for $-12)\ 300\ (-25=-u$

$bu$

0

$\cdot\cdot$

60

0

---

Case 2. Divide $dg=40$ by $g=5$

$g)\ dg\ (d$ the Quote; for 5$)\ 40\ (8=d$

$dg$

0

0

---

Case 3. Divide $-ch=-200$ by $h=20$

$h)\ -ch\ (-c$ the Quote for 10$)\ -200\ (-20=-c$

$-ch$

0

00

---

*Prop.* 2. To divide a Compound by a simple Quantity.

Case 1. Divide $2chu+cd=10000+80$ by $c=20$

$c)\ 2chu+cd\ (2hu+d$

$2chu$

$+cd$

$cd$

(0) Rests.

Quote

$20)\ 10080\ (504$

$=2hu+d$.

Case

**Case 2.** Divide $ccd+cb=1600+200$ by $-c=-20$

| $-c)\ ccd+cb\ (-cd-b$ | $-20)\ 1800\ (-90$ Quote |
|---|---|
| $ccd$ | $=-cd-b$ |
| $+cb$ | |
| $+cb$ | |
| $0$ | |

**Case 3.**
Divide $-2cbu-cd=-10000-80$ by $-c=-20$

| $-c)\ -2cbu-cd\ (2bu+d$ | $-20)\ -1008\ (50$ Quote $4$ |
|---|---|
| $-2cbu$ | $=2cb+d$ |
| $-cd$ | |
| $-cd$ | |
| $0$ | |

*Prop.* 3. To divide compound Quantities by Compound.

**Case 1.** Divide $ggbb+ggg+bbb+gb=3675$ by $gg+b=35$

| $gg+b)\ ggbb+ggg+bbb+gb\ (bb+g$ Quote. | $35)\ 3675\ (105=bb+g$ Quote. |
|---|---|
| $ggbb+bbb$ Deduct | $175$ |
| $ggg-bbb+bbb+gb$ rests of the (Dividend. | $(0)$ |

Or $=ggg+gb$, if Contracted.
$ggg+gb$ Deduct.
$0$ Rests.

**Case 2.** Divide $ggbb+ggg+bbb+gb=3675$
by $-bb-g=-105$

| $-bb-g)\ ggbb+ggg+bbb+gb\ (-gg-b$ | $-105)\ 3675\ (-35=$ Quote. |
|---|---|
| $ggbb+ggg$ | $=-gg-b.$ |
| $=$ Quote | $525$ |
| $0+bbb+gb$ | $0$ |
| $+bbb+gb$ | |
| Rests $(0)$ | |

**Case 3.** Divide $-ggbb-ggg-bbb-gb$
by $-gg-b$

| $-gg-b)\ -ggbb-ggg-bbb-gb\ (bb+g=$ the Quote. | $-35)\ -3675\ (105$ |
|---|---|
| $-ggbb-bbb$ | $175$ |
| $-ggg+bbb-bbb-gb=$ the Reſt of (the Dividend. | $0$ |

Or $-ggg-gb$ if Contracted.
$-ggg-gb$
$(0)$ Reſts.

*General*

### *General Rule as to the Signs.*

It is eaſily ſeen by the nine Caſes above, that if you divide Quantities by thoſe having like Signs with thoſe divided, then the Signs of thoſe put in the Quote are + or Affirmative; but if the Signs of the Dividend and Diviſor are unlike, then the Sign of thoſe Quantities put in the Quote are Negative.

### *General Rule as to the Quantities.*

Proceed in the Method of common Diviſion, putting thoſe Quantities in the Quote which are not in the Diviſor, as in the laſt Caſe; I ask how often $-gg$ in $-ggbb$? the Anſwer therefore is $bb$, which I put in the Quote, and multiplying the Diviſor by $bb$, is $=-ggbb-bbb$ (in the laſt Caſe above,) ſo the Remainder is $-ggg+bbb$; (becauſe the Sign muſt be changed of $bbb$, to deduct it from $-ggg$;) then becauſe here is $+bbb-bbb$, they deſtroy one another, (as is ſhewed under the word *Contraction* in the firſt Section of this Chapter,) ſo there reſts but $-ggg-gh$: And I find I can have $-gg$ of the Diviſor in $-ggg$ of the Dividual $+g$, by which multiplying the Diviſor $-gg-h$, produceth $-ggg\ gh$, which deducted from the laſt compound Quantity contracted, leaves (o) or nothing for a Remainder.

But though this is pretty enough, and ſerves to ſhew the Coherence of Diviſion in *Algebra*, with that in other kinds of Arithmetick; yet it is far from falling in uſe, ſo much as where the Diviſor in Species will not divide any part of the Dividend; or eſpecially where it wont divide it all, but ſome part; and the Reſt as a Fraction of the Diviſor being put under the remaining part of the Dividend.

*The firſt Kind of more common Caſes in* Diviſion *are theſe.*

To divide $gh$ by $r$, quoteth $\frac{gh}{r}$; ſo $pr+st-m$ by $c$, is $\frac{pr+st-m}{c}$, and alſo $cd-c+m$ by $pr+s-tu$, is $\frac{cd-c+m}{pr+s-tu}$, &c. which are done by only putting the Dividend over the Diviſor.

*The*

*The ſecond Kind of more common Caſes in* Diviſion *are.*

Where the Diviſor is found in ſome Quantities of the Dividend, but not in all: As $su+stu+g$ being divided by $s+st$, will ſtand as in the Margin.

So alſo to divide $msu+pru+gh-rs$ by $ms+pr$.

$s+st)\ su+stu+gh\ (u+\frac{gh}{s+st}$
$su+stu$ = the Quote.
$+gh$ Reſts.

$ms+pr)\ msu+pru+gh-rs\ (u+\frac{gh-rs}{ms+pr}$
$msu+pru$ = the Quote.
$gh-rs$ Reſts.

The Truth of theſe two Caſes is eaſily demonſtrated, by ſuppoſing the Species to repreſent any Numbers. But becauſe I ſhall a little further have Occaſion to ſhew how to ſolve Equations by Diviſion, I ſhall refer the Numeral Work thereto.

A learned Friend put me in Mind of giving this for my laſt Example in *Diviſion*.

$a+b)\ a^7+b^7\ (a^6-a^5b+a^4b^2-a^3b^3+a^2b^4-ab^5+b^6$ = the Quote, which
$a^7+a^6b$ $a+b$ (I prove thus.

$-a^6b+b^7$
$-a^6b-a^5b^2$

$+a^5b^2+b^7$
$+a^5b^2+a^4b^3$

$-a^4b^3+b^7$
$-a^4b^3-a^3b^4$

$+a^3b^4+b^7$
$+a^3b^4+a^2b^5$

$-a^2b^5+b^7$
$-a^2b^5-ab^6$

$+ab^6+b^7$
$+ab^6+b^7$

0

$a^7-a^6b+a^5b^2-a^4b^3+a^3b^4-a^2b^5+ab^6$
$+a^6b-a^5b^2+a^4b^3-a^3b^4+a^2b^5-ab^6+b^7$

Sum $=a^7+b^7$ = the Product contracted.
= the Dividend.

And

And *Note*, That the Differences of any two like Powers of two Quantities, will always be divided by the Difference of the Quantities without any Remainder, as $a^4 - b^4 \div a - b = a^3 + a^2b + ab^2 + b^3$.

2. When the Exponents of the Powers are odd Numbers, the Sum of the Powers can be divided by the Sum of the Quantities without Remainder, as $a^3 + b^3 \div a + b = a^2 - ab + bb$. And as in the above Example at large.

3. Also when the Powers are even, the Difference of the Powers can be divided by the Sum of the Quantities; as $a^6 - b^6 \div a + b = a^5 - a^4b + a^3b^2 - a^2b^3 + ab^4 - b^5$.

## Sect. VI. *Of raising the Powers of Numbers and Quantities, and of Extracting the Roots.*

1. By the following Table you have the several Ways of expressing the Powers with their Exponents, both of Numbers and Species; what is said of the Powers of 9 being expressed by their Exponents, being to be taken also in the other Roots 8, 7, 6, 5, 4, 3, 2, and 1, for as $9^{10}$ is the 10th Power of 9, which by the Table is 3486784401: So is $8^{10}$ = the 10th Power of 8, which you see is 1073741824, which are the two various Ways of expressing the Powers of Numbers.

2. For the Powers of Quantities, whether Simple as those of $u$, or compounded by Multiplication as those of $ub$, viz. $uu$ or $u^2$, and $uuu$ or $u^3$, &c. for the second and third Powers of $u$, or of $ub$, whose Square is $uubb$, or better $u^2b^2$, the Cube $= u^3b^3$; you have the best way of Notation in this Table.

*A* TABLE *shewing the several Ways of expressing the Powers, both of Numbers and Species.*

| Exponents. | 1st Power. | 2d Power. | 3d Power. | 4th Power. | 5th Power. | 6th Power. | 7th Power. | 8th Power. | 9th Power. | 10th Power. |
|---|---|---|---|---|---|---|---|---|---|---|
| | 1 Root or | Square or | Cube or | Biquadrat or | Surfolid or | Squared Cube or | 2d Surfolid or | Squared Biquadrat or | Cubed Cube or | Squared Surfolid or |
| | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| | 2 | 4 | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 |
| | 3 | 9 | 27 | 81 | 243 | 729 | 2187 | 6561 | 19683 | 59049 |
| | 4 | 16 | 64 | 256 | 1024 | 4096 | 16384 | 65536 | 262144 | 1048576 |
| | 5 | 25 | 125 | 625 | 3125 | 15625 | 78125 | 390625 | 1953125 | 9765625 |
| | 6 | 35 | 216 | 1296 | 7776 | 46656 | 279936 | 1679616 | 10077696 | 60465176 |
| | 7 | 49 | 343 | 2401 | 16807 | 117649 | 823543 | 5764801 | 40353607 | 282475249 |
| | 8 | 64 | 512 | 4096 | 32768 | 262144 | 2097152 | 16777216 | 134217728 | 1073741824 |
| | 9 | 81 | 729 | 6561 | 59049 | 531441 | 4782969 | 43046721 | 387420489 | 3486784401 |
| Or more Brief. | 9 | $9^2$ | $9^3$ | $9^4$ | $9^5$ | $9^6$ | $9^7$ | $9^8$ | $9^9$ | $9^{10}$ |
| | 10 | $10^2$ | $10^3$ | $10^4$ | $10^5$ | $10^6$ | $10^7$ | $10^8$ | $10^9$ | $10^{10}$ |
| In Species thus: | *n* | *nn* | *nnn* | *nnnn* | *nnnnn* | *nnnnnn* | *nnnnnnn* | *nnnnnnnn* | *nnnnnnnnn* | *nnnnnnnnnn* |
| Or rather | *n* | $n^2$ | $n^3$ | $n^4$ | $n^5$ | $n^6$ | $n^7$ | $n^8$ | $n^9$ | $n^{10}$ |
| If Compound thus: | *nb* | *nnbb* | *nnnbbb* | &c. | . . . | . . . | . . . | . . . | . . . | . . . |
| Or better thus: | *nb* | $n^2b^2$ | $n^3b^3$ | $n^4b^4$ | $n^5b^5$ | $n^6b^6$ | $n^7b^7$ | $n^8b^8$ | $n^9b^9$ | $n^{10}b^{10}$ |
| | *bcd* | $b^2c^2d^2$ | $b^3c^3d^3$ | $b^4c^4d^4$ | $b^5c^5d^5$ | $b^6c^6d^6$ | $b^7c^7d^7$ | $b^8c^8d^8$ | $b^9c^9d^9$ | $b^{10}c^{10}d^{10}$ |

*And the Notation of Powers with fractional Exponents is thus:*

| Roots. | Squares. | Cubes. | Biquadrat. | Surfolid. | Squared Cube. | Second Surfolid. | Squared Biquad. | Cubed Cube. | Squared Surfolid. |
|---|---|---|---|---|---|---|---|---|---|
| $a^{\frac{1}{2}}$ | $a$ | $a^{1\frac{1}{2}}$ | $a^2$ | $a^{2\frac{1}{2}}$ | $a^3$ | $a^{3\frac{1}{2}}$ | $a^4$ | $a^{4\frac{1}{2}}$ | $a^5$ |
| $a^{\frac{1}{3}}$ | $a^{\frac{2}{3}}$ | $a$ | $a^{1\frac{1}{3}}$ | $a^{1\frac{2}{3}}$ | $a^2$ | $a^{2\frac{1}{3}}$ | $a^{2\frac{2}{3}}$ | $a^3$ | $a^{3\frac{1}{3}}$ |
| $a^{\frac{1}{4}}$ | $a^{\frac{2}{4}}$ | $a^{\frac{3}{4}}$ | $a$ | $a^{1\frac{1}{4}}$ | $a^{1\frac{2}{4}}$ | $a^{1\frac{3}{4}}$ | $a^2$ | $a^{2\frac{1}{4}}$ | $a^{2\frac{2}{4}}$, &c. |
| Or with Literal Exponents, thus: | | | | | | | | | |
| $a^{\frac{m}{n}}$, | $a^{\frac{2m}{n}}$, | $a^{\frac{3m}{n}}$, | $a^{\frac{4m}{n}}$, | $a^{\frac{5m}{n}}$, | $a^{\frac{6m}{n}}$, | $a^{\frac{7m}{n}}$, | $a^{\frac{8m}{n}}$, | $a^{\frac{9m}{n}}$, | $a^{\frac{10m}{n}}$, &c. |

Some

## *Some* Obſervations *upon this Table.*

1. Any two Exponents added, give that of the Product of the reſpective Powers: As where the Root is 3, and the Exponents 4 and 5. I ſay, the Sum of 4 and 5 =9, which is equal to the Exponent of the Rectangle of the reſpective Powers 243×81=19683.

2. Double any Exponent, gives the Exponent of the Square of the reſpective Power, as in the 4th Power of 5, which is 625. I ſay, double the Exponent is 8, which is the Exponent of the Rectangle of 625×625, *viz.* of 390625.

3. I obſerve that 3, 4, *ec* times any Exponent, gives the Exponent of the 3d, 4th, *&c.* Powers: As three times the Exponent 2 is 6, ſo the Cube of the Powers 9, 16, *ec* under the Exponent 2, gives 729, 4096, *ec* the reſpective Powers under the Exponent 6.

4. If an 8th Power be to be divided by a 2d Power, the Exponent 2 taken from the Exponent 8, leaves the Exponent 6; ſo any of the Numbers under the Exponent 8, (as ſuppoſe 256,) divided by any of thoſe reſpectively under the Exponent 2, (as for inſtance 4,) the Quote will be reſpectively the Powers under the Exponent 6, (in this Example 64.)

5. The Cube *ec* Root of any Power, is the Power reſpectively of the Exponent of that Power given, divided by 3*ec*; ſo the Cube Root of 262144 (its Exponent being 9, a third of which is 3,) is 64, found under the Exponent 3, and againſt the Power given. So likewiſe the Surſolid Root of 1024, (or any of the Numbers under the Exponent 10,) is 4, (or the reſpective Numbers under the Exponent 2, the Quote of 10 divided by 5; for $4^5$=1024 the Proof. So much for raiſing the Powers of Quantities compounded by *Multiplication*; I proceed to ſhew,

III. *How to raiſe the Powers of Quantities compounded by* Addition *or* Subſtraction, *i. e. from a Binomial or Reſidual Root, and to extract Roots.*

$a+b$ = the Root or 1ſt Power.
$a+b$

$aa+2ba+bb$ = the Square or 2d Power of $a+b$.
$a+b$

$aaa+2baa+bba$
$+\ baa+2bba+bbb$

$a^3+3baa+3bba+bbb$ = the Cube or 3d Power of $a+b$.
$a+b$

$a^4+3ba^3+3bbaa+ab^3$
$+\ ba^3+3bbaa+3ab^3+b^4$

$a^4+4ba^3+6b^2a^2+4ab^3+b^4$ = the Biquadrat or 4th Power of $a+b$.
$a+b$

$a^5+4ba^4+6b^2a^3+4a^2b^3+ab^4$
$+\ ba^4+4b^2a^3+6a^2b^3+4ab^4+b^5$

$a^5+5ba^4+10b^2a^3+10a^2b^3+5ab^4+b^5$ = the Surſolid or 5th Power of $a+b$.
$a+b$

$a^6+5ba^5+10b^2a^4+10a^3b^3+5a^2b^4+ab^5$
$+\ ba^5+5b^2a^4+10a^3b^3+10a^2b^4+5ab^5+b^6$

$a^6+6ba^5+15b^2a^4+20a^3b^3+15a^2b^4+6ab^5+b^6$ = the Squared Cube of $a+b$.
$a+b$

$a^7+6ba^6+15b^2a^5+20a^4b^3+15a^3b^4+6a^2b^5+ab^6$
$+\ ba^6+6b^2a^5+15a^4b^3+20a^3b^4+15a^2b^5+6ab^6+b^7$

$a^7+7ba^6+21b^2a^5+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7$ = the 2d Surſolid of $(a+b$.

$a^8+8ba^7+28b^2a^6+56a^5b^3+70a^4b^4+56a^3b^5+28a^2b^6+8ab^7+b^8$ = the 8th $(\text{Power}$.

$a^9+9ba^8+36b^2a^7+84a^6b^3+126a^5b^4+126a^4b^5+84a^3b^6+36a^2b^7+9ab^8+b^9$ $(=$ the 9th Power.

$a^{10}+10ba^9+45b^2a^8+120a^7b^3+210a^6b^4+252a^5b^5+210a^4b^6+120a^3b^7+$
$45a^2b^8+10ab^9+b^{10}$ = the 10th Power of $a+b$.

*Obſervations*

*Obſervations from this laſt Table of Powers.*

1. That the Sum of the Square of the Parts of any Number or Quantity more the double Rectangle is = the Square of the whole, as *per Euclid.*

2. That the foregoing Example of Powers may ſerve as a Table for any one to know any Power of any other Binomial Roots by, tho' of other Letters, *mutatis mutandis*; the Unciæ and Powers being the ſame as in this.

3. That the 10th Power is eaſily produced from the 9th, or any ſubſequent Power from the preceding, by firſt finding the Unciæ as *per* the Rules foregoing, or as under the 5th Head following, and conſidering the Courſe of the Powers that precede that which you would find: By which Method I have produced the 8th, 9th, and 10th Powers without any Multiplication.

4. And that the Reader may not want a proper Way of expreſſing in Words any Power, I ſhall give an Example how the 10th Power is properly Read; which is thus:

The ſquared Surſolid of $u$, more 10 times the Rectangle of $b$ in the cubed Cube of $u$; more 45 times the Square of $b$ in the ſquared Biquadrat of $u$; more 120 times the 2d Surſolid of $u$ in the Cube of $b$; more 210 times the ſquared Cube of $u$ in the Biquadrat of $b$; more 252 times the Surſolid of $u$ in the Surſolid of $b$; more 210 times the Biquadrat of $u$ in the ſquared Cube of $b$; more 120 times the Cube of $u$ in the 2d Surſolid of $b$; more 45 times the Square of $u$ in the ſquared Biquadrat of $b$; more 10 times $u$ in the cubed Cube of $b$; more the ſquared Surſolid of $b$.

5. *A very brief Way of finding the Unciæ of any Powers by that of the next preceding Power is this*; (which I have not ſeen given before.)

*A Second Way of finding the Unciæ.*

Add the Unciæ of the 1ſt and 2d Members (for Example of the 9th Power to find the Unciæ of the 10th) together they make 10, which is that of the 2d Member of the 10th Power, 36 in the 3d, and 9 in the 2d Members of the 9th Power is the 3d Member in the 10th or 45; 36 and 84 in the 3d and 4th Members is 120 belonging to the 4th Member of the 10th Power, *&c.* of the reſt.

*And the Powers themſelves, and their proper Exponents are found by the like Method*, obſerving, That in which of the 2 Powers ſoever added,

added, is the greateſt Exponent, that Exponent muſt be put in the Sum.

Thus to make the 10th Power by theſe Rules from the 9th. The Unciæ of the 1ſt and 2d Members is 10= the 2d in the 10th; and the Sum of the Powers is $bu^9$; (becauſe 9 is found the higheſt Exponent of $u$ and $b$ is only the 1ſt Power.) So $10bu^9$ is the 2d Member of the 10th Power. The 3d Member of the 10th is the 2d and 3d Members of the 9th, *&c.* done as in that laſt mentioned.

And this will be made plain by a Numeral Example of finding the Unciæ to the 15th Power of a Binomial, by an eaſy Addition of only two Numbers; which Unciæ, with thoſe of all the intermediate, are found much ſpeedier than that of the 15th Power alone can be by the Method before-mentioned; which requires Subſtraction, Multiplication, and Diviſion, and thoſe pretty tedious ones before thoſe of the 15th Power can be exhibited by this Method.

*A New Table of the Unciæ to the 15th Power of a Binomial Root, made by Addition only.*

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | add | add | add | add | add | add | add | add | add | add | add | add | add | |
| 1 | 1 | | | | | | | | | | | | | | |
| 1 | 2 | 1 | | | | | | | | | | | | | |
| 1 | 3 | 3 | 1 | | | | | | | | | | | | |
| 1 | 4 | 6 | 4 | 1 | | | | | | | | | | | |
| 1 | 5 | 10 | 10 | 5 | 1 | | | | | | | | | | |
| 1 | 6 | 15 | 20 | 15 | 6 | 1 | | | | | | | | | |
| 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | | | | | | | | |
| 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 | | | | | | | |
| 1 | 9 | 36 | 84 | 126 | 126 | 84 | 36 | 9 | 1 | | | | | | |
| 1 | 10 | 45 | 120 | 210 | 252 | 210 | 120 | 45 | 10 | 1 | | | | | |
| 1 | 11 | 55 | 165 | 330 | 462 | 462 | 330 | 165 | 55 | 11 | 1 | | | | |
| 1 | 12 | 66 | 220 | 495 | 792 | 924 | 792 | 495 | 220 | 66 | 12 | 1 | | | |
| 1 | 13 | 78 | 286 | 715 | 1287 | 1716 | 1716 | 1287 | 715 | 286 | 78 | 13 | 1 | | |
| 1 | 14 | 91 | 364 | 1001 | 2002 | 3003 | 3432 | 3003 | 2002 | 1001 | 364 | 91 | 14 | 1 | = the Unciæ the 14 Pow |
| 1 | 15 | 105 | 455 | 1365 | 3003 | 5005 | 6435 | 6435 | 5005 | 3003 | 1365 | 455 | 105 | 15 | 1 = the Unciæ the 15 Pow |

*This Method, to me, is intirely New, and my own Contrivance: But ſee a Third Way a little farther.*

6. It is obſervable, that the Unciæ encreaſe till you come to the middle, and then decreaſe; there being the ſame Unciæ to the Right-hand of the higheſt, as towards the Left; ſo that when you

you have found half the Unciæ, you have all: Obſerving farther, That there are two higheſt Unciæ in the middle of the odd Powers, as the 3d, 5th, 7th, &c. Powers.

And that the Exponents of ſeveral Members after you come to the higheſt, do change, as in the 10th Power in the former Table above; the Member toward the Left-hand of the higheſt is $210u^6b^4$; but toward the Right of the higheſt, the Power is $210u^4b^6$, which is the ſame, only the Exponent's changed: So the 2d Member towards the Left-hand is $120u^7b^3$; but 'tis $120u^3b^7$: The 3d $=45u^8b^2$; but 'tis $45u^2b^8$. } toward the Right.
The 4th $=10u^9b$; but 'tis $10ub^9$, &c. }

7. You may ſee that four of the Unciæ, *viz.* the two firſt and two laſt are always known in all Powers, being one; and the Exponent of the Power. And by theſe Rules the Unciæ, the Powers, and Exponents of any Binomial Root may be found with much leſs Trouble than the common Way, or any I have ſeen before.

8. But the Unciæ of any Term, or Member of any of the Powers in the Table above, may be found without any of the previous Unciæ of Powers or their Terms, by eaſy Operations in Multiplication and Diviſion, as by theſe 6 Examples.

*A Third Way of finding the Unciæ.*

| Term or Member. | Power. | Factors. | Products. | | Quote or Unciæ required. |
|---|---|---|---|---|---|
| 4 of the | 4 | 1,2,3 = | 6 | the Diviſor | = 4 |
| | | 4,3,2 = | 24 | the Dividend | |
| 4 | 5 | 1,2,3 = | 6 | the Diviſor | = 10 |
| | | 5,4,3 = | 60 | the Dividend | |
| 5 | 10 | 1,2,3,4 = | 24 | the Diviſor | = 210 |
| | | 10,9,8,7 = | 5040 | the Dividend | |
| 7 | 10 | 1,2,3,4,5,6 = | 720 | the Diviſor | = 210 |
| | | 10,9,8,7,6,5 = | 151200 | the Dividend | |
| 5 | 15 | 1,2,3,4 = | 24 | the Diviſor | = 1365 |
| | | 15,14,13,12 = | 32760 | the Dividend | |
| 4 | 16 | 1,2,3 = | 6 | the Diviſor | = 560 |
| | | 16,15,14 = | 3360 | the Dividend | |

Here it may eaſily be ſeen that I tell the Figures from one to within one of the Term given, and multiply them together *for a Diviſor*.

And

And telling as many from the Power given backward; I multiply them one in another for a Dividend; whence the Unciæ of any Term of any Power ariſeth by Diviſion.

For ſuppoſe $p$ = the Exponent of any Power, and $t$ = the Term, and $u$ = the Unciæ ſought, the Rule will be

$\frac{p \times p-1 \times p-2 \times p-3, \&c.}{t-1 \times t-2 \times t-3 \times t-4, \&c.} = u$ (in this Example the Unciæ of the 5th Term or Member of the 10th Power = 210.)

9. Any Number may be ſquared by the Canon exhibited in the Square, or 2d Power of the above Table of the Powers raiſed from a Binomial Root. For Example; What is the Square of $1234^9$?

The Analytical Square or Canon directing us how to work, is $uu+2ub+bb$. Now ſuppoſe 1200, the 2 firſt Figures = $a$

And 34 the 2 next the Right-hand = $b$

It follows, That $u^2 = 1200^2 = 1440000$

$2ub$ or $u \times b \times 2$ is = 81600 = $1200 \times 34 \times 2$

$bb$ (or $34 \times 34$) is = 1156

Sum = 1522756 = $uu+2ub+bb$. = $1234^2$

10. And by the Cube or 3d Power we have a Rule how to Cube any Number. Example: What is the Cube of 2232?

The Canon = $u^3+3u^2b+3ub^2+b^3$.

I ſay 2000 = $u$

200 = $b$

8000000000 = $uuu$ or $2000^3$

2400000000 = $3uub$ = $2000^2 \times 200 \times 3$

240000000 = $3ubb$ = $2000 \times 200^2 \times 3$

8000000 = $bbb$ = $200^3$.

10648000000 = Sum or $2200^3$ or $uuu$ in the 2d Operation; where

2200 = $a$

and 30 = $b$

43560000 = $3uub$ = $2200^2 \times 30 \times 3$

594000 = $3ubb$ = $2200 \times 900 \times 3$

27000 = $bbb$ = $30^3$.

11089567000 = Sum, or $2230^3$. So in the 3d Operation 2230 =

and 2 =

2983740 = $3uub$ = $2230^2 \times 2 \times 3$

26760 = $3ubb$ = $2230 \times 4 \times 3$

8 = $bbb$ = $2^3$.

11119431168 = $2232^3$ = $uuu+3uub+3ubb+bbb$.

Thi

This proves that the third Power in the foregoing Table is the true Cube of $a+b$, as is here demonstrated by Numbers working with the Species exactly according to the Canon, without having regard to the way of placing Numbers in common *Multiplication*.

11. But that which renders the Powers of a Binomial Root more admirably useful is, that they shew plainly the Reason of all those mysterious Rules given in Extracting the Root of any Power, as the Square, Cube, Biquadrat, &c. Roots in the former Part of this Book mentioned: For Example,

In Extracting the Square Root of 1522756.

| | | |
|---|---|---|
| | . . . . | Canon for the Square Root. |
| | 1522756 (1000 $=a$ | $=aa+2ab+bb$. |
| | 1000000 $=aa$ deduct. | |
| ÷ this Rest by $2a=2000$) | 522756 Rests, (200 $=b=$ the Quote. | |
| | 400000 $=2ab$. | |
| | 40000 $=bb$. | |
| | 440000 $=2ab+bb$, deduct from the last Rest. | |
| ÷ by $2a=2400$) | 82756 remains, & now 1000 + 200 $=1200=a$ | |
| | (30 $=b$, the Quote. | |
| | 72000 $=2ab$. | |
| | 900 $=bb$. | |
| | 72900 $=2ab+bb$, deduct from the last Re- | |
| | (mainder. | |
| ÷ by $2a=2460$) | 9856 $=$ Rest; & now 1230 $=$ the 3 Roots $=a$. | |
| | (4 $=b=$ the Quote or last in the root sought. | |
| | 9840 $=2ab$. | |
| | 16 $=bb$. | |
| | 9856 $=2ab+bb$, deduct from the last Rest, | |
| | 0 Rests. | |

And the Reason of Extracting the Cube Root, will appear in the following Example, proceeding according to this Canon $aaa+3aab+3ab^2+b^3$.

Extract the Cube Root of 1111943111 68 (2000=*a*.
8000000000 =*aaa*, deduct.

12006000) 3119431168 resteth, which divide by $3a+3aa$,
(200=*b*. (gives 200=*b*.
6000 =3*a*, the Treble Root.
12000000 =3*aa*, = the Treble Square of
(the Root *a*.
12006000 = $3a+3aa$, the Divisor.
8000000 =*bbb* or $200^3$.
2400000000 =3*abb*=6000×40000.
2400000000 =3*aab*=4000000×200×3.
2648000000 =*bbb*+3*abb*+3*aab*=the subtrahend
(to deduct from the last resolved.
14526600) 471431168 resteth the second Resolvend.
(30=*b*, =the Quote.
6600 = Treble Root 2200=3*a*.
14520000 =3*aa*=$2200^2$×3.
14526600 =3*a*+3*aa*=the Divisor for 2d
(Resolvend, gives 30=*b*.
27000 =*bbb*=$30^3$.
5940000 =3*abb*=6600×900.
435600000 =3*aab*=4840000×30×3.
441567000 =*bbb*+3*abb*+3*aab*=the subtrah.
14925390) 29864168 = third Resolvend Rests.
(2=*b*, the Quote.
6690 =3*a*, the Treble Root =2230×3.
14918700 =3*aa*=4972900×3.
14925390 =3*a*+3*aa*, the Divisor for the 3d
(Resolvend.
8 =*bbb*=$2^3$.
26760 =3*abb*=6690×4.
29837400 =3*aab*=14918700×2.
29864168 =*bbb*+3*abb*+3*aab*, the Sum or Sub-
(trahend, which taken from the 3d Resolvend.
(0) there Rests (0)
So the Root is found 2000+200
+30+2, And

And the Proceſs according to the Canon of the third Power of $a+b$, is repeated three times, *viz.*

| | | | | | |
|---|---|---|---|---|---|
| 1ſt, | 2000 = $a$, and | 200 = $b$ | or | 2000 | |
| 2dly, | 2200 = $a$, and | 30 = $b$ | + | 200 | = 2232, the Root required. |
| 3dly, | 2230 = $a$, and | 2 = $b$ | + | 30 | |
| | | | + | 2 | |

Theſe Examples, according to the 2d and 3d Powers of the Binomial Roots in the Table above, fully ſhew the Reaſon of the Method uſed in the Extraction of the Square and Cube Roots. And by the Obſerving of the Canon given by the Powers, the Root of any other, even the ſquared Surſolid may be extracted: As I did that of the Biquadrat in *Sect.* 6. of *Chap.* 1. Article the 2d; having never ſeen it done but only proceeding purely as the 4th Power directs.

IV. The Roots of Algebraick Quantities are either thoſe of Rationals or Surds. 1. The Roots of Rational Quantities, whether theſe Quantities be compounded by Multiplication of Single, Binomial, or Reſidual Roots are thus expreſs'd.

1. How Single Roots of Powers are expreſs'd in Species and Numbers.

| | Powers. in Species. | in Numb. | Their Roots. Species. | Numb. |
|---|---|---|---|---|
| Square | $uu$ = | 4 | $u$ = | 2 |
| Cube | $bbb$ = | 27 | $b$ = | 3 |
| Biquadrat | $c^4$ = | 256 | $c$ = | 4 |
| Surſolid | $d^5$ = | 3125 | $d$ = | 5 |
| Squared Cube | $h^6$ = | 46656 | $b$ = | 6 |
| 2d Surſolid | $k^7$ = | 823543 | $k$ = | 7 |

2. How Binomials and Reſidual Roots are extracted.

| Binomial Powers. Species. | N°. | Their Roots. Species. | N°. | Reſidual Powers. Species. | N°. | Their Roots. Species. | N°. |
|---|---|---|---|---|---|---|---|
| $uu+2ub+bb$ = | 25. | $u+b$ = | 5 | $dd-2du+uu$ = | 9. | $d-u$ = | 3 |
| $uu+2u+1$ = | 9. | $u+1$ = | 3 | $bb-2b+1$ = | 4. | $b-1$ = | 2 |
| $16hh+8hk+kk$ = | 961. | $4h+k$ = | 31 | $16hh-8hk+kk$ = | 289. | $4h-k$ = | 17 |
| $bbuu+2buu+uu$ = | 64. | $bu+u$ = | 8 | $bbuu-2buu+uu$ = | 16. | $bu-u$ = | 4 |



2*dly*, Surd

3*dly*, Surd Roots, or Ropts of Surd Quantities are expreſs'd thus by their own Radical Signs.

## Simple Surd Quantities.

| | Powers. Roots. | Powers. Roots. |
|---|---|---|
| Square Root of | $z = \sqrt{z}$ or in N°. | $171 = \sqrt{171}$ |
| Cube Root | $l = \sqrt[3]{b}$ . . . . . | $315 = \sqrt[3]{315}$ |
| Biquadrat. | $c = \sqrt[4]{c}$ . . . . . | $517 = \sqrt[4]{517}$ |
| Surſolid | $d = \sqrt[5]{d}$ . . . . . | $735 = \sqrt[5]{735}$ |
| Squared Cube | $g = \sqrt[6]{g}$ . . . . . | $379 = \sqrt[6]{379}$ |
| 2d Surſolid | $b = \sqrt[7]{b}$ . . . . . | $1231 = \sqrt[7]{1231}$ |
| Squared Biquad. | $k = \sqrt[8]{k}$ . . . . | $339 = \sqrt[8]{339}$ |
| Cubed Cube | $l = \sqrt[9]{l}$ . . . . | $1129 = \sqrt[9]{1129}$ |
| Squared Surſol. | $m = \sqrt[10]{m}$ . . . | $1057 = \sqrt[10]{1057}$ |

## Compound Surd Quantities.

| Powers. | Roots. |
|---|---|
| $z+b$ . . . . . | $\sqrt{z+b}$ |
| $z+b+c$ . . . . . | $\sqrt[3]{z+b+c}$ |
| $z+b+c+d$ . . . . . | $\sqrt[4]{z+b+c+d}$ |
| $z+b+c+d-g$ . . . . . | $\sqrt[5]{z+b+c+d-g}$ |
| $z+b+c-d+g+b$ . . . . . | $\sqrt[6]{z+b+c-d+g+b}$ |
| $z+b+c-d+g-b-k$ . . . . | $\sqrt[7]{z+b+c-d+g-b-k}$ |
| $z+b-c+d+g+b+k+l$ . . . | $\sqrt[8]{z+b-c+d+g+b+k+l}$ |
| $z+b+c+d-g-b+k+l+m$ . . | $\sqrt[9]{z+b+c+d-g+k+l+m}$ |
| $17r+gb-bc+m$ . . . . . | $\sqrt[10]{17r+gb-bc+m}$ |

It muſt be obſerved, That the Sign connecting the Reſidual Root is always the ſame with that of the double Rectangle, *i. e.* Negative as under the 2d above.

Having fully ſhewn the Nature of Raiſing the Powers of Quantities, and Extracting the Roots, and illuſtrated the ſame by Numbers,

Numbers, it naturally leads me to *The Arithmetick of Surd Quantities*; which I shall also demonstrate by Numbers.

## SECT. VII. *The Algorithm of Surd Quantities.*

In regard there is often Occasion for Working the Roots of Species or Numbers, as by Addition, Substraction, Multiplication, *&c.* of them; and that it would be tedious and less accurate to find near the real Roots of Numbers, and impossible to be done of all Algebraick Quantities; therefore the Business of this Section is to give Rules for such Operations in Powers without regarding the Roots farther than by the Radical Signs prefixed: So that the compleat Powers of Quantities, or Numbers may hereby be added, substracted, *&c.* as well, (and much sooner) than if the real Roots were known, and the Work done by them. See Article 7. Sect. 1. of this Chapter.

### I. *Reduction of Surd Quantities.*

Reduction is here taught first, because Quantities are required to be reduced to a common Radical Sign before those that have different Signs can be multiplyed, divided, *&c.*

Example 1. Reduce $\sqrt{u}$ and $\sqrt[4]{c}$ to Surds having a common Radical Sign; or Surds Heterogeneal to Homogeneal. See the Operation.

$\underline{4_2 \quad 4}$ The common Radical Sign.
$\underline{2, \quad 1}$ = The lowest Terms of the given Radical Signs or Exponents.
$\underline{\sqrt[2]{u}, \quad \sqrt[4]{c}}$ = The Surds given to be reduced.
$\underline{\sqrt[4]{uu}, \sqrt[4]{c}}$ = The Answer or the Surds given in a common Radical Sign.

By the Operation 'tis plain, that first I take half the Radical Sign 2 (given) and put it over the other given, or 4; and half the given Radical Sign 4, and place it over the other given or 2, which are the lowest Terms of the given Radical Sign.

2*dly*, I multiply the Radical Signs given by those standing over respectively, and they produce 4 = the common Radical Sign required.

3*dly*, I

3*dly*, I raiſe the Surd Roots given to the Powers of the loweſt Terms of the given Radical Signs which ſtand over the reſpective Roots; as *u* raiſed to the 2d Power is *uu*, *c* in the 1ſt Power is $=c$, and placing the common Radical Sign before this *uu* and the *c*, I have the Anſwer $\sqrt[4]{uu}$ and $\sqrt[4]{c}$.

Example 2d ſhall be in Numbers, as to reduce $\sqrt[2]{9}$ and $\sqrt[4]{16}$ to a common Radical Sign.

See the Work by the ſame Rule as above; and in Multiplication and Addition 'tis ſhewn how theſe prove each other, or how the Totals agree, and conſequently prove the Truth of this Method of Reduction.

4, 4=Common Radical Sign.
2, 1=Leaſt Terms of given Radical (Signs, placed interchangeably.
$\sqrt[2]{9}, \sqrt[4]{16}$=Surds given.
$\sqrt[4]{81}, \sqrt[4]{16}$=Surds required, or thoſe given (reduced.

## II. *Multiplication of Simple Surds.*

In this Article I ſhall ſhew,

1. How to multiply a Surd by a Rational Number or Quantity.
2. To multiply a Surd in others of the ſame kind, or the ſame Radical Sign.
3. To multiply Surds together having different Radical Signs.

### 1. *To multiply a Surd in a Rational Number or Quantity.*

This is expreſſed two ways, for Example, to multiply $\sqrt[4]{c}$ by *b*, the Anſwer is very often expreſſed thus, $b\sqrt[4]{c}$ or *b* in the Biquadrat Root of *c*. But actually to multiply them is done by this

*Rule.*] Raiſe *b* up to the Power of the Surd given, as here *b* in the fourth Power is *bbbb*, and putting the ſame Radical Sign before, is $\sqrt[4]{bbbb}$, which being multiplied by $\sqrt[4]{c}$, gives $\sqrt[4]{bbbbc}$.

*Or in Numbers*, ſuppoſe $c=16$, and $b=8$, then $b^4=4096$, and $\sqrt[4]{16}$ multiplied in $\sqrt[4]{4096}=\sqrt[4]{65536}$ the Product, and that Root being Extracted is $=16$, the Product in a Rational Number.

To

To prove both which Ways of working, by raiſing the Power of the Rational Number, to be Genuine,

I ſay, $c$ being $=16$, its $\sqrt[4]{}$, $=2$, which multiplied by $b=8$, gives 16 as before.

2. *To multiply a Surd Number by a Surd, which are Homogeneal, or when the Radical Signs are the ſame.*

*Rule.*] Multiply the one by the other, and the Product is the Anſwer, prefixing one of the Radical Signs given.

*Example.* Multiply $\sqrt[3]{dr}$ by $\sqrt[3]{u}$, and the Product is $=\sqrt[3]{dru}$, and putting $u=8$; $r=9$; and $d=3$. It is thus done in Numbers, $\sqrt[3]{27}$ by $\sqrt[3]{8}=\sqrt[3]{216}=6$ the Product.

*Proved thus:* By the real Roots without Surds. I ſay $\sqrt[3]{27}=3$; and $\sqrt[3]{8}=2$, and 2 times 3 is 6, as before in the Numerical Way laſt above, which is the ſame as the Literal.

3. *To multiply Heterogeneal or Surds, having different Radical Signs.*

*Rule.*] Reduce them to a common Radical Sign, and then work as in the laſt.

*Example.* Multiply $\sqrt[4]{c}$ by $\sqrt[2]{u}$. In a common Radical Sign, the Surds are $\sqrt[4]{uu}$ and $\sqrt[4]{c}$; whoſe Product is $\sqrt[4]{cuu}$.

*Or in Numbers:* $c=16$; and $u=9$; or $\sqrt[4]{16}$ to be multiplied by $\sqrt[2]{9}$, which in a common Radical Sign are $\sqrt[4]{16}$ and $\sqrt[4]{81}$, whoſe Product is $\sqrt[4]{1296}=6$.

*Rule proved thus:* By the real Roots, for $\sqrt[4]{16}=2$; and $\sqrt[4]{81}=3$; and $3\times2=6$.

## III. Diviſion *of Surd Quantities.*

There is nothing of Difficulty in this Rule; for if the Quantities of the Dividend can be divided by thoſe of the Diviſor, and the Radical Signs are the ſame, work as is taught in *Diviſion of Algebra*, or of whole Numbers; as $\sqrt{ubc} \div$ by $\sqrt{bc}$ the Quote $=\sqrt{u}$, &c.

*Or*

*Or in Numbers*; $\sqrt{144} \div \sqrt{9} = \sqrt{16} = 4$, or $= 12 \div 3$. For Proof,

2. If the Radical Signs are not the ſame, they may be reduced to a common Radical Sign, and then divide the Power of the Dividend by the Diviſor; or put it over that of the Diviſor, as if the $\sqrt[4]{cuu}$ be divided by $\sqrt[2]{u}$. In a common Radical Sign they are $\sqrt[4]{cuu}$ for the Dividend, and $\sqrt[4]{uu}$ for the Diviſor; ſo the Anſwer is $\sqrt[4]{\frac{cxx}{xx}} = \sqrt[4]{c}$. And this proves the laſt Example in the laſt Rule, and is farther proved by dividing the Numeral Rectangle there by one Factor, for the Quotient will be the other Factor.

## IV. Addition *of Surd Quantities.*

This Rule is placed after *Multiplication*, becauſe that is often uſed in this Rule of *Addition*. I ſhall ſhew

1. How to add when there are the ſame Quantities and Radical Signs.

2. When there are different Quantities or Species, but the ſame Radical Signs, which Quantities are ſuppoſed Commenſurable.

3. How to add Quantities, having the ſame Species, but different Radical Signs, or when both the Radical Signs are different, and the Quantities incommenſurable.

1. To add $\sqrt[4]{u}$, $\sqrt[4]{u}$, and $\sqrt[4]{u}$.

*Rule.*] Multiply any 1 of the Surds by the whole Number of them, as here $\sqrt[4]{u} \times 3 = \sqrt[4]{81u}$, (obſerving the firſt Rule of *Multiplication* laſt above,) which is the Anſwer.

*Or in Numbers*: Add $\sqrt[4]{16}$, $\sqrt[4]{16}$, and $\sqrt[4]{16}$. Here 3 the Number of Roots raiſed to the fourth Power is 81; and $\sqrt[4]{16} \times \sqrt[4]{81} = \sqrt[4]{1296} = 6$ proved thus; $\sqrt[4]{16} = 2$, and 3 times 2 is 6, or $2+2+2$, which proves the Rule.

2. When two unequal Surds are Commenſurable, and have the ſame Radical Signs, and when divided by their greateſt common Meaſure, are reduced to ſuch Quantities as are the compleat Powers of ſome Quantities or Numbers; they may be added thus, ſuppoſe $\sqrt[3]{40}$ to be added to $\sqrt[3]{135}$.

*2d Rule.*]

*2d Rule.*] Divide the Surds by their common Measure $\sqrt[3]{5}$, and they are reduced to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, the Cube Roots of which are 2 and 3; the Sum of which is 5, which Sum of the Roots multiplied is $5\sqrt[3]{5}=\sqrt[3]{625}$ the Answer.

*And for Proof*, Extract the Cube Roots of the Surds given in the common way of Extraction, and you'll find the Sum $=\sqrt[3]{625}$ $=8.55$.

*How to add Quantities having the same Surds, but different Radical Signs; or when the Radical Signs are either different or the same, and the Surds are incommensurable.*

*Rule.*] There is no Occasion for farther Operation, than to put the Radical Sign + between the given Surd Roots: As

To add $\sqrt{cc}$ to the $\sqrt[3]{cc}$ and $\sqrt[4]{cc}$; the Sum is $\sqrt{cc}+\sqrt[3]{cc}+\sqrt[4]{cc}$.]

Or the Sum of $\sqrt{5}$, the $\sqrt[3]{6}$ and $\sqrt[4]{13}$, is $=\sqrt{5}+\sqrt[3]{6}+\sqrt[4]{13}$.

Or $\sqrt{3}+\sqrt{7}+\sqrt{12}$, *&c.* is the Sum of such Surds as $\sqrt{3}$, $\sqrt{7}$ and $\sqrt{12}$, *&c.*

There is a Rule which some pretend helps us to the Sum of these kind of Surds, when the Radical Sign is $\sqrt[2]{}$. The Rule is this; *To the Square Root* of the Sum of the Squares of the Surds given, add the double Rectangle of these Surds, and the Sum is the Answer.

*Example.* To $\sqrt{12}$, add $\sqrt{8}$; according to the Rule, the Sum of the Squares of the Surds is $=20$, its Root $\sqrt{20}$; and the double Rectangle of the Surds is $2\sqrt{96}=8\times12\times4=\sqrt{384}$, by the first Example of *Multiplication* of Surds. So by this Rule, the Sum of $\sqrt{12}+\sqrt{8}$ is reduced to $\sqrt{20}+\sqrt{384}$. I must confess I do not see the Use of the Rule, nor of the Alteration of the Surds, since to say the Answer is $\sqrt{12}+\sqrt{8}$ is more true and intelligible, than to call the Sum $\sqrt{20}+\sqrt{384}$.

## V. Subtraction *of Simple Surds.*

1. When the Surd Roots are commensurable or reduceable to Rational Quantities, as in the second Example in *Addition*; work

as is there directed, but when you come to the Roots, inſtead of taking their Sum, take their Difference, and multiply as in the ſame Example. To know the Difference between $\sqrt[3]{40}$ and $\sqrt[3]{135}$: Theſe $\div 5$ reduces them to $\sqrt[3]{8}$ and $\sqrt[3]{27}$, whoſe Roots are 2 and 3, and the Difference of thoſe Roots is 1, which multiplied by the common Meaſure $\sqrt[3]{5}$, gives $\sqrt[3]{5}$ for the Remainder or Difference.

But when the Surds are incommenſurable, as in the laſt Example in *Addition*, you have nothing to do but place the Sign (—) between the Surds given; ſo the Difference of $\sqrt{b}$ above $\sqrt{g}$, is $\sqrt{b}-\sqrt{g}$.

## VI. Addition *and* Subſtraction *of Compound Surds.*

1*ſt*, For commenſurable Surds, as thoſe under the ſecond Rule of *Addition* above, take this Example.

To $\sqrt{45}+\sqrt{294}$
Add $\sqrt{125}-\sqrt{96}$ Sum $=\sqrt{320}+\sqrt{54}$.

Here the Sum of $\sqrt{45}$ and $\sqrt{125}=\sqrt{320}$; and of $\sqrt{294}$ and $-\sqrt{96}=\sqrt{54}$ by the ſaid ſecond Rule.

2*dly*, For mixt Numbers, as Surds and Rational Numbers; ſee this Example.

| | |
|---|---|
| To $331+\sqrt[4]{768}-\sqrt[4]{80}$ | Or from $331+\sqrt[4]{768}+\sqrt[4]{80}$ |
| Add $153+\sqrt[4]{243}+\sqrt[4]{1280}$ | Take $153+\sqrt[4]{243}-\sqrt[4]{1280}$ |
| Sum $=484+\sqrt[4]{7203}+\sqrt[4]{80}$ | Refts $=178+\sqrt[4]{3}+\sqrt[4]{1296}$ |

I need give no farther Examples, becauſe the *Addition* and *Subſtraction* of theſe Surds are done by the ſelf-ſame Rules as thoſe called *Simple Surds*, as by the Rule under the ſecond Example, and in *Subſtraction* of thoſe Surds.

The ſecond Example laſt foregoing is done thus: To add $\sqrt[4]{768}$ to $\sqrt[4]{243}$; I divide them by the common Meaſure $\sqrt[4]{3}$, which reduceth them to $\sqrt[4]{256}$ and $\sqrt{81}$, whoſe Roots are 4 and 3, whoſe Sum

Sum is 7p. so $7\sqrt[4]{3}$ is the Answer, or $7^4\text{R}\sqrt{3} = \sqrt{7203}$, and so of the Reft.

*The Demonſtration of the Rule for this laſt, as under* Article IV. Rule 2.

This I ſhall do by this plain Inſtance of Numbers adapted on purpoſe, as in the Margin; where $\sqrt{64} + \sqrt{64}$, are given to have their Sum found.

$\sqrt{4})\ \sqrt{64}+\sqrt{64} =$ the ſuppoſed Surds given.
Quotes $\sqrt{16} = 4$ Roots.
Quote $= +\sqrt{16} = 4$

$8 =$ Sum of the Roots.
and $8\sqrt{4} = \sqrt{256} = 16$ the Anſwer.
$= \sqrt{64}+\sqrt{64} = 8+8.$

1. Theſe divided by $\sqrt{4}$, are reduced to the Root $\sqrt{16}+\sqrt{16}$, whoſe real Roots are $4+4=8$. And $\sqrt{4}\times 8$, as by the firſt Rule of *Multiplication* above, gives the $\sqrt{256}$, whoſe Square Root being 16, is = the Sum of the Roots of the two Surds given.

## VI. Multiplication *of Compound Surds.*

*Three Examples or Varieties done in Species two Ways.*

*The three Examples in Species proved by Numbers.*

Example 1.

To multiply $b \sqrt{ac}$ in $ac+\sqrt{b}$ } See the Operation, alſo in Numbers { $4 \sqrt{64}$ / $2+\sqrt{16}$ } Multiply as the Species.

$abc\sqrt{ac}+b\sqrt{abc}$ = the Prod. or in Numbers $= 8\sqrt{64}+4\sqrt{64}\sqrt{6} = 192$

Or reduced into all Surds. $= \sqrt{aaabbccc}+\sqrt{abbbc}$

For $4\times 8\ (=\sqrt{64}) = 32$
& that $32\times 2+4\ (=\sqrt{16}) = 6 = 192$

Example 2.

To multiply $b \sqrt{ac}$ in $ac\sqrt{b}$

Example 2d proved thus:

$4\sqrt{64}$ } Multip'y as the
Multiply by $2\sqrt{16}$ } Species are.

$abc\sqrt{abc}$ = the Product; or in Numbers $= 8\sqrt{64}\sqrt{16} = 256$

Or reduced to Surds. $= \sqrt{aaabbbccc}$

For $4, \times, 8\ (=\sqrt{64}) = 32 = 256$
And $32\times\ 2\times 4\ (=2\sqrt{16}) = 8 = 256$

Example 3.

To multiply $b+\sqrt{ac}$ in $ac+\sqrt{b}$

$abc+ac\sqrt{ac}+b\sqrt{b}+\sqrt{abc}$

Or more in Surds $=abc+\sqrt{aaaccc}+\sqrt{bbb}+\sqrt{abc}$

Or in Numbers thus: $4+\sqrt{64}$ × in $2+\sqrt{16}$ } By the 3d Example.

$=8+2\sqrt{64}+4\sqrt{16}+\sqrt{64}\sqrt{16}=72$

And so 'tis in Reality for $4+\sqrt{64}=12$

And $2+\sqrt{16}=6$

And $12\times 6=72$

1. *Note*, That in multiplying $b$, multiplied in the Square Root of $ac$, by $ac$ more the Square Root of $b$; I say $b\sqrt{ac}$ by $ac=abc$ $\sqrt{ac}$, and stands so in the Example 1. More $b$ (in the Multiplicand) $\sqrt{abc}$, &c.

2. *Note*, In like manner by the Numbers, Example 1. I say 2 times $4\sqrt{64}$ is $8\sqrt{64}$. 2*dly*, $4\sqrt{64}$ by $\sqrt{16}$, is $=4\sqrt{64}\sqrt{16}$; so the Product is $=8\sqrt{64}+4\sqrt{64}\sqrt{16}$. Now the Square Root of 64 is 8, and of 16, 4, so that this Product is $8\times 8=4\times 8\times 4=192$: And that this is right multiplied, will appear by the Numbers given, or $4\times$ in 8, (which 8 is the Square Root of 64,) gives 32 for the Value of the Multiplicand.

And 2, more 4, (which 4 is the Square Root of 16,) makes 6, which is the Value of the Multiplier; so 6 times 32 is $=192$ as before, which is a full Proof of the Rule by the first Example, and the rest are performed and proved much after the same manner, which it would be needless to insist farther upon.

*Example* 4. To multiply $\sqrt[3]{u-cd}$ by $\sqrt[3]{u+cd}$: See the Operation in the Margin.

In Numbers demonstrated thus; $u=8, c=1, d=2$.

Then $\sqrt[3]{8-2}$

by $\sqrt[3]{8+2}$

$\sqrt[3]{64}-\sqrt[3]{64}+\sqrt[3]{64}-4$

$\sqrt[3]{64}-4=$ the Sum or Product contracted.

Now the Cube Root of $64=4$, and $4-4=0$

So the $\sqrt[3]{8}=$ the Multiplicand $=2$, and $2-2=0\times 2$ in the Multiplier $=0$ (the Proof.

$\sqrt[3]{u-cd}$

$\sqrt[3]{u+cd}$

$\sqrt[3]{uu}-\sqrt[3]{cccdddu}+\sqrt[3]{cccdddu}-ccdd$

Prod. $=\sqrt[3]{uu}+\sqrt[3]{cccdddu}-\sqrt[3]{cccdddu}-ccdd$

Product contracted $=\sqrt[3]{uu-ccdd}$

5. Hence

5. Hence it appears, that if you multiply two Surd Numbers together, whose Radical Signs are $\sqrt[2]{}$, the Square Root of the Product is the Answer in a Rational Number. But if the Root cannot be Extracted without a Remainder, divide the Product by the greatest Square Number, that will do it without Remainder; so the Root of that Divisor in the Quote is the Answer, as in the first Case, $\sqrt{16}$ in $\sqrt{9} = \sqrt{144} = 12$ the Answer. And in the second Case, $\sqrt{72}$ in $\sqrt{8} = \sqrt{576}$, which $\div 144$ quoteth 4, and the $\sqrt{144} = 12$; so that $12\sqrt{4}$ is the Answer.

## VII. Division *of Compound Surds.*

This will appear very easy to those who understand common Division in Species, as before taught, and Multiplication of Surds.

*Example*, Divide $\sqrt{bu} + \sqrt{du}$ by $\sqrt{u}$; see the marginal Work.

Quote.

$\sqrt{u}$) $\sqrt{bu} + \sqrt{du}$ ($\sqrt{b} + \sqrt{d}$

$\sqrt{bu}$

$\sqrt{du}$

$\sqrt{du}$

0 Rests.

*Example* 2. To divide $abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}$ by $b + \sqrt{ac}$, or in Numbers, $8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}$, $\sqrt{16}$ by $2 + \sqrt{16}$.

Done Literally thus:

$b + \sqrt{ac}$) $abc + ac\sqrt{ac} + b\sqrt{b} + \sqrt{abc}$ ($ac + \sqrt{b}$ = the Quote.

$abc + ac\sqrt{ac}$

$b\sqrt{b} + \sqrt{abc}$

$b\sqrt{b} + \sqrt{abc}$

Rests = 0

Done Numerally thus:

$2 + \sqrt{16}$) $8 + 2\sqrt{64} + 4\sqrt{16} + \sqrt{64}$ $\sqrt{16}$ ($4\sqrt{64}$ = Quote.

$8 + 4\sqrt{16}$

$2\sqrt{64} - 4\sqrt{16} + 4\sqrt{16}$

Or Contracted, and the Rest brought down. } $= 2\sqrt{64} + \sqrt{64} \cdot \sqrt{16}$

$2\sqrt{64} + \sqrt{64} \sqrt{16}$

0 Remains.

There

There is nothing of Difficulty in these Operations, observing to change the Signs of what you subtract: As in the Numeral Example, I am to take $4\sqrt{16}$ from $2\sqrt{64}$, therefore I say that $2\sqrt{64}-4\sqrt{16}$ is the Remainder, to which if I bring down $4\sqrt{16}$, the $+$ and $-$ destroy the $4\sqrt{16}$: So that bringing down to the said Remainder $2\sqrt{64}$, the rest of the Dividend, which is $\sqrt{64}\sqrt{16}$, I find that $2+\sqrt{64}$ is contained in $2\sqrt{64}$; $\sqrt{64}$ times, which I put in the Quote, and multiply and deduct, so the Remainder is (0).

*And the Truth is thus proved*, 1*st*, In that it agrees with the Literal Example. 2*dly*, As it is the Product of Example the Third foregoing, divided by the Multiplier. And 3*dly*, And most plainly thus:

*The Value of the several Members of the Numeral Dividend are,*

| | |
|---|---|
| 8 . . . . . | = 8 |
| $2\sqrt{64}$, or $2\times 8$ . . . . . | = 16 |
| $4\sqrt{16}$, or $4\times 4$ . . . . . | = 16 |
| $\sqrt{64}\sqrt{16}$, or $8\times 4$ . . . . . | = 32 |

Value of the Divisor $2+\sqrt{16}=2+4=6$) 72 = Sum (12 = Quote
(= $4+8=4+\sqrt{64}$ for Proof.

## Sect. VIII. *Algebraical Fractions.*

Although the Business of Fractions in Species are often used in the Solution of Analytical Questions: Yet since the Reader of this Part is supposed to have perfectly learnt *Reduction of Vulgar Fractions*, Chap. 2. Sect. 1. and there being so very little Difference between the Method of working Fractions in Numbers and Species, therefore I shall not need to enlarge much on this Section.

1. *Note*, The fifth Case is done thus: Multiply the Quantity given by the proposed Denominator, and place the Product over the Denominator for the Answer.

2. And the sixth Case is only placing the Integeral Quantity given as a Numerator, and a Unit for a Denominator.

3. And that the Quantities omitted, which are found in both the Terms, brings a Fraction into its lowest Terms.

*The*

## *The Cases of* Reduction *of* Algebraical Fractions.

| Number of Cases given. | To reduce. | Examples, two to each Case. | The Answers. | Done by Rules Ch. 2. Sect. 1. & Cases. |
|---|---|---|---|---|
| 1 | Mixt Quantities to Fractions. | $ub+\frac{b}{c}$ | $\frac{bcu+b}{c}$ | 1 |
| | | $u+\frac{b+c}{d}$ | $\frac{du+b+c}{d}$ | |
| 2 | An improper Fraction to a whole or mixt Quantity. | $\frac{bcu+b}{c}$ | $bu+\frac{b}{c}$ | 2 |
| | | $\frac{du+b+c}{d}$ | $u+\frac{b+c}{d}$ | |
| 3 | A Fraction to its lowest Terms. | $\frac{ubbd}{ubc}$ | $\frac{bd}{c}$ | 4 |
| | | $\frac{14udc+21dpu}{7dru}$ | $\frac{2c+3p}{r}$ | |
| 4 | Fractions of different to common Denominators. | $\frac{u}{c}, \frac{m}{r}, \frac{s}{t}$ | $\frac{urt}{crt}, \frac{cms}{crt}, \frac{crs}{crt}$ | 6 |
| | | $\frac{u+c}{m}, \frac{b+d}{gg}$ | $\frac{ggu+cgg}{ggm}; \frac{bm+dm}{ggm}$ | |
| 5 | A whole Quantity to a Fraction of the same Value, whose Denominator is given. | $b+c$ to that of Denominator $r$ | $\frac{br+cr}{r}$ | |
| | | $u+m$ to that whose Denom. $=r+s$ | $\frac{ru+mr+su+ms}{r+s}$ | |
| 6 | An Integeral Quantity to the Form of a Fraction. | $bu$ | $\frac{bu}{1}$ | |
| | | $cu-pr+hh$ | $\frac{cu-pr+hh}{1}$ | |
| 7 | A Compound to a simple Fraction. | $\frac{u}{b}$ of $\frac{c}{d}$ of $\frac{r}{ns}$ | $\frac{cru}{bdns}$ | |
| | | $\frac{3ub+cd}{m}$ of $\frac{r-5st}{b+n}$ | $\frac{3bru+cbr-15bstu-5cdst}{bm+mn}$ | 3 |

## II. Addition, Subſtraction, Multiplication, *and* Diviſion *of* Algebraical Fractions; *two* Examples *to each Rule.*

| | Propoſitions or Examples. | Solutions or Anſwers. | Rules. |
|---|---|---|---|
| Addition | 1. To $\frac{am}{n}$ add $\frac{2bb}{m}$ | Sum $= \frac{amm+2bbn}{mn}$ | Reduce the Fractions to a common Denominator, and add the Numerators as in Vulgar Fractions. Note, $-gr$ and $+gr$ deſtroy each other. |
| | 2. To $\frac{n-r}{g}$ add $\frac{r}{g+n}$ | $\frac{gn-gr+nn-rn+gr}{gg+gn}$ | |
| Subſtraction | 1. From $\frac{amm+2bbn}{mn}$ Take $\frac{2bb}{m}$ | Reſts $= \frac{am}{n}$ | Reduce to a common Denominator and work as in Vulgar reducing the Remainders to their loweſt Terms |
| | 2. From $\frac{gn-gr+nn-rn+gr}{gg+gn}$ Take $\frac{r}{g+n}$ . . . . . | Reſts $\frac{n-r}{g}$ | |
| Multiplication | 1. $\frac{2bn}{c}$ by $\frac{b}{n}$ | Product $\frac{2bbn}{cn}$ | Multiply the Numerators give $2bbn$, and the Denominator $= cn$ as in Vulgar Fraction reducing $n+\frac{b}{r}$ into a Fraction, as by the firſt Reduction |
| | 2. $n+\frac{b}{r}$ by $\frac{3mp}{s}$ | Prod. $= \frac{3mprn}{rs}+\frac{3bmp}{rs}$ | |
| Diviſion | 1. Divide $\frac{2bbn}{cn}$ by $\frac{b}{n}$ | Quote $= \frac{2bn}{c}$ | Work as in Vulgar, and reducing the Quotes to loweſt Terms, ſhall be as you ſee. |
| | 2. $\frac{3mprn+3bmp}{rs}$ by $\frac{3mp}{s}$ | Quote $= n+\frac{b}{r}$ | |

Theſe *Examples* above are very eaſy to ſuch as have ſufficiently acquainted themſelves with *Vulgar Fractions*, and the two Examples in *Subſtraction* prove thoſe in *Addition*, as thoſe in *Diviſion* do

do thoſe in *Multiplication*; there is no need to illuſtrate theſe by Numbers, becauſe that is done by comparing theſe with the like Examples in *Vulgar Fractions*. So that I proceed to

## SECT. IX. *Rules to reſolve Simple Equations.*

The Buſineſs of this is, (when unknown Quantities are intermixt with known,) to bring the unknown Quantity to poſſeſs ſolely one Side of the Equation (commonly that toward the Left-hand) now theſe complex Quantities being ſo either by the unknown Quantity, being added to, ſubſtracted from, multiplied in, or divided by known, or by moſt or all of theſe together; therefore Equations are reſolved, or the Value of the unknown Quantity is diſcovered by *Addition*, *Subſtraction*, *Multiplication*, *Diviſion* and *Evolution*, &c. and that in a contrary Way to that by which the known and unknown Quantities are connected and placed together, *i. e.* thoſe of *Subſtraction* ſolved by *Addition*; *Addition* by *Subſtraction*; *Multiplication* by *Diviſion*; and thoſe divided and ſtanding Fraction-ways are ſolved by *Multiplication*, as by theſe Examples.

### Simple Equations Reſolved.

1. *By Addition* (*Note* $b=20$, $c=12$, $d=8$) And that $x$ is put for the unknown Quantity required.

| Literally | Numerally |
|---|---|
| $x-d=b-c$ | $x-8=20-12$ |
| Add $d$ — $d$ Add | Add 8 — 8 Add |
| Sum $x=b-c+d$ the Anſwer. | Sum $x=20-12+8=16$ |

2. *By Subſtraction.*

| | |
|---|---|
| $x+d=b+c$ | $x+8=20+12$ |
| Deduct $d$, — $d$ Deduct | Deduct 8, — 8 Deduct |
| Reſts $x=b+c-d$ the Anſwer. | $x=20+12-8=24$ |

3. *By Multiplication.*

| | |
|---|---|
| $\frac{x}{4}=\frac{dc}{b}$ | $\frac{x}{4}=\frac{96}{20}$ |
| $x=\frac{4dc}{b}$ = the Anſwer. | Anſwer $x=\frac{96\times4}{20}=19\frac{1}{5}$ |

4. *By Diviſion.*

$du+cu=5b$

$u=\frac{5b}{d+c}$ the Anſwer.

$8u+12u=20\times5$

$u=\frac{100}{8+12}=5$

---

5. *By Addition, and Subſtraction, or Tranſpoſition*

$u+c-d=b$

$u=b-c+d$ the Anſwer.

$u+12-8=20$

$u=20-12+8, =16$

---

6. *By Addition, Subſtraction, and Multiplication.*

$\frac{u-d}{2}+c=b-d$

$u-d+2c=2b-2d$

$u=2b-d-2c$ the Anſwer.

$\frac{u-8}{2}+12=20-8$

$u-8+24=40-16$

$u=40-16+8-24$

Or $u=8$ . . . .

---

7. *By Addition, Subſtraction, Multiplication, and Diviſion.*

$\frac{cu+du}{b}+d-b=5c$

$cu+du+bd-bb=3bc$

$cu+du=3bc-bd+bb$

$u=\frac{3bc-bd+bb}{c+d}$ the Anſwer.

$\frac{12u+8u}{20}+8-20=3\times12$ or $36$

$12u+8u+20\times8-20\times20=36\times20$

$12u+8u=720-160+400$

$u=\frac{720-160+400}{12+8}$ (or $u=48$)

*Eight General Rules for reſolving Simple Equations, applied to the foregoing Examples.*

1. *When one or more known Negative Quantities are connected with an unknown Affirmative, they are ſeparated, and the unknown made to poſſeſs ſolely one Side of the Equation (which is the ſame as to ſay that ſuch Equations are Reſolved) by* Addition *only.* So in the Example of reſolving by *Addition*, I add the Quantity connected with ($u$), as ($d$) to both Parts of the Equation, and the Sum is very plainly $u=b-c+d$.

*The Reaſon of which is plain, and ſhews the Foundation of Tranſpoſition:* (which Word ſee in the Alphabetical Account near the beginning of this Section) For adding the ſame Quantity or Number to both Parts of the Equation does preſerve the Equality; but this Advantage

vantage is gained thereby, that $+d$ added to $-d$, deſtroys the Quantity $-d$, ſo that it vaniſhes and leaves $u$ in Poſſeſſion of one Side of the Equation, and that $(+d)$ being added to the other part of the Equation, makes $b-c+d$, to which $(u)$ is therefore found equal. And the Reaſon appears ſtill more plain in the Numerical Operation; for if $u-8$ be $=8$, (or $20-12$,) it muſt follow, that $u=8+8$ or 16, and this is no more than what is taught in *Subſtraction of intire natural Numbers.*

2. *When one or more known affirmative Quantities are connected with an unknown, that Equation is reſolved by ſubſtracting ſuch known Quantities from each part of the Equation.* Thus in the Example of the Reſolving by *Subſtraction*, $(d)$ taken from $+d$ (which is connected with $u$) leaves nothing but $(u)$ on that Side of the Equation; and $d$ deducted from the other part of the Equation, there reſts $b+c-d$, to which therefore $(u)$ is equal, and the Equation is Reſolved.

*The Reaſon of this is evident* from what was ſaid of adding in the firſt Rule above; *for if equal Numbers or Quantities be either added to or taken from equal, the Sums or Remainders will be equal.* And the Reaſon plainly appears alſo in the Numerical Work; for if $u+8$ be $=32$, it follows that $u=32$, abating only the 8.

3. When in an Equation, there are ſeveral known Quantities on the ſame Side with the unknown, ſome Affirmative, and ſome Negative; it follows from the above Rules, that you may tranſpoſe ſuch known Quantities to the other Side of the Equation, changing their Signs, as in the fifth Example: And this you may do both ways to the Right, or thence toward the Left-hand of the $=$, and that with either known or unknown Quantities, keeping cloſe to the Rules above, your own Reaſon, and the Nature of the Queſtion: I mean, you will not think it reaſonable to make the unknown Quantity a Negative, by tranſplacing it, when the Reaſon and Nature of the Queſtion does dictate, that what is on the other Side of the Equation, is ſo far from being equal to $-u$, that it ought to be at leaſt equal to $u$, or to ſome abſolute Number or Quantity which $-u$ is not.

4. When in an Equation, the unknown Quantity is divided by ſome Number or known Quantity, poſſeſſing one Side of the Equation; then every Member of ſuch Equation muſt be multiplied by the Denominator of the Fraction, whereof the unknown Quantity is the Numerator, as in the third Example; I multi-

ply $\frac{u}{4}$ by 4, produceth $u$; and $\frac{dc}{b}$ by 4, produceth $\frac{4dc}{b}$; and so the Equation is resolved, the unknown Quantity being cleared and found $= \frac{4dc}{b}$, as the Numeral Work farther demonstrates; for equal Quantities multiply'd by equal, produce equal, as well as adding and subtracting.

*The Reason is very evident:* For if a fourth Part of $u$ be $= \frac{dc}{b}$, it must follow, that the whole ($u$) must be equal to four times the $\frac{dc}{b}$; and the like holds for any other Divisor of the unknown Quantity, or Denominator as aforesaid.

5. Hence it appears, that in multiplying by the Denominator of such a Fraction, 1*st*, *That every Member of both Parts of an Equation must be multiplied.* And 2*dly*, *That the Numerator of that Fraction is multiplied by putting it down as a whole Number*, as $\frac{1}{2}x$ by 2, is $= 1$; $\frac{3}{4}$ by 4, is $= 3$; $\frac{5}{6}$ by 6, is $= 5$, *&c.*

6. *When Quantities known and unknown stand multiplied together in an Equation possessing one Side thereof, such Equations are solved by* Division; *in which Case the Quantities so multiplied in the unknown, are to be your Divisor*; as in the fourth Example or Division above. To separate $u$ (in the two Members $du+cu$) from the known Quantity $d$ and $c$; I divide both Parts of the Equation by $d+c$, and the Quotient is $u = \frac{5b}{d+c}$; *for equal Quantities divided by one and the same Quantity, must exhibit equal Quotients*; and at the same time you free one Side of the Equation of complex Quantities, and shew the known, which the unknown Quantity is equal to. Thus in the Numeral Example, the Matter is plain, for where $8u+12u = 100$; $20u$ is $= 100$; therefore $u$ only must be equal to $\frac{100}{20}$, which is the same as in the Process, according to the Rule above; for $8u+12u \div 8+12 = u$; and $20 \times 5$, or $100 \div 8+12 = 5$.

*But Note*, That when you have two Members, one mixt with a known Quantity, the other intirely unknown; in this Case, the known $+$ or $-1$ is your Divisor, as if $8u-u = 49$, then $u = \frac{49}{8-1}$; and if $6u-u = 25$, then $u = \frac{25}{6-1} = 3$.

7. When (as in the seventh Example) 'tis found that unknown Quantities are so mixt with known, as to have Products, Quotients,

ents, Sums, and Remainders; you have nothing to do but to work as by theſe foregoing Rules, as in $\frac{cu+du}{b}+d-b=3c$: For if the whole is multiplied by $b$, it gives the next Step $cu+du+bd-bb=3bc$. And if $bd-bb$ be tranſpoſed, (as being all known Quantities,) the third Step ſtands thus; $cu+du=3bc-bd+bb$. And if the third Step be divided by $c+d$, (becauſe thoſe are the known Quantities that are multiplied in the unknown,) the fourth Step or Quote, which is the Anſwer is $u=\frac{3bc-bd+bb}{c+d}$; each of which Steps appears ſtill more plain in the Literal Work, which ſhews in this laſt Equation $u=48$.

8. There is one other Thing to be obſerved, but that is rather in compoſing or forming the Equation, than in the Reſolution, *i. e.* If $u.\ c::b.\ d.$ what is the Value of $u$?

Here the ſecond Step is found, as being the Rectangle of the two Extreams, which is equal to that of the means. And dividing by $d$, gives the Canon. See the Words *Analogy* in the beginning of this Chapter, and *Equation*. And if you would reduce an Equation into an Analogy, it is as $u$ to $c$ or $b$; ſo is the other to $d$ in the two Examples above, or

$u.\ c::b.\ d$
$ud=bc$
$u=\frac{cb}{d}$ the Anſwer.

$\left\{\begin{array}{l} u.\ b::c.\ d \\ u.\ c::b.\ d, \text{ \&c.} \end{array}\right.$

## Sect. X. *Reſolution of Quadratick Equations.*

What ſimple and adfected Quadraticks are, is ſhewed in the foregoing Definition of Terms near the beginning of this Chapter; where under the Words *Simple Quadraticks*, I have ſaid enough relating to that kind. As for adfected Quadratick Equations, (which Word ſee,) there are by ſome Algebriſts reckoned three Sorts, but I make but two really diſtinct; to one of which, all other adfected Equations are reducible. I ſhall give Examples,

1*ſt*, Of the two diſtinct Kinds of Square adfected Equations.
2*dly*, Of thoſe reducible thereto, and how performed.

II. *Shew how theſe Equations are Reſolved by compleating the Square.*

III. *How Reſolved by Subſtitution; how done four different Ways, two of which (being the beſt) are new.*

1*ſt*, The

1*ſt*, The two diſtinct Forms of Quadratick Equations are theſe only.

| | In Species. | In Numbers. |
|---|---|---|
| 1*ſt*, | $uu+dz=bc$ . . . . . | $uu+6u=135$ |
| 2*dly*, | $uu-du=\frac{bc}{5}$ . . . . . | $uu-6u=27$ |

*2dly, Equations reducible to one of the two Forms above, are theſe and ſuch like.*

| Equations given. | | Reduced to theſe Quadraticks. |
|---|---|---|
| 1ſt, | $du-uu=m$ | $uu-du=-m$, as the 2d Form. |
| 2dly, | $u+b=\frac{c-d+b}{2cu}$ | $uu+bu=\frac{c-d+b}{2c}$, as the 1ſt Form. |
| 3dly, | $u=\frac{c+db}{3bu}+b$ | $uu-bu=\frac{c+db}{3b}$, as the 2d Form. |
| 4thly, | $u=\frac{cu+tu}{3bzu}-bb$ | $uu+bbu=\frac{c+t}{3b}$, the 1ſt Form *cc.* |

*Notes upon theſe Reductions.*

1*ſt*, That the firſt is performed by changing the Signs of all the Members, (as is directed in Rule the third foregoing to *Simple Equations*,) which makes $-m=-du+uu$, which is the ſame as to ſay $uu-du=-m$, as in the ſecond Form; though ſome make this a diſtinct Sort, but with no more Reaſon than the three ſubſequent; for the Square is compleated, the Root extracted, and the Canon produced exactly as in one of the two Kinds above-mentioned.

2*dly*, The ſecond Equation above is reduced, as taught in the 4th and 6th Rules to the Examples of *Simple Equations*, multiplying by $2cu$, and dividing by $2c$ each part of the Equation.

3*dly*, The third Equation above is reduced by the ſecond, fourth, and ſixth Rules to the Examples of *Simple Equations*, tranſpoſing $b$, multiplying by $3bu$, and dividing by $3b$.

4*thly*, The fourth is reduced by multiplying by $3buu$, expunging $z$ in each Member, and dividing the reſt by $3b$.

II. *The*

## II. *The Steps of resolving Quadratick Equations by compleating the Square.*

| Equations proposed. | Squares compleated. 1*st.* | Roots extracted. 2*dly.* | Canons exhibited. 3*dly.* |
|---|---|---|---|
| $uu+du=bc$ | $uu+du+\frac{dd}{4}=bc+\frac{dd}{4}$ | $u+\frac{d}{2}=\sqrt{bc+\frac{dd}{4}}$ | $u=\sqrt{bc+\frac{dd}{4}}-\frac{d}{2}$ |
| $uu-du=\frac{bc}{5}$ | $uu-du+\frac{dd}{4}=\frac{bc}{5}+\frac{dd}{4}$ | $u-\frac{d}{2}=\sqrt{\frac{bc}{5}+\frac{dd}{4}}$ | $u=\sqrt{\frac{bc}{5}+\frac{dd}{4}}+\frac{2}{d}$ |
| $uu+2cu=3b$ | $uu+2cu+cc=3b+cc$ | $u+c=\sqrt{3b+cc}$ | $u=\sqrt{3b+cc}-c$ |
| $uu+u=2b$ | $uu+u+\frac{1}{4}=2b+\frac{1}{4}$ | $u+\frac{1}{2}=\sqrt{2b+\frac{1}{4}}$ | $u=\sqrt{2b+\frac{1}{4}}-\frac{1}{2}$ |
| $uu+6u=135$ | $uu+6u+9=135+9$ | $u+3=\sqrt{135+9}$ | $u=\sqrt{135+9}-3$ |
| $uu-6u=\frac{135}{5}$ | $uu-6u+9=\frac{135}{5}+9$ | $u-3=\sqrt{\frac{135}{5}+9}$ | $u=\sqrt{\frac{135}{5}+9}+3$ |
| $uu+u=90$ | $uu+u+\frac{1}{4}=90+\frac{1}{4}$ | $u+\frac{1}{2}=\sqrt{90\frac{1}{4}}$ | $u=\sqrt{90\frac{1}{4}}-\frac{1}{2}$ |

*Rules to be observed in this Method (which is the best) of the Resolution of Quadratick Equations, applied to the Examples above.*

1*st*, *Whatsoever known Quantities are multiplied in the Root, or unknown, they are called the Co-efficients*; as $d$, $2c$, and 1 in the second Terms of the four first Examples: In the fourth of which there being no Co-efficient, therefore 1 must be taken for it, because a Unit is supposed to stand before every Quantity, as $u$ is $= 1u$, $c=1c$, &c.

2*dly*, *To complcat any Square having only the two first Members given*, is done by adding the Square of half the Coefficient: Thus in the first and second Examples, half the Coefficients $\frac{d}{2}$,

(the

(the Square of which is $\frac{dd}{4}$,) added to the two other Members ſtanding towards the Left-hand, the compleat Squares are $uu+du+\frac{dd}{4}$ and $uu-du+\frac{dd}{4}$. And in the third and fourth Examples $2c$ and $1$ are the Coefficients, the half of which are $c$ and $\frac{1}{2}$; the Squares of which Halves are $cc$ and $\frac{1}{4}$, the former of which compleats the Square of the two Members towards the Left-hand of the third Example, and the latter (or $\frac{1}{4}$) added, doth compleat the Square of the fourth Example: See at the beginning of this Chapter.

3*dly*, And becauſe this Addition of the Square of half the Coefficients is made to one part of the Equations, where the unknown Quantities are; therefore it muſt be added alſo to the other part of the Equations, *That ſo the Equality may be continued, (as is ſaid in the firſt and ſecond Rules to the Examples of* Simple Equations,) and thus the ſecond Column is made in all the ſeven Examples above.

4*thly*, The third Column from the Left-hand, are the Roots of thoſe in the ſecond.

1ſt, *For in all compleat Squares, the Square Roots are exactly thoſe of the firſt and third Member.*

2*dly*, *The Sign connecting the Quantities of the Root, is always the ſame as that before the ſecond Term of the compleat Square, or of the Equation given:* Thus the Roots of

| Squares. | Roots. |
|---|---|
| $uu+du+\frac{dd}{4}$ in the ſecond Column, | is $u+\frac{d}{2}$ in the third Column. |
| $uu-du+\frac{dd}{4}$ is . . . . . | $= u-\frac{d}{2}$ |
| $uu+2cu+cc$ . . . . . | $= u+c$ |
| $uu+u+\frac{1}{4}$ . . . . . | $= u+\frac{1}{2}$, &c. |

And for a Proof, any of theſe Roots being multiplied in it ſelf, produces the reſpective Square above.

3*dly*, The

*3dly*, The Root of the 1ſt Part being extracted, in order to diſcover the Value of the ſingle unknown Quantity; therefore the Roots of the Quantitys known, ſtanding on the other part of the Equation, (as is ſaid under the 3d of adding the Square of half the Co-efficient) muſt have their Roots extracted; which is done by prefixing the Radical Sign, becauſe they are generally Surds.

*4thly*, To reduce the 3d Column to the 4th, or to the Canons or Anſwers, you have nothing to do but to tranſpoſe the Root of the 3d Member of the compleat Square, as $\frac{d}{2}$, $c$, and $\frac{1}{2}$ in the 4 firſt Examples changing the Sign as taught.

*A farther Demonſtration or Proof of the Method for the Solution of Quadraticks, by the Numeral Operations above.*

In the Numeral Work theſe Numbers are equal to the Letters in the Specious, *viz.* $d=6$, $b=45$, $c=3$, and $u$ is found equal to 9.

Here (proceeding by the Rules for Solution above) in the Equation $uu+6u=135$, $u$ in the 4th Column is found $=\sqrt{135+9}-3$.

Now $135+9$ is $=144$, the Square Root of which is 12, whence take 3, and the reſt is $=9$, ſo $u=9$.

And $u$ being found $=9$ by the Canon, grounded upon the Method of Proceſs aboveſaid; if ſuppoſing $u=9$, the 2 Parts of the Equations propoſed prove equal, it fully proves the Truth of the Method for Solution.

So $uu+6u=81+54$ in the 5th Equation given, is $=135=$ the other part of that Equation given.

$uu-6u=81-54$ in the 1ſt part of the 6th Equation is $=27=^{1}_{3}{}^{5}$ the 2d part of that Equation.

And $uu+u=81+9$ in the 1ſt part of the 7th Equation is $=90$ in the 2d part of that Equation, *&c.*

Thus I have given the moſt plain and demonſtrative Rules and Examples for the underſtanding and proving the Method of ſolving Square Equations by the way of compleating the Square. I proceed to ſhew,

## III. *The Method of Solving Quadratic Equations by Substitution.*

| N° of Steps. | *Steps towards the Answer.* | *How each Step is produced.* |
|---|---|---|
| 1 | $uu + cu = 18d$ .... | = *Equation given.* |
| 2 | for $u$ is put $a - \frac{c}{2}$ in this 1st Case. | |
| 3 | $aa - ac + \frac{cc}{4}$ ..... | $\left\{\begin{array}{l} = \square a - \frac{c}{2} = uu \\ \text{or } a - \frac{c}{2} \end{array}\right.$ ☞ |
| 4 | $ac - \frac{cc}{2}$ ..... | $= a - \frac{c}{2} \times c - cu$ |
| 5 | $aa - xc + \frac{cc}{4} + ac - \frac{cc}{2} = 18d$ | = 3d Step + 4th. |
| 6 | $aa - \frac{cc}{4} = 18d$ ..... | = 5th Contracted. |
| 7 | $a = \sqrt{\frac{4}{18d + \frac{cc}{4}}}$ ..... | = 6th $uu$, and $\frac{cc}{4}$ + |
| 8 | $x + \frac{c}{2} = \sqrt{18d + \frac{cc}{4}}$ | $u + \frac{c}{2}$ put for $a$ |
| 9 | $u = \sqrt{18d + \frac{cc}{4}} - \frac{c}{2}$ | $\frac{c}{2}$ in the 8 + |

*A 2d Example see towards the right hand.*

| N° of Steps. | *Process toward an Answer.* | *Rules for each Step.* |
|---|---|---|
| 1 | $uu - cu = 9d =$ | *the Equation given.* |
| 2 | for $u$ is put $a + \frac{c}{2}$ in this 2d Case | $u = a + \frac{c}{2}$ } |
| 3 | $aa + ac + \frac{cc}{4}$ ..... | $\left\{\begin{array}{l} = \square a + \frac{c}{2} = uu \\ \text{or } a + \frac{c}{2} \end{array}\right.$ ☞ |
| 4 | $- ac - \frac{cc}{2}$ ..... | $= a + \frac{c}{2} \times -c$ |
| 5 | $aa + ac + \frac{cc}{4} - ac - \frac{cc}{2} = 9d$ | = 3d + 4th Steps |
| 6 | $aa - \frac{cc}{4} = 9d$ ..... | = 5th Contracted |
| 7 | $a = \sqrt{\frac{4}{9d + \frac{cc}{4}}}$ ..... | = 6th $uu$ and $\frac{cc}{4}$ + |
| 8 | $u - \frac{c}{2}\sqrt{9d + \frac{cc}{4}}$ .... | $\left\{\begin{array}{l} u - \frac{c}{2} \text{ put for} \\ \text{in the 7th Step} \end{array}\right.$ |
| 9 | $u = \sqrt{9d + \frac{cc}{4}} + \frac{c}{2}$ | $= -\frac{c}{2} +$ in the 8th Step |

I have here inserted the Operations at large, to make them more plain and easy; which the 3d and 6th Columns from the right hand do so fully contribute to, that I have little to add, only,

1. To put the Reader in mind, That the Signs connecting the substituted Quantities are always contrary to that which connecteth the 2 Members of the Square given, as is plain by the 2 Examples above.

2. That any Letter as well as $a$ (if it be not in the Equation given) + or less half the Co-efficient, as above, may be substituted.

3. In the 8th Step I omit $a$, and substitute in place thereof $u + \frac{c}{2}$ in the 1st Example, because if $a - \frac{c}{2} = u$, then by transposing $- \frac{c}{2}$, $a$ is $= u + \frac{c}{2}$, and in the 2d Example $a = u - \frac{c}{2}$.

A

### *A New and Better Way.*

But tho this is the only Way I know extant of ſolving theſe Equations by Subſtitution; *yet I am ſure 'tis a better, to make the Vowel and half the Co-efficient in the Quantitys given to be your Subſtitute:* and then it will fall right at the laſt, remembring that half the Co-efficient is to be added or deducted, and 'twill ſave ſome Steps of working. For example, putting $u - \frac{d}{2}$ for $u$, in the Equation $uu + du = bc$.

Here it only needs to be noted, that when the Sign connecting the 2 firſt Terms of the Equation given is +, I make the half Co-efficient added to the Surd Root of the Anſwer —; and the contrary.

$$uu - du + \frac{dd}{4} = \square\ u - \frac{d}{2}$$

$$du - \frac{dd}{2} = u - \frac{d}{2} \times d.$$

$$uu - \frac{dd}{4} = bc; \left\{\begin{array}{l}\text{the Sum of the 2}\\ \text{laſt contracted.}\end{array}\right.$$

$$u = \sqrt{bc + \frac{dd}{4}} - \frac{d}{2} = \text{the Canon}$$

But in truth this way of ſolving Equations by Subſtitution, is at the beſt not ſo brief and eaſy a Method as that by compleating the Square; for if that be well conſidered, there ariſes from it,

IV. *Theſe Examples and Rules for Solving moſt Quadratic Equations by Inſpection only.*

| *Examples.* Note, $c=3$, and $u$ found $=9$. | *Demonſtrations of the Truth of the Solutions or Canons.* |
|---|---|
| 1. Equations propoſed. $uu + 10u = 57c$<br>Canon .... $u = \sqrt{57c + 25} - 5$ | $\sqrt{57c + 25} - 5 = 9$: ſo $u = 9$ as *per* Canon 1.<br>and $57c = 171$: ſo is $uu + 10u = 171$ as *per* Equation 1. |
| 2. Equations propoſed. $uu - 6u = 9c$<br>Canon ..... $u = \sqrt{9c + 9} + 3$ | $\sqrt{9c + 9} + 3 = 9$: ſo $u = 9$ as *per* Canon 2.<br>$9c = 27$: ſo is $uu - 6u = 27$ as *per* Equation 2. |

*The Rules for Solving a Quadratic Equation by Inſpection.*

1. What I mean here, *by Inſpection*, is the putting down only the Canon or Anſwer without any previous Operation: To do which, take

take the Root of the Square of the unknown Quantity, which here is *u*.

2. Make the Sign of Equation, and to the right hand of that, place the Root of the known Quantity or Quantitys given, with the Sign √ before, as in Equation the 1ſt, $u = \sqrt{57c}$.

3. Square half the Co-efficient, and add to the laſt mentioned Quantity with the Sign + always before it, and it will be $u = \sqrt{57c + 25}$.

4. To the laſt add half the Co-efficient, when the Sign is —, and ſubſtract it when the Sign before the Quantity where the Co-efficient is, is +; as in this Example I put —5, becauſe 10*u* is +10*u*: ſo is the Canon $u = \sqrt{57c + 25} - 5$.

And thus you may give the Canon for any Quadratic Equation when reduced into either of the two Forms above.

Here is the proper place for giving the Geometrical Conſtruction of Simple and Quadratic Equations; which I omit for theſe four Reaſons: 1. Becauſe it is done by ſo many other Authors. 2. For that this Treatiſe is deſigned to be as eaſy as poſſible, in order to inſtruct and encourage the Tyro, rather than to make a ſhew of Learning. 3. It cannot be expected, where ſo many things are contained in one ſmall Volume. And, 4. It is ſufficiently proved and demonſtrated above, by comparing the Parts of the Canon and thoſe of the Equation given, as grounded on the Solution, eſpecially in the Numeral Operations.

## Sect. XI. *Containing for Exerciſe, ſome Queſtions reſolv'd by Simple and Quadratic Equations, and by various Poſitions.*

I. By *Simple Equations.* I had the following propoſed to me, which the Queriſt my Friend had a real Occaſion to know, as falling in his Buſineſs.

*Problem* 1.] 365 Pounds is received for the Uſe of a publick Building, and the Receiver was to have 4 *d per* Pound for what he paid in: what was the Poundage, and what paid in?

*Steps.*

| Steps. | *Done Literally thus.* | *The Column of Rules to both Ways* | *Numerally thus:* | Steps. |
|---|---|---|---|---|
| 1 | $u$ = the Poundage, and $b$ is = 365 | | $u$ = the Poundage. | 1 |
| 2 | $b-u$ = the Sum paid in. | | 365 — $u$ = the Sum paid in. | 2 |
| 3 | $60u = b-u$ | | $60u = 365-u$ | 3 |
| 4 | $61u = b$ ..... | — $u$ in 3d + the 4th ÷ 61 | $61u = 365$ | 4 |
| 5 | $u = \frac{b}{61}$ the Canon. | | $u = \frac{365}{61}$ = the Canon. | 5 |
| | So $\frac{b}{61}$ = the Poundage. | | $365 \div 61 = l.\ 5 : 19 : 8$ | |
| | And $b - \frac{b}{61}$ = the Sum paid in. | | and $365 - \frac{365}{61} = 359 : 0 : 4$ | |

*Problem* 2.] *The whole Length of a May-Pole is* 16 *Yards, the major part of which is broke by the Wind, tho not quite off, but so that the Top of the Pole extends from the Fracture to the Ground at* 8 *Yards distant from the Appearance of the Pole above the Surface of the Earth in which it was placed: how much is broken off, and how much is standing?*

I put $16 = k$ and $8 = n$ —

| Steps. | | *Rules*, viz. | | Steps. |
|---|---|---|---|---|
| 1 | $u$ = the Perpendicular (or (Part standing. | | $u$ = the part standing. | 1 |
| 2 | $k-u$ = Hypothenuse. | | $16-u$ = the Hypothenuse | 2 |
| 3 | $nn + uu = kk - 2ku + uu$ | $n$ ⊙ + 1st ◎ + 2d ◎ | $8^2 + u^2 = 16^2 - 2\times 16u + u^2$ | 3 |
| 4 | $2ku = kk - nn$ | $2ku +$, $nn +$ | $2\times 16u = 16^2 - 8^2$ | 4 |
| 5 | $u = \frac{kk-nn}{2k}$ = the Canon | 4th ÷ $2k$ | $u = \frac{16^2-8^2}{2\times 16}$ = the Canon | 5 |
| | *Demonstration of both.* | | | |
| | $k^2 = 256 - n^2 = 64 = 192 \div 2k = 32 = u = b.$ | | $16^2 = 256 - 8^2 = 64 = 192 \div 32 = 6$ = the part (standing. | |
| | Vid. *Euclid*'s Elem. *Lib.* 1. *Prop.* 47. | | | |

*Problem* 3.] *Numbers in Geometrical Progression being given to find the Rule for the Aggregate of any Series. This is shewn in* Progression, *Chap.* 2. *page* 82.

*Problem* 4.] *Supposing the Pendulum of a Clock which vibrateth Seconds (or* 60 *times in a Minute) be* 39.2 *Inches in length; how long must a Pendulum*

*dulum be to, vibrate as often in a Minute as it hath Inches in length?* This also was a Question propoſed for me to anſwer, which I did thus:

I put $k=60$ Seconds: and $n=39.2$ Inches.

| Steps. | $n=$ the N° of Vibrations requir'd | *Rules for the Work.* | $u=$ N° of Vibrations. |
|---|---|---|---|
| 1 | | | |
| 2 | $k\,k.\quad n.\qquad u\,u.\quad \frac{k\,k\,n}{u\,u}$ the Analogy. | Or $uu.\,kk::n.\frac{kkn}{uu}$ in a direct Proportion. | $60^2.\,39.2:uu.\,39.2\times60^2 \div uu$ |
| 3 | $u=\frac{k\,k\,n}{u\,u}$ ..... | $uu\,(\omega)=\frac{kkn}{uu}$ or $\sqrt{uu}=$ &c. | $u=\frac{60^2\times39.2}{u\,u}$ |
| 4 | $u\,u\,u=k\,k\,n$ | 3d Reduced. | $uuu=60^2\times39.2$ |
| 5 | $u=\sqrt[3]{k\,k\,n}$ | $=\sqrt[3]{}$ of the 4th. | $u=\sqrt[3]{60\times39.2}$ *viz.* of (141120 |

$kk=3600\times n\,(=39.2)=141120$; and $\sqrt[3]{141120}=52$ near.

$\sqrt[3]{141120}=52=$ the Vibrations in a Minute, And Inches for the Length of ſuch a Pendulum.

*Problem* 5.] *What Age is that Perſon when* $\frac{2}{3}$ *of the Time from his Birth is equal to* $\frac{7}{8}$ *of what he wants of* 60?

I put $60=k$, $\frac{2}{3}=n$, and $\frac{7}{8}=t$.

| Steps. | $u=$ the Perſon's ſuppoſed Age. | *Rules for the Steps.* | $u=$ his Age. |
|---|---|---|---|
| 1 | | | |
| 2 | $k-u$ the time he wants of 60. | or $60-u$ | $60-u$ |
| 3 | $\frac{u}{n}=\frac{k-u}{t}$ | $u\div n$, & $k-u\div t$ | $u\div\frac{2}{3}=60-u\div\frac{7}{8}$ |
| 4 | $u=\frac{kn-nu}{t}$ | the 3d × by $n$ | $u=60\times\frac{2}{3}-u\times\frac{2}{3}\div\frac{7}{8}$ |
| 5 | $tu=kn-nu$ | the 4th × by $t$ | $\frac{7u}{8}=60\times\frac{2}{3}-u\times\frac{2}{3}$ |
| 6 | $tu+nu=kn$ | $nu$ in 5, + | $\frac{7u}{8}+\frac{2u}{3}=60\times\frac{2}{3}$ |
| 7 | $u=\frac{k\,n}{t+n}=$ the Canon. | 6th $\div$ $t+n$ | $u=60\times\frac{2}{3}\div\frac{7}{8}+\frac{2}{3}$ |

So that $\frac{kn}{t+n}=$ the Age; and $k-\frac{kn}{t+n}=$ it wants of 60.

And $\frac{kn}{t+n}\div n=k-\frac{kn}{t+n}\div t$ for Proof.

$u=25\frac{3}{3}\frac{5}{7}$ *Years*, the Age, and $60-25\frac{3}{3}\frac{5}{7}=34\frac{2}{3}\frac{2}{7}$ *Years*, the want of 60.

And $25\frac{3}{3}\frac{5}{7}\div\frac{2}{3}=34\frac{2}{3}\frac{2}{7}$ *Years* $\div\frac{7}{8}$ for a Proof.

*Problem 6.*] *Two Persons,* A *and* B, *have each a certain Number of Pounds, so that if* B *give* A 3 *of his, then they will have each a like Number: But if* A *give* B *three of his Pounds, then* B *will have* 5 *times as many as* A: *What Number of Pounds had each at first?*

I put $n=3$ and $k=5$.

| | | | |
|---|---|---|---|
| 1 | $u=B$ hath at first. | $u$ suppoſed | $u=B$ hath at firſt. |
| 2 | $u-2n=A$ hath at firſt. | | $u-3\times 2$ |
| 3 | $u-n=B$ will have. | 1ſt $-\frac{2n}{2}$ | $u-3$ |
| 4 | $u-n=A$ will have. | *ibid.* as 3d. | $u-3$ |
| 5 | $u-3n=A$ will after have. | $=u-2n-n$ | $u-9$ |
| 6 | $u+n=B$ will then have. | | $u+3$ |
| 7 | $ku-3kn=u+n$ | 5th $\times k+$ 6th | $5u-3\times 5\times 3=u+3$ |
| 8 | $ku-u=n+3kn$ | $u+$ & $3ku+$ | $5u-u=3+3\times 5\times 3$ |
| 9 | $u=\frac{n+3kn}{k-1}$ the Anſwer. | 8th $\div k-1$ | $u=\frac{3+3\times 5\times 3}{4}$ Anſwer. |
| | $\frac{n+3kn}{k-1}=4n$, which $B$ had. And $A$ had $2n$: for $2n-n=n$: and $n+4n=5n$. | | $\frac{3+3\times 5\times 3}{4}=12$, which $B$ at 1ſt had. And $A$ had 6: for $6-3=3$; and $3+12=5\times 3=15$. |

*Problem 7.*] *A Hare,* 50 *of her Steps or Paces before a Greyhound, takes* 4 *Steps to the Dog's three; but then the Dog ſteps as much at twice as the Hare does at thrice: How many Steps muſt the Dog make before he catch the Hare?* This old Algebraic Queſtion I anſwer thus:

I put $k=50$, $n=3$, $s=2$, and $b=4$.

| | | | |
|---|---|---|---|
| 1 | $u=$the ſuppos'd Anſwer (or Dog's Steps. | $u$ | |
| 2 | $n.\ b::u.\ \frac{bu}{n}$ all the Hare's (actual Steps | | $3.\ 4::u.\ \frac{4u}{3}$ |
| 3 | $n.\ s::\frac{bu}{n}+k.\ \frac{bsu}{nn}+\frac{ks}{n}$ the (Dog's Steps. | | $3.\ 2::\frac{4u}{3}+50.\ \frac{8u}{9}+\frac{100}{3}$ |
| 4 | $u=\frac{bsu}{nn}+\frac{ks}{n}$ | | $u=\frac{8u}{9}+\frac{100}{3}$ |
| 5 | $nnu-bsu=kns$ | the 4th $\times nn$, &c. | $9u-8u=\frac{900}{3}$. *Ergo* |
| 6 | $u=\frac{kns}{nn-bs}$ the Canon for (Anſwer. | the 5th $\div nn-bs$ | $u=\frac{900}{3}=300$ Steps, the (Anſwer. |

*Problem* 8.] A Ciſtern that hath four Vents of different Dimenſions containeth 72 Barrels of Liquid: If one Vent or Cock be opened, the Liquid will be ſpent in 6 Hours; if a ſecond be opened alone, it will run out in 8 Hours; if the third Cock be turned alone, the Liquid will be exhauſted in 9 Hours; and the fourth Cock or Vent is ſo ſmall, that if it alone be opened, it will be twelve Hours before the Ciſtern is empty: Now the Queſtion is, *in what Time the Ciſtern will be void of Liquid*, if all the four Vents be opened at the ſame Moment of Time. This Queſtion (which I have not, that I know of, ſeen before anſwer'd by Algebra) I ſolve thus; tho there is ſomewhat a ſhorter way: but I deſign it plain to the Learner, and if 72 or $k$ be omitted and ſuppoſed 1, it will come to the ſame Canon, as you may make tryal at your leiſure, for $k$ is thrown out even in this Example.

For 72 I put $k$, $6=n$, $8=s$, $9=t$, $12=r$.

| | $u$ = the unknown Time or ſuppoſed (Anſwer. | | $u$ |
|---|---|---|---|
| 1 | $u.\ k :: u.\ \frac{ku}{n}$ | | $6.\ 72 :: u.\ \frac{72u}{6}$ |
| 2 | $s.\ k :: u.\ \frac{ku}{s}$ | | $8.\ 72 :: u.\ \frac{72u}{8}$ |
| 3 | $t.\ k :: u.\ \frac{ku}{t}$ | | $9.\ 72 :: u.\ \frac{72u}{9}$ |
| 4 | $r.\ k :: u.\ \frac{ku}{r}$ | | $12.\ 72 :: u.\ \frac{72u}{12}$ |
| 5 | So that $\frac{ku}{n}+\frac{ku}{s}+\frac{ku}{t}+\frac{ku}{r}=k$ | | $\frac{72u}{6}+\frac{72u}{8}+\frac{72u}{9}+\frac{72u}{12}=72$ |
| 6 | $\frac{u}{n}+\frac{u}{s}+\frac{u}{t}+\frac{u}{r}=1$ | {5th Contracted or: by $k$} | $\frac{u}{6}+\frac{u}{8}+\frac{u}{9}+\frac{u}{12}=1$ |
| 7 | $u+\frac{nu}{s}+\frac{nu}{t}+\frac{nu}{r}=n$ | the 6th $\times n$ | $u+\frac{6u}{8}+\frac{6u}{9}+\frac{6u}{12}=6$ |
| 8 | $su+nu+\frac{nsu}{t}+\frac{nsu}{r}=ns$ | the 7th $\times s$ | $8u+6u+\frac{6u\times 8}{9}+\frac{6u\times 8}{12}=6\times 8$ |
| 9 | $stu+ntu+nsu+\frac{nstu}{r}=nst$ | the 8th $\times t$ | $8u\times 9+6u\times 9+6u\times 8+\frac{6u\times 8\times 9}{12}=6\times 9$ |
| 10 | $rstu+nrtu+nrsu+nstu=nrst$ | the 9th $\times r$ | $8u\times 9\times 12+6u\times 9\times 12+6u\times 8\times 12+6u\times 8\times 9=6\times 8\times 9\times 12$ |
| 11 | $u=\frac{nrst}{rst+nrt+nrs+nst}$ = Anſw. | the 10th ÷ all that are $\times u$ | $u=\frac{6\times 8\times 9\times 12}{8\times 9\times 12+6\times 9\times 12+6\times 8\times 12+6\times 8\times 9}$ |

And $\frac{nrst}{rst+nrt+nrs+nst}=\frac{5184}{2520}$ Hours.

Numerat. = 5184; Denom. = 2520 = 2:3:2:3:5 which is the accurate Anſwer.

*Problem 9.*] How shall I find out a Canon for discovering the Discount of Money? See the Operation.

$d$ = Days to come. $c$ = 100 *l.*
$p$ = Principal. $y$ = a Year or 365 Days.
$r$ = Rate of Interest. $u$ = the Discount sought.

$$y.\ r :: d.\ \frac{dr}{y}$$

$$\text{And } c + \frac{dr}{y}.\ \frac{dr}{y} :: p.\ u$$

$$cu + \frac{dru}{y} = \frac{dpr}{y}$$

or $cuy + dru = drp$, and consequently $u = \frac{dpr}{cy + dr}$ the Canon for Discount.

*Problem* 10.] To find a Canon for discovering the present Worth of Money due at the end of any Days. Symbals as above.

$$y.\ r :: d.\ \frac{dr}{y}$$

$$c + \frac{dr}{y}.\ c :: p.\ u$$

$$cu + \frac{dru}{y} = cp$$

$cuy + dru = cpy$; and consequently $u = \frac{cpy}{cy + dr} =$ the Canon for present Worth.

These Canons apply'd under *Sect.* 8. *Chap.* 3. in finding the Discount of Money much sooner than by the common Way of the Rules of Proportion.

## II. *To resolve Questions by Quadratic Equations, which they form.*

*Problem* 1.] There are three Numbers in Geometrical Proportion continued; the Mean $=42$, the Difference between the Extremes $=112$: What are the two Extremes?

I put $k=112$; $n=42$.

| | | | |
|---|---|---|---|
| 1 | $u=$ the leſſer Extreme. | | $u=$ the leſſer Extreme. | 1 |
| 2 | $u+k$ the greater. | | $u+112=$ the greater. | 2 |
| 3 | $uu+ku=nn$ | 2d × 1ſt $=n^2$ | $uu+112u=42^2$ | 3 |
| 4 | $u=\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}$ *Anſw.* | 3d $w$; and $\frac{k}{2}+$ | $u=\sqrt{42^2+56^2}-56$ | 4 |

| | |
|---|---|
| $\sqrt{nn+\frac{kk}{4}}-\frac{k}{2}=14=$ the leſſer. | $\sqrt{42^2+56^2}-56=14$ the leſſer. |
| And $14+k=$ the greater Extreme. | And $14+112=126$ the greater. |

And for Proof $14\times126=42^2=1764$.

*Problem* 2.] To divide a Line into Extreme and Mean Proportionals: or (which is one Uſe thereof) to find the Side of the Dodecahedron by having that of the Octahedron given, whereof this is the greater Proportional Part.

A Line is thus divided, when the Square of the greater Part is $=$ the Rectangle under the whole Line and the leſſer Part.

I put $k=$ to the Side of the Octahedron $(=ab)$ 120. 


| | | | |
|---|---|---|---|
| 1 | $u=$ the greater part (or the ſide of the Dodecahedr. | | $u=ac$ of the Line $ab$. | 1 |
| 2 | $k-u=$ the leſſer part (as $cb$) | | $120-u=cb$ of the Line $ab$. | 2 |
| 3 | $uu=kk-ku$ | the 1ſt ⊙; and $k\times$ in 2d | $uu=120^2-120u$ | 3 |
| 4 | $uu+ku=kk$ | $k\,u+$ | $uu+120u=120^2$ | 4 |
| 5 | $u=\sqrt{kk+\frac{kk}{4}}-\frac{k}{2}=ac$ (Anſw. | the 3d $ww$ and $\frac{k}{2}+$ | $u=\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}$ | 5 |

| | |
|---|---|
| So that $u=ac$ of the Line $ab=$ the Side requir'd. | And $\sqrt{120^2+\frac{120^2}{4}}-\frac{120}{2}=74.16$ (near. |

Pro-

*Problem* 3.] A Merchant sold a Ship for *l.* 2400, and gained after the Rate of what the Ship cost by laying out 10000 Pounds: What did the Vessel cost him?

I put $s = 2400$, and $t = 10000$.

| | | | |
|---|---|---|---|
| 1 | $u$ = the supposed Cost. | | $u$ | 1 |
| 2 | $s - u$ = the Gain. | | $2400 - u$ | 2 |
| 3 | $u. s - u :: t. \frac{st - tu}{u}$ | | $u. 2400 - u :: 10000. \frac{2400 - u \times 10000}{u}$ | 3 |
| 4 | $u = \frac{st - tu}{u}$ | {or the 1st = 4th Proportional.} | $u = \frac{2400 - u \times 10000}{u}$ | 4 |
| 5 | $uu + tu = st$ | 4th $\times u$ and $tu +$ | $uu + 10000\,u = 24000000$ | 5 |
| 6 | $u = \sqrt{st + \frac{tt}{4}} - \frac{t}{2} = 2000$ | 5th Step $ww$, &c. | $u = \sqrt{24000000 + 25000000} - 5000 (= 2000$ | 6 |

And for Proof 2000. 400 :: 10000. 2000

*Problem* 4.] Admit $h\,i\,n$ (*Fig.* 9. *Plate* C.) to be the Curve of an Hyperbola, and that there are given

$t\ (= i\,o)$ the *Latus transversum* $= 7.4$
$l\ (= i\,h)$ the *Latus rectum* $= 1.76$
$o\ (= z\,h)$ the greatest Ordinat $= 4.68$

What is the Length of the Abscissa, $(i\,z)$ for which I put $(u)$ because unknown?

| | | | |
|---|---|---|---|
| 1 | $oo = lu + \frac{luu}{t}$ = the Property of the Curve. | | $4.68^2 = 1.76 \times u + \frac{1.76 \times uu}{7.4}$ | 1 |
| 2 | $ltu + luu = oot$ | 1st × by $t$ | $1.76 \times 7.4 \times u + 1.76 \times uu = 4.68^2 \times 7.4$ | 2 |
| 3 | $uu + tu = \frac{oot}{l}$ | 2d ÷ by $l$ | $uu + 7.4 \times u = \frac{4.68^2 \times 7.4}{1.76}$ | 3 |
| 4 | $u = \sqrt{\frac{oot}{l} + \frac{tt}{4}} - \frac{t}{2} = 6.7$ | the 3d $ww$, & $\frac{t}{2} +$ | $u = \sqrt{\frac{4.68^2 \times 7.4}{1.76} + \frac{7.4^2}{4}} - \frac{7.4}{2} = 6.7$ | 4 |

*Problem* 5.] What Number is that, whose Square multiply'd by 10, being added to its Biquadrate, the Sum makes 30251?

I put $k=30251$, and $10=n$.

| | | | |
|---|---|---|---|
| 1 | $u$=the Answer unknown. | | $u$ |
| 2 | $u^4+nuu=k$ | | $uuuu+10uu=30251$ |
| 3 | $uuuu+nuu+\frac{nn}{4}=k+\frac{nn}{4}$ | 2d $+\frac{n^2}{2}$ or $+\frac{nn}{4}$ | $u^4+10uu+\frac{10^2}{4}=30251+\frac{10^2}{4}$ |
| 4 | $uu+\frac{n}{2}=\sqrt{k+\frac{nn}{4}}$ | 3d $ww$ | $uu+\frac{10}{2}=\sqrt{30251+\frac{10^2}{4}}$ |
| 5 | $u=\sqrt{\sqrt{k+\frac{nn}{4}}-\frac{n}{2}}$ | $\left\{\begin{array}{l}\text{4th Step } ww\\ \text{and } \frac{n}{2}+\end{array}\right\}$ | $u=\sqrt{\sqrt{30251+\frac{10^2}{4}}-\frac{10}{2}}=13$ |

And for Proof $13^2=169\times10=1690+13^4\ (=28561)=30251$.

---

The last Canon for the Answer $=u$, is thus to be read: That if half $n$ be deducted from the Square Root of $k+\frac{nn}{4}$, the Square Root of the Remainer is $=u=13$.

*Problem* 6.] A Farmer left his Son and a Nephew *l.* 400 to be so divided between them, that their Parts being alternately divided by each other, the Sum of the Quotients will be 30; but the greater part he devised to his Son: What did he leave to each?

Some make this Question, which I propose as above, to be solved by Various Positions, others by the Doctrine of Surds: But I am sure it is most naturally reducible to the 2d of my 2 Cases of Quadraticks before mention'd. And because I have not seen it done before, I shall insert the Solution at large as it is most easily understood, and that by the Numeral Way; but shall wave the Literal Method on purpose that the Reader may try his own Proficiency therein.

$u=$

| | | |
|---|---|---|
| 1 | $u$ = the Son's Share. | |
| 2 | $400-u$ = the Nephew's Share. | $400-$ the 1ſt Step. |
| 3 | $\frac{u}{400-u}+\frac{400-u}{u}=30$ | 1ſt ÷ 2d, & 2d ÷ by 1ſt |
| 4 | $u+\frac{160000-800u+uu}{u}=12000-30u$ | the 3d × by $\overline{400-u}$ ; the 4th × by $u$ |
| 5 | $un+160000-800u+uu=12000u+30uu$ | |
| 6 | $160000=12800u-32uu$ | the 5th contracted, and $2uu-800u+$ |
| 7 | $5000=400u-uu$ ; and *ergo* | the 6th ÷ by 32 |
| 8 | $uu-400u=-5000$ an Equation of the (2d ſort. | 5000 (in the 7th + and $400u-uu+$ |
| 9 | $uu-400u+40000=35000$ | $\frac{400}{2}$ ⊙ and added to each part of the Equation. |
| 10 | $u-200=\sqrt{35000}$ | the 9th $w$ |
| 11 | $u=\sqrt{35000}+200$. | $-200$ in the 10th + |

And $\sqrt{35000}+200$ = Son's Share = *l.*387.083 ; and $400-387$, &c.
$=12.917$

And for Proof $387.083 \div 12.917 = 29.966$ ; and $12.917 \div 387.083$ $= .034$ ; and $.034+29.966=30$ : which anſwers in every part the Requiſits.

## III. *The Reſolution of Queſtions which require Various Poſitions.*

Having in *Sect.* 9, & 10. given Rules for ſolving Simple and Quadratic Equations ; there was no need in *Sect.* 11. that I ſhould repeat any thing farther than what I have done in the middle Column of Rules : But in this Head of Various Poſitions having ſaid nothing thereon before, I ſhall make it as plain as I can.

1. Then

1. Then by various Positions we mean Questions which have more Numbers than 1 sought for, and are discovered by assuming a Vowel for each Number sought.

2. And we proceed in the Work first with one unknown Number, not regarding the rest but as known: And when we have found the first unknown Quantity equal to some others, we omit that Quantity, and take instead thereof what it is equal to. Examples, and Rules thereupon, will make the matter plain.

*Problem* 1.] There is a Composition of 20 Integers, whose total Value is 82; but there are 2 different prized Ingredients in the 20, 1 of 4, the other of $4\frac{1}{4}$: How much of each Simple is there in the said Composition, and what the Value of each Ingredient in the whole of it, to make up the 82?

I put $q=20$, $4=k$, $4\frac{1}{4}=n$, and $82=t$.

| Steps. | | *Rules for the Operations.* | | Steps. |
|---|---|---|---|---|
| 1 | $u=$ the Quantity at 4 | | $u$ | 1 |
| 2 | $a=$ the Quantity at $4\frac{1}{4}$ | | $a$ | 2 |
| 3 | $ku+an=t$ | the 1st and 2d $\times k$ & $n$ | $4u+4\frac{1}{4}a=82$ | 3 |
| 4 | $ku=t-an$ | $a\,n$ + | $4u=82-4\frac{1}{4}a$ | 4 |
| 5 | $u=\frac{t-an}{k}$ * | 4th $\div$ by $k$ | $u=\frac{82-4\frac{1}{4}a}{4}$ | 5 |
| 6 | $\frac{t-an}{k}+a=q$ | {2d part of 5th, $+a=q$} | $\frac{82-4\frac{1}{4}a}{4}+a=20$ | 6 |
| 7 | $t-an+ak=kq$ | the 6th $\times$ by $k$ | $82-4\frac{1}{4}a+4a=80$ | 7 |
| 8 | $t-kq=an-ak$ | $-an+ak+$ in 7th | $82-80=4\frac{1}{4}a-4a$ | 8 |
| 9 | $a=\frac{t-kq}{n-k}=$ the Canon. | the $8\div n-k$ | $\frac{82-80}{4\frac{1}{4}-4}=a$ | 9 |

Hence appears the Canon to find the Quantity at $4\frac{1}{4}$, *viz.* $\frac{82-80}{4\frac{1}{4}-4}$

$=2\div\frac{1}{4}=8$. and $20-8=12=a$.

For Proof of which, 12 at $4=48$; and 8 at $4\frac{1}{4}=34$: Now $48+34=82=$ the total Value.

*Problem* 2.] A Person dying, left by Will to 2 Sons each a Sum of Money; so that if to 5 times what he left the elder you add 3 times what

what he gave to the younger, the Sum will be 1675 : And if from 7 times the Share of the elder Brother you deduct 8 times the Share of the younger, the Remainer will be 210: What Sum did the Father leave to each ?

I put $s=1675$, $d=210$, $k=5$, $n=3$, $m=7$, and $t=8$.

| | | | | |
|---|---|---|---|---|
| 1 | $u$=the Legacy of the (Elder. | | $u$ | 1 |
| 2 | $a$=that of the Younger. | | $a$ | 2 |
| 3 | $ku+an=s$. | 1ſt × $k$; 2d × $n$ = $s$ | $5u+3a=1675$ | 3 |
| 4 | $u=\frac{s-an}{k}$ * | 3d ÷ $k$; & $an$ + | $u=\frac{1675-3a}{5}$ * | 4 |
| 5 | $\frac{ms-amn}{k}-at=d$ | {2d part, 4th × by $m-at=d$} | $\frac{1675\times7-3a\times7}{5}-8a=210$ | 5 |
| 6 | $ms-amn-akt=dk$ | 5th × by $k$ | $1675\times7-3a\times7-8a\times5=210\times5$ | 6 |
| 7 | $ms-dk=amn+akt$ | {$amn-akt$ + and $dk$ +} | $1675\times7-210\times5=3a\times7+8a\times5$ | 7 |
| 8 | $\frac{ms-dk}{mn+kt}=a=175$. *Ergo* | 7th ÷ $mn+kt$ | $\frac{1675\times7-210\times5}{3\times7+8\times5}=a=175$, and | 8 |
| 9 | $\frac{s-175n}{k}=u=230$. | 4th compared with 8th. | $\frac{1675-175\times3}{5}=u=230$ | 9 |

And for Proof $230\times5+175\times3=1675$. And $230\times7-175\times8=210$, as *per* the Queſtion.

*Problem* 3.] A Surveyor having meaſured three Fields of Encloſure, hath loſt the Contents : But remembers, that if 847 Acres be added to the 1ſt, the Sum will be equal to that of the other two Fields ; if 847 Acres be added to the 2d Encloſure, the Sum will be double

double to the Content of the 1ſt and 3d; and if 847 Acres be added to the 3d Field, it will make treble the 1ſt and 2d: How many Acres is in each Field?

I put $a$=1ſt, $e$=2d, $u$=3, and $k$=847.

| | | | | |
|---|---|---|---|---|
| 1 | $a+k=e+u$; ſo that $a=e+u-k$ | | $k$ ← in 1ſt. | $a+847=e+u$ or $a=e+u-847$ |
| 2 | $e+k=2a+2u$ | $e=2a+2u-k$ | $k$ ← in 2d. | $e+847=2a+2u$ $e=2a+2u-847$ |
| 3 | $u+k=3a+3e$ | $u=3a+3e-k$ | $k$ ← in the 3d. | $u+847=3a+3e$ $u=3a+3e-847$ |
| 4 | $e=2e+2u-2k+2u-k$ | | {the 1ſt × in 2 +2 $u$ − $k$ in 2d.} | $e=2e+2u-\overline{847\times2},+2u-847$ |
| 5 | $e=3k-4u$ | | {the 4th contracted.} | $e=\overline{847\times3},-4u$ |
| 6 | $u=9k-12u+3u-3k+9k-12u-k$ | | {the 5th × 3 + 3$u$ − $k$ ←, the 5th × 3 − $k$.} | $u=\overline{847\times9},-12u+3u-\overline{847\times3}+847\times9-12u-847$ |
| 7 | $u=14k-21u$ | | 6th Step contracted | $u=847\times14-21u$ |
| 8 | $u=\frac{7k}{11}$ | * | {21$u$ + in the 7th and ÷ 22, &c.} | $u=\frac{847\times7}{11}=539$ *Acres* = 3d Field. |
| 9 | $e=3k-\frac{28k}{11}$ or $\frac{5k}{11}$ | * | {=3$k$ − 4$u$ as in the 5th, and $u$ in the 8th is $=\frac{7k}{11}\times4=$9th} | $e=\frac{847\times5}{11}=385$ = 2d Field. |
| 10 | $a=3k-\frac{21k}{11}-k$ | | {$a$ = as *per* 1ſt compar. with Steps 8, & 5.} | $a=\overline{847\times3}-\frac{847\times21}{11}-847$ |
| 11 | $a=2k-\frac{21k}{11}=\frac{k}{11}$ | * | {the 10th Step contracted.} | $a=\frac{847}{11}=77$ = the 1ſt Field. |

Now $\frac{k}{11}=a$ the 1ſt Field $=77$ *Acres*

$\frac{5k}{11}=e$ the 2d Field $=385$

$\frac{7k}{11}=u$ the 3d Field $=539$

And for Proof, *Acres*

$77+847=924=385+539$ } =the 2d and 3d.

$385+847=1232=77+539\times2$ } =2d and 3×2.

$539+847=1386=77+385\times3$ } =1ſt and 2d×3.

*Note*

*Notes on the three Operations to the laſt three Problems above.*

The middle Column has ſuch plain Directions, as may ſuffice a docile Reader; but that I may render every thing plain, I ſhall obſerve that

*In the firſt Operation*, every Step is as eaſy as in common ſimple Equations, till you come to the ſixth Step, which is formed thus:

Since $u$ (an unknown Quantity in the 5th Step) is found $= \frac{t-an}{k}$, and ſince $u+a$ (the 2 Quantities of the different Ingredients in the 20) is equal to $q=20$; therefore that I may get rid of $u$, I take what it is equal to, and ſay $\frac{t-an}{k}+a=q$, and then proceeding by the common Rules laid down, I find $a=\frac{t-kq}{n-k}$, which are all known Quantitys: And this Canon doth ſhew $a=8$.

Therefore $u$ muſt be $20-8=12$. Or you may find it by a more artificial way, by having recourſe to the 5th Step, where $u=\frac{t-an}{k}$ *ergo* $u=\frac{t-8n}{k}$, which you'll find $u=12$.

*In the 2d Operation*, I proceed as in common ſimple Equations to the 4th Step, and there, becauſe $u=\frac{s-an}{k}$ I omit $u$, and take what it is equal to, $\frac{s-an}{k}$, which according to the Tenor of the Queſtion being multiply'd by $m$, and made leſs by $at$, the Remainder is $=d$; and ſo proceeding by the uſual Rules, as before taught, I find $a=\frac{ms-dk}{mn-kt}=175$: And if you have recourſe to the 4th Step, and inſtead of $an$ there, put $175n$ ($a$ being found $=175$) you will have $u=\frac{s-175n}{k}$, or $u=230$. And

*In the 3d Operation*, the three firſt Steps are compoſed according to the very Words of the Problem, and the 2d part of thoſe Steps are the ſame, only having $k$ tranſpoſed towards the right, there is ſhew'd what the 3 ſeveral unknown Quantitys are ſimply equal to.

Now for the 4th Step I argue thus: Conſidering that in the 2d Step $e=2a+2u-k$, and that $a$ is $=e+u-k$, therefore $e=2e+2u-2k$ (which is twice the firſt Step, $=2a$ in the 2d) $+2u-k$, which are the reſt of the 2d Step.

Which 4th Step is in the 5th reduced to $3k-4u$.

Then I proceed to diſcover the Value of $u$ by compoſing the ſixth Step, thus: $u$ being in the third Step towards the right hand

$= 3a + 3e - k$; and the 1ſt Quantity in what $a$ is $=$, being $e$; and $e$ being in the 5th Step $= 3k - 4u$, therefore I ſay 3 $a$'s is firſt 3 times $3k - 4u$, *i. e.* $9k - 12u$: and the Remainer of what $a$ is $=$, being $u - k$, *ergo* that $\times$ by the 3, I ſay $3a = 9k - 12u + 3u - 3k$. Now the reſt of what $u$ is $=$ being $3e$, and $e$ in the ſaid 5th Step being $3k - 4u$; therefore (to make the Proceſs plain) I repeat $3e$ or $9k - 12u$. And there being farther $- k$ to make up what $u$ is $=$; therefore I ſay in the 6th Step $u = 9k - 12u + 3u - 3k + 9k - 12u - k$, which contracted and reduced, is eaſily found (as *per* the middle Column in the Example) $u = \frac{7k}{11}$. Therefore $e$ in the 5th Column; I put $= 3k$, $- 4$ times what $u$ is found equal to in the 8th Step.

Laſtly, $a$ being $= e + u - k$; I ſay it is $= e$ in the 9th Step, $+ u$ (in the 8th Step) $- k$, or $a = 2k - \frac{21k}{11}$, which is $= \frac{k}{11}$.

## Sect. XII. *Concerning Cubical Equations.*

A Cubical Equation may be known by having commonly (for the firſt Term) the Cube of the unknown Quantity; the 2d Term, being the Square of that Quantity multiply'd in a Coefficient or ſome known Quantity; the third is the unknown Quantity ſought in a known, as thus and the like; $uuu + cuu + du = k$, $u$ being the Root or Quantity ſought.

A Cubical Equation is formed from 1, 2 or 3 Binomials, *&c.* involved or multiply'd in each other; repreſenting either a compleat Cube or Parallelopiped, whoſe Baſe is either a Geometrical Square, or a Parallelogram: in which laſt, if $u + b$ be the greater Side, $u - b =$ the leſſer Side of the Baſe, and $u + d =$ the Length; the Quantitys multiply'd produce this Cubical Equation, *viz.* $uuu + duu - bbu - bbd = k$ the Content. To find the Dimenſions, which depend on the Value of $u$, it muſt be done by the Rules following.

2. In a Quadratic Equation, by the Methods of Reſolution foregoing, you gain 1 Root; and by dividing the Equation given by that Root you find the other, if there be 2 different Roots.

Thus in the Equation $uu - 16u = -63$, being reſolved gives $u = 9$: And if the $uu - 16u + 63$ (the 63 being tranſpoſed) be divided by $u - 9$ (the 9 being tranſpoſed) the Quote will be $u - 7$ or $u = 7$. So are $u - 9$ and $u - 7$ found the 2 Roots or Dimenſions of that Equation, *i. e.* 9 and 7.

I. To

3. I. *To resolve Cubic Equations by Tentation and Depression.*

Having by tryal, as in the following Example, found 1 Root, you must divide the Cubic Equation given by that Root, and that will depress the Equation into a Quadratic; which Quadratic you may find the Roots of, as is shewn last above.

*Example* 1.] What are the 3 Roots or real Dimensions of this Cubical Equation $u\,u\,u - 57\,u\,u + 1040\,u = 6000$?

To answer this; I must first find by tryal a Root, which being multiply'd, *&c.* according to the given Equation, will be 6000.

*To which end I suppose* 10 *to be* 1 *Root* $= u$.

Then $uuu$ (or $10^3$) is $= 1000$
The 3d Term $1040\,u = 10400$ } Sum $= 11400$
2d Term $- 57\,uu$ (or $57 \times 10^2$) deduct. $= 5700$ } Rests 5700

The Remainer should be 6000, but 'tis only 5700; therefore the 10 supposed is too little.

*2dly, I suppose therefore a Root to be* $12 = u$.

Then $u\,u\,u$ (or $12^3$) is $= 1728$
$+ 1040\,u \ldots\ldots = 12480$ } Sum $= 14208$
$- 57\,uu$ (or $57 \times 144$) $\ldots\ldots\ldots = 8208$ } Rests 6000

So that I find $12 = u$ or $u - 12$ to be 1 Root.

By which Root having divided the Cubic Equation given, I depress it to the Quadratic Equation $u\,u - 45\,u = -500$. Which Equation being resolved, gives $u = 25$, or $u - 25$ for another Root. And dividing that Quadratic by this $u - 25$, the Quote or 3d Root is found $u - 20$, or $u = 20$. See the Operation.

$u-12)\,uuu-57uu+1040u-6000\,(=uu-45u=500=$ *the Quote or Quadratic.*

$uuu-12uu$

$-45uu+1040u$
$-45uu+540u$

$500u-6000$
$500u-6000$

or $u = \sqrt{.6.25} + 22.5 = 25$ *Quadratic resolved, (or a 2d Root.*

$u-25)\,uu-45u+500\,(u-20$ *(= a 3d Root.*

$uu-25u$

$-20u+500$
$-20u+500$

0

So that the 3 Roots being found 12, 20 and 25; you may prove the Truth by putting $u$ equal to each, transposing the Roots, and multiplying the Quantitys one in another: for the 2d Rectangle will be the Cubical Equation given, transposing the Member next the right hand of the last Product.

1 *Note*, That in a Cubic Equation if the ſingle Root or Square of the unknown Quantity be multiply'd in more than one known Quantity, with Signs between the Rectangles, as ſeveral Members of the Equation; where the ſaid unknown Root is found ſquared in 2 *&c.* ſuch Members, they make but 1 Term: and ſo do all thoſe where the unknown Root is found ſingle make another; as $u^3 - u^2b - u^2d + uc + ur + d^3 = 0$. In this $-u^2b - u^2d$ is but 1 Term of the Equation, and $uc + ur$ is one other Term.

*The Proof of the foregoing Work.*

$u - 25$
$u - 20$

$uu - 25u$
$\quad - 20u + 500$

$uu - 45u + 500$
$u - 12$

$uuu - 45uu + 500u$
$\quad - 12uu + 540u - 6000$

$uuu - 57uu + 1040u - 6000$
or $uuu - 57uu + 1040u = 6000 =$ (*the Equation given, for Proof.*

2 *Note*, That in the Cubic ſuppoſe $u^3 = uc + ur + d^3 = 0$; the 2d Term here is ſaid to be wanting, and $uc + ur$ is the 3d Term.

*Example* 2.] Admit this Equation were to be reſolved; $uuu - 30.5uu + 309u = 1039.5$: I ſuppoſe 1 Root = 12. and working as by the Equation, I find

$u^3 = 12^3 = 1728$
$-30.5u^2 = -4392$
$+309u = 3708$

} The Sum as *per* Addition of Algebra is 1044, which is more than the 1039.5.

2*dly*, I therefore ſuppoſe a leſſer Number, *viz.* 11 = a Root, then

$u^3 = 11^3 = 1331$
$-30.5uu = -3690.5$
$+309u = 3399$

} The Sum of which is = 1039.5 = the Sum in the Equation given: ſo that 11 is 1 Root.

3*dly*, Dividing the Equation given (with the 1039.5 tranſpoſed) by $u - 11$, I depreſs it to the Quadratic $uu - 19.5u = -94.5$

4*thly*, This Quadratic being reſolved is $u = \sqrt{95.0625} - 94.5 + 9.75 = 10.5$ a 2d Root.

5*thly*, Dividing this Quadratic Equation by the 2d Root $= u$ or $u - 10.5$, the Quote or 3d Root is found 9. And for Proof, the 3 Roots 9, 10.5, and 11, (putting each $= u$, and tranſpoſing it) multiply'd one in another, will give the Cubic Equation propoſed to have its Root extracted, *viz.* $u^3 - 30.5u^2 + 309u = 1039.5$

From theſe 2 Examples it may be obſerved, That whether you happen to pitch upon the leaſt Root (upon your Tryals) as in the firſt Example, or upon the greateſt of the 3, as in the 2d Tryal of the 2d Example; the Method of Proceſs is the ſame: and ſo it would, if you happen firſt upon the middle Root.

Theſe

These 2 Examples are of Regular Cubics having all their Terms.

4. I shall give one Example of an Irregular Solid Equation: as suppose $uuuu + .10uu = 20.61293$.

To resolve this, I can easily see that 2 is more than the Root; for 10 times the Square of 2 (alone, without the Surfolid) is 40. Therefore I take 1.5 for the Root, and (working with that according to the Equation) I find it too great; I therefore try 1.4, which is also a small matter too much: so that working according to the Equation with 1.3, I find it just $= 20.61293$. So that I discover the Root to be $1.3 = u$. for $1.3^5 + 10 \times 1.3^2 = 20.61293$.

5. *Biquadratic Equations* are resolved after the same manner as the Regular Cubics in the 2 first Examples: First, by supposing a Root till you pitch upon the right, and depressing the Biquadratic thereby into a Cubic; and then finding another Root by Tentation whereby to divide the Cubic to depress it to a Quadratic, *&c.* as in the said 2 first Examples above, for all Regular Biquadratics (or such as have all their Terms.) Thus, *for example*, to resolve or find the Roots of the Biquadratic $uuuu - 29uuu + 308uu - 1420u = 2400$.

(1.) By Tentation I find 8 to be 1 Root, and dividing the Equation given by $u - 8$, I depress it to the Cubical Equation $uuu - 21uu + 140u = 300$.

(2.) I make tryal again, and find 10 to be another Root; so dividing the Cubic Equation by $u - 10$, I depress it to the Quadratic $uu - 11u = -30$, which being resolved, gives a 3d Root or $u = 6$.

(3.) Dividing the said Quadratic Equation by $u - 6$, I find the Quote to be $= u - 5$ or $u = 5$, the 4th Root: so are 8, 10, 6 and 5 the 4 Roots required.

There are 3 other Ways of resolving or finding the Roots of all Cubic and Biquadratic Equations, done more Mathematically: as,

II. By Substitution, Deduction, and Division.

III. By way of Construction, with the help of the Parabola and Circle.

IV. By Approximation or Converging Series.

### II. *To resolve Cubic Equations by Substitution.*

It has been shew'd how to resolve Quadratics by Substitution, and much after the same manner is a Canon found whereby to resolve Cubic, *&c.* Equations.

*Example* 1.] Admit the Value of $u$ is required in this following Equation, *viz.* $uuu + duu - gu = k$, or $uuu + 120uu + 300u = 3714544$: Here 'tis plain that $d$ is put for the given 120; $g$ for 300; and $k$ for 3714544.

The

The first Step toward the Answer is to substitute $b+c$ in the place of $u$ in every Term of the Equation, in the Power that ($u$) is put.

*Then will this be the Canon for Resolution.*
$bbb+3bbc+3bcc+ccc$ (instead of $uuu$)
$bbd+2bcd+ccd$ .... (instead of $duu$)
$-bg-cg$ (in the place of $-gu$)

The whole Process and Rules are in the Margin, whereby $u$ is found $=124$.

1. Having your Canon before you, and also that $d=120$, $g=300$, proceed as the Canon directs you, first pointing over every third after the first Digit of the given Number; the 3 Points shewing the Root sought, will consist of 3 places, and therefore the 1 ($=$the first $b$) is 100.

2. You must compose a Number to deduct from that given, by taking those in the Canon which have not $c$ found in them; as $b^3+bbd-bg=2170000$, which deducted reduceth the Number given to 1544544, which is your first Dividend, as by Step 7.

3. Take for a Divisor (in order to discover the Value of $c$) the Sum of the Numbers answering to the Sumbals mix'd with or $\times$ by $c$, such are those in the 8th, 9th, 10th, 11th, and 12th Steps; which Sum is

| Step. | *An Example of the Resolution of a Cubic.* | |
|---|---|---|
| | . . . | |
| 1 | 3714544 (100$=b$ | |
| 2 | 1000000$=bbb$ | Add. |
| 3 | 1200000$=bbd$ | |
| 4 | 2200000$=bbb+bbd=1+2$d Step. | |
| 5 | 30000$=-bg$ deduct from the 4th. | |
| 6 | 2170000 rests, which deduct from the 1st Step. | |
| 7 | 1544544 $\div$ $=$the 1st $-$ the 6 Step $\div$ by 12 | |
| 8 | 30000$=3bb$ | Add. |
| 9 | ..300$=3b$ | |
| 10 | 24000$=2bd$ | |
| 11 | ..120$=d$ | |
| 12 | $-$300$=-g$ | |
| 13 | $\div$ 54120$=$the Sum, or Divisor. | |
| | (20$=c$ | |
| 14 | 600000$=3bbc$ | Add. |
| 15 | 120000$=3bcc$ | |
| 16 | ..8000$=ccc$ | |
| 17 | 480000$=2bcd$ | |
| 18 | 48000$=ccd$ | |
| 19 | $-$6000$=-cg$ | |
| 20 | 1250000$=$Sum deduct from the 7th Step. | |
| 21 | 294544 $\div$, rests of $b$, or 1st Step. | |
| | Now 120$=b$ | |
| 22 | 43200$=3bb$ | Add. |
| 23 | ..360$=3b$ | |
| 24 | 28800$=2bd$ | |
| 25 | ..120$=d$ | |
| 26 | $-$300$=-g$ | |
| 27 | $\div$ 72180$=$Sum, by which to $\div$ the 21 Step. | |
| | (4$=c$ | |
| 28 | 172800$=3bbc$ | Add. |
| 29 | ..5760$=3bcc$ | |
| 30 | ....64$=ccc$ | |
| 31 | 115200$=2bcd$ | |
| 32 | ..1920$=ccd$ | |
| 33 | $-$1200$=-cg$ | |
| 34 | 294544$=$Sum, deduct from the 21 Step. | |

54120 your Diviſor, which may be had in the 6 Figures next the left hand of the Dividend 2 times, (which 2 muſt be 20, becauſe it is the middle of the 3 that will be in the Root, as is obſerved above) this 20 is $= c$.

4. You muſt not make any farther Uſe of the Quantitys in the 1ſt, 2d, and 4th Steps, but having found the Value of $c$, you proceed to find your 2d Subtrahend in the 20th Step with the ſeveral Quantitys in the Canon; which you ſee in the 14th, 15, 16, 17, 18, and 19th Steps.

5. And having found your 2d Diviſor by the very ſame Quantitys that you had the 1ſt, (and obſerving that here $b = 120$, and not to 100) you may eaſily finiſh the Operation by the Example, which is only a Repetition of the Method above ſhew'd, and you find the 3 ſeveral Roots put together is $124 = u$ required: which I ſhall prove both from the Equation given, and by the Canon, to be the true Anſwer.

1. *By the Equation* $u^3 = 124^3 = 1906624$
$+120uu = 1845120$
$-300u = -37200$

Sum $= 3714544 = k$ in the Equation given.

2. *The Proof by the Canon* $bbb + 3bbc + 3bcc + ccc + bbd + 2bcd + ccd - bg - cg$.

*Note*, $b = 120$; $c = 4$
$d = 120$; $g = 300$

| | | | |
|---|---|---|---|
| $bbb$ | $= 1728000$ | | |
| $+3bbc$ | $= 172800$ | | |
| $+3bcc$ | $= 5760$ | | |
| $+ccc$ | $= 64$ | Sum | $= 3751744$ |
| $+bbd$ | $= 1728000$ | $-bg$ | $= 36000$ |
| $+2bcd$ | $= 115200$ | $-cg$ | $= 1200$ |
| $+ccd$ | $= 1920$ | | |

$-37200$

$3714544$ reſts $= k$

Biquadratics are reſolved after the ſame Method, raiſing the Power of $b + c$ (which is ſubſtituted) accordingly in making the Canon; and proceeding by that, as has been in the laſt Example obſerved.

*Example* 2.] Of an irregular Cubic Equation, as ſuppoſe $uuu + 32334u = 823975 = k$, or $u^3 + fu = k$.

1. Make your Canon as before, ſubſtituting $b + c$ in the 2 Members of the Equation (as $u$ is in power) in the place of $u$.

2. Where the Equation has not all its Terms, and the Co-efficient is large, you muſt put ſuch a Number firſt in the Root,

Root, as being multiply'd by the Coefficient, and added to its own Cube, the Sum will be as near as poſſible to, and leſs than that given on the right ſide of the Equation; as in the Example finding 50 $=b$ much too great (as in the 4th Step) I try $30=b$, which I ſtill find too much, as in the 7th Step. Then putting $20=b$, I find its Cube as in the 8th Step, and the Coefficient 32334 by 20, the Sum is deducible as in the 10th Step from the 1ſt, and then you proceed with the Remainer as in the laſt Example, and the Value of $u=25$.

| Step. | |
|---|---|
| | The Canon is $bbb+3bbc+3bcc+ccc+bf+cf=k$. |
| 1 | 823975 (80 $=b$ (30 $=b$ (20 $=b$ |
| 2 | $125000=bbb$ |
| 3 | $1616700=bf$ |
| 4 | $1741700=bbb+bf$ > 1ſt Step, and cannot be deducted. |
| 5 | $27000=30^3=b^3$ |
| 6 | $970020=bf$ |
| 7 | $997020=b^3+bf$ > 1ſt Step. |
| 8 | $8000=b^3=20^3$ |
| 9 | $646680=bf$ |
| 10 | 654680 deduct from the 1ſt Step. |
| 11 | 169295 ÷ reſts of the 1ſt Step. |
| 1 | $1200=3bb$ } |
| 13 | $60=3b$ } Add |
| 14 | $32334=f$ } |
| 15 | ÷ 33594 = the 3 laſt Steps a Diviſor. (5 $=c$ |
| 16 | $6000=3bbc$ } |
| 17 | $1500=3bcc$ } Add. |
| 18 | $125=ccc$ } |
| 19 | $161670=cf$ } |
| 20 | 169295 Sum, deduct from 11th Step. |
| | 0 reſts. |

3. It ſometimes happens where the Coefficient is very large, and the Root of the higheſt Power (or Value of $u$) is ſmall, that you muſt make that Figure 1ſt put in the Root one place (or more) leſs than the Points over the Number given (as in the 1ſt Step above) denote. See the following Example.

*Example* 3.] Admit $u^3+82334\,u=2073975$, what is the Value of $u$? 82334 being $=f$, and $b+c$ ſubſtituted in the room of $u$.

Here

Steps; w...

Here 'tis plain the Number given admits of 3 Cubical Points, but the Root $u$ (or $b+c$) is but 25.

1. For if I ſhould put 1 as the Root of 2 next the left hand; that 1 (there being 3 Points) muſt be 100. Now if I multiply the Co-efficient by 100, *i. e.* if I ſuppoſe two Cyphers put towards the right hand of it, it will be 8233400, wch (without the Cube of the 100, which muſt as *per* the Examples above be added) is much too great to be taken from the 2073975 in the firſt Step.

| Step. | *The Canon as before* $= b^3 + 3b^2c + 3bcc + c^3 + bf + cf$ |
|---|---|
| 1 | 2073975 (20 $= b$ |
| 2 | 8000 $= b^3$ |
| 3 | 1646680 $= bf$ |
| 4 | 1654680 Sum, from the 1ſt Step. |
| 5 | 419295 reſts of Step 1ſt. |
| 6 | 1200 $= 3bb$ |
| 7 | 60 $= 3b$ |
| 8 | 82334 $= f$ |
| 9 | 83594 $=$ Sum or Diviſor. (5 $= c$ |
| 10 | 6000 $= 3bbc$ |
| 11 | 1500 $= 3bcc$ |
| 12 | 125 $= ccc$ |
| 13 | 411670 $= cf$ |
| 14 | 419295 $=$ Sum, from the 5th Step. |
| | 0 reſts. |

2. Seeing a leſſer Digit than 1 cannot be put firſt, or in hundreds place in the Root; I muſt therefore conclude, that the Root will have but two places.

3. I therefore ſuppoſe 10 in the Root, by putting 1 Cypher towards the right hand of the Co-efficient (to repreſent $bf$) which would be 823340, which (by doubling the 8 next the left hand I find 16) may eaſily be taken from 20, (next the ſame left hand of the Number given) I therefore put the double of 10 (or 20) in the Root, and ſo proceed; all the difficulty in this caſe being to put the firſt true Figure in the Root: to which end I have not ſeen any Rules that contribute to do ſo eaſily and certainly as theſe above.

*Example* 4.] *Of Reſolving Cubics by Subſtitution*, &c.

Admit $uuu + 350u^2 = 1225125$. Or that $uuu + duu = k$; and that $b + c$ is put $= u$.

This Example is of an irregular Cubic compoſed of 2 Members different from the former; where tho there be 3 Cubical Points (or Ternaries) yet the Root has but 2 places, *viz.* 55.

For if we ſuppoſe $b =$ but 1 in the 3d place (or 100) you will find $bbb + bbd$ to be much more than can be taken from the Number pointed; and $10 = b$ is much too little: therefore I try $50 = b$, and find that to be the Root, that produces (by its Cube added to $bbd$) the next Number to and leſs than 1225125 given, and therefore I proceed as by the Canon, as directed in Example the 1ſt, laſt foregoing.

| Step. | |
|---|---|
| | The Canon is $bbb + 3bbc + 3bcc + ccc = u^3$<br>$bbd + 2bcd + ccd = diuu.$ |
| 1 | 1225125 (50 $= b$ |
| 2 | 125000 $= bbb$ |
| 3 | 875000 $= bbd$ |
| 4 | 1000000 = Sum. |
| 5 | 225125 = 1ſt—4th Step, a Dividend. |
| 6 | 7500 $= 3bb$ |
| 7 | 150 $= 3b$ |
| 8 | 35000 $= 2bd$ |
| 9 | 350 $= d$ |
| 10 | 43000 = Sum, a Diviſor.<br>(5 $= c$ |
| 11 | 37500 $= 3bbc$ |
| 12 | 3750 $= 3bcc$ |
| 13 | 125 $= ccc$ |
| 14 | 175000 $= 2bcd$ |
| 15 | 8750 $= ccd$ |
| 16 | 225125 = 11 + 12 + 13 + 14 + 15 thSteps. |
| | reſts 0 5th—16th Step. |

## III. *To reſolve Cubic Equations by way of Conſtruction.*
Vid. *Plate* A. *Fig.* 27.

This being done by the Parabola and Circle, it is neceſſary to ſhew the making of the Parabola, &c.

A Parabola is one of the 5 Sections of a Cone, being made by cutting a Cone by a Plane parallel to the Side which is oppoſite to that wherein you cut; *and it is Geometrically formed thus:*

1. Draw the Line ($t\ v\ F\ \odot\ d\ y$) the Axis of the Section. Then croſs the ſame at right Angles with a Line at pleaſure, as ($a\ p$) and according as you would have the ſame (or if the $a\ p$, which is called the

the *Latus rectum*, or Parameter, be given) ſuppoſe 1 inch, ſet half an inch from *F* to *a*, and to *p*.

Then take half of one of thoſe Halfs (or $\frac{1}{4}$ of the Parameter, or of 1 inch) and ſet it from *F* to *v*, and from *v* to *t*, ſo is *F* = the Focus, and *v* the Vertex of the Parabola.

Then draw as many Lines as you pleaſe parallel to *a p* (as *o o l* &c.) with a Ruler on purpoſe for drawing parallel Lines; and the Diſtance *t* to *o* in the Axis extends from *F* the Focus to *l* in the ſame Parallel forming the Curve (*v p l l l*, &c.) which you may draw through the Points (*l*, *l*, &c.) And having found the Curve on one ſide of the Axis, as *v l*, ſet it off on the ſame Parallel, on the other ſide of the Axis, as *v o*, *v o*, &c. and draw the Curve (*o o v*) ſo is the Section finiſhed, and (*a F p*) is the *Latus rectum*, *o o l* an Ordinat Applicate (~~or whole Ordinat~~) *o o* the ~~Semi-Ordinat~~, (which is commonly called the Ordinat) any (*v o*) in the right Line is called an Abſciſſa.

Now ſuppoſe a Cubic Equation where $\left.\begin{matrix} uuu + buu - pu - q = 0 \\ \text{Or } uuu + 3uu - 2u - 2 = 0 \end{matrix}\right\}$ ſo $\begin{cases} b = 3 \\ p = 2 \\ q = 2 \end{cases}$

1*ſt*, To find the Value of *u*. From the ſame Scale by which you ſet off the *Latus rectum* of your Parabola, take 3 (the Co-efficient multiply'd in the Square of the Root ſought) and applying it at right Angles to the Axis (*v y*) towards the right hand becauſe + 3, it will cut *h* in the Curve.

2*dly*, Draw a Line at pleaſure cutting a ſuppoſed Line (*v h*) in the middle at right Angles, as *q q*.

3*dly*, Where that Line interſects the Axis, as at ⊙ ſet half *p* downward, becauſe — *p*, *viz.* 1, from the ſame Scale extends from ⊙ to *n*.

4*thly*, On (*n*) erect a perpendicular Line (*n m*) which will cut the infinite Line *q q* in *r*.

5*thly*, From *r* ſet half *q* = 1 towards the right hand becauſe — *q*, and it extends (by the ſame Scale) from *r* to *s*, which is the Center of the Circle ſought.

6*thly*, Extend the Compaſſes from *s* to *h*, and deſcribe the Arch (λ *e h*) which cutting the Curve at (*e*) the perpendicular Line (*e e*) meaſured on the ſame Scale, gives *u* = 1, which is + 1 or affirmative, becauſe the Section (*e*) is in the right hand part of the Curve. See *Philoſoph. Tranſactions*, N° 188 & 190; Dr. *Haley*'s Method.

## Sect. XIII. *Approximation or Converging Series.*

WHAT is meant hereby, is ſhewn under the words *Converging Series* in the foregoing Alphabetical Account at the beginning of this Chapter, and will be farther explained by the Rules and Examples following: as $u - ub + ub^2 - ub^3 + ub^4 - ub^5$, &c. is a Series; which I multiply by $u^2 - b^2$ as followeth.

$$\begin{array}{l} u - ub + ub^2 - ub^3 + ub^4 - ub^5 \\ u^2 - b^2 \\ \hline u - b + u b^2 - u b^3 + u b^4 - u b^5 + u b^6, \text{ \&c.} \\ \qquad - u b^2 + u b^3 - u b^4 + u b^5 - u b^6, \text{ \&c.} \\ \hline u - b = \text{the Product.} \end{array}$$

Here it is plain, that if the Series was continued never ſo far, the Negatives would deſtroy the Affirmatives: ſo that it had been the ſame in the Product, if you had run it only to $u - b$ there. But my Intent in this Section is not to ſhew the way of repreſenting or expreſſing by a Series, a Product, Root, &c. for tho that be a pretty Speculation, yet 'tis not ſo eaſy to diſcover the Roots, &c. therefrom. But I chuſe rather here to ſhew the great Uſe of Converging, in finding with much eaſe and expedition the Real Roots themſelves.

II. *To find the Square Root,* as ſuppoſe of 13.

Here I will put $s$ for the Square or Surd Number given; $r$ = the 1ſt two Figures in the Root; $a$ = the 2d Approach to the true Root; and $e$ = the 3d Approach; the ſeveral Approaches being the coming nearer and nearer to the Truth of the Root by ſo many Diviſions, by 1 of which ſeveral places in the Root are gained, as is ſhewed under the fifth Head following.

If in the Example above it be conſidered what Number ſquared will be next to and leſs than 13, or what 2 Digits will be next to

13.00

13.00 and put $r$ for thoſe 2 places it will then be $s - rr \div 2r = a$ the 2d Approach.

$$\overline{s - \overline{r+a}^2} \div \overline{r+a \times 2} = e \text{ the 3d Approach, } \mathcal{C}c.$$

Or in Numbers $13 - 3.6^2 \div 3.6 \times 2 = .00555 = a$ the 2d Approach, $3.6 = r$.

$$13 - 3.60555^2 \div 3.60555 \times 2 = .000001275\,46 = e,$$

and $r + a + e$. Or $3.6 + .00555 + .000001275\,46$ which is $= 3.605551275464 - =$ the Square Root required to 13 places, (but ſee more under General Head the 5th following.) Or $e$ will be $= \frac{s - rr - 2ra - aa}{2r + 2a}$ which facilitateth the Operation as to ſquaring 3.60555, $\mathcal{C}c$.

Or inſtead of 3.6 at the 1ſt Approach, you may, as in the common Way of Extraction, put 3, whoſe Square is next to and leſs than that of any other Digit deducible from 13.

*Prop.* 2. *To find the Square Root of a larger Number*, as of 371.00071 $= s$, $\mathcal{C}c$. as before.

$371.00071 = s$. $19 = r$ the 1ſt Approach.
$-361 = rr$

$10.00071 \div 2r\ (= 38) = .26 = a$ the 2d Approach, and
$r + a = 19.26$

Again $371.00071 - 19.26^2 = .05311 \div 19.26 \times 2 = 0013787 = e$ the 3d Approach.

And $r + a + e = 19 + .26 + .0013787 + = 19.2613787 =$ the Root ſought.

So by the ſame Rules of Converging Series the Square Root of 4712345 is found 2170.7936+

### III. *Approximation, or the Extraction of the Cube Root by the Method of Converging Series.*

I ſhall alſo proceed in this with as much Plainneſs as I can, obviating what only is to the purpoſe.

1. Suppoſe then I would know the Cube-Root of $13 = s$, for the firſt in the Root which is known here I put $r$. Then the Proceſs, as in common Extraction, (which is the beſt way in this Caſe, as being moſt accurate and certain for the 2d in the Root) will be

$$s - r^3 \div 3rr = R = .3 \text{ the 2d in the Root.}$$

Or ſuppoſing 2.3 put in the Root together (for I can eaſily ſee that 2.3 is the Root of the next Cube Number to and leſs than 13.000) it will be as before to find the 2d Approach, $2.3 = r =$ the 1ſt Approach.

Then

Then $s - r^3 \div 3r = .05 =$ the 2d Approach, and now $2.35 = R$.
And $s - R^3 \div 3RR = .001333+$; = the 3d Approach. So is
$2.351333+$ = the Root.

Or in Numbers more plainly thus;
$13 - 2.3^3 (= 12.167) = .833, \div 2.3^2 \times 3 (= 15.87) = .05$ the 2d Approach.
And $13 - 2.35^3 (= 12.977875) = .022125 \div 2.35^2 \times 3 (= 16.5675$
$= .001333+$ = the 3d Approach.

And the 1ſt Approach + the 2d + the 3d = 2.351333+, which is the Cube Root of 13, done with near 200 Figures fewer than in the common Way of Extraction.—— But ſee *Head* VIII. following.

2. And if we put $r =$ the known part of the Root, and $a$ for each new Approach, you have

$rr + 2ra + aa =$ the Square of $r + a$
$r^3 + 3r^2a + 3ra^2 + a^3 =$ the Cube of $r + a$
So likewiſe $r^4 + 4r^3a + 6r^2a^2 + 4a^3r + a^4 =$ the Biquadrate of $r + a$
And $r^5 + 5r^4a + 10r^3a^2 + 10r^2a^3 + 5ra^4 + a^5 =$ the Surſolid of $r + a$

And as in the Square and Cube Roots it appears to be (putting $s$ for the Number given to have its Root extracted)

$s - rr \div 2r = a$ the ſeveral Approaches in the Square Root.
$s - r^3 \div 3rr = a =$ the Approaches in the Cube Root.

*So for the Biquadrate and Surſolid Roots it will be*

$s - r^4 \div 4r^3 = a =$ the Approaches to the Biquadrate Root.
And $s - r^5 \div 5r^4 = a =$ the Approaches to the Surſolid Root.

**Hence it appears, That the known part of the 2d Quantity next the left hand in the above-mentioned Powers of $r + a$, is your Diviſor. I come next,**

### IV. *To illuſtrate by Numeral Examples the Extraction of the Biquadrate and Surſolid Roots by way of Converging Series.*

1. *To find the Biquadrate Root* of 314. The Proceſs is (the 1ſt $r$ being $= 4$)

1ſt, $314 - r^4 (= 256) = 58 \div 4r^3 (= 256) = .2 = a$. So is 4.2 a new $r$.
2dly, $314 - r^4 (= 311.1696) = 2.8304 \div 4r^3 (= 296) = .009$; ſo is $4.209 =$ a new $r$.

And by repeating the ſame Method with that laſt $r$, you have $4.2099+$ for the Biquadrate Root of 314.

2. *To*

2. *To find the Surfolid Root of* 314 (the firſt $r=3$) the Proceſs will be

1ſt, $314-3^5\ (=243)=71\div 5r^4\ (=405)=.1=a$. So is 3.1 a new $r$.

2dly, $314-3.1^5\ (=286.29)=27.7185\div 461.75\ (=5r^4)=.05=a$, and $3.15=$ a new $r$.

3dly, $314-3.15^5\ (=310.1364)=3.8636\div 5r^4\ (=492.28)=.007=a$. So is $3.157=$ a new $r$.

And proceeding in the ſame Method with that new $r$, the Surfolid Root of 314 is found $3.1578+$

But there is yet

## V. *Another Way of Converging (for Example) to the Square Root.*

For as in the 1ſt Example in the Square Root in this Sect. the Canon was (putting $s=$ the Square Number or Surd given, and $r$ for each Approach to the true Root) $\frac{s-rr}{2r}=r$ the 2d, *&c.* (or here to $a$) ſo in this Method I am going to ſhew, the Canon is $\frac{s+rr}{2r}=r$ the 2d, *&c.*

Now ſuppoſe $s=13$, it follows that the firſt $r$ (or Figure in the Root) is 3, ſo that $13+3^2\div 6=3.66=r$ the 2d.

$13+3.66^2\div 7.32=3.605955=r$ 3d.

And $13+3605955^2\div 7.21191=3.60555129=r$ 4th.

Note, *The Diviſors are always the double of the* r *next before.*

From the laſt Example you may note,

1. That though the preceding Number be falſe, yet by every Repetition of Proceſs according to the Canon it convergeth or comes nearer to the true Root or Value of $r$.

2. That the Places of the Digits in the ſeveral Approaches are true, according to the Progreſſion whoſe Ratio is 2, and the 1ſt Term $=1$. So *Places.*

In the 1ſt Approach 1 *viz.* 3
2 ..... 2 ... 3.6
3 ..... 4 ... 3.605
4 ..... 8 ... 3.60555 12+
$=r=$ the Square Root of 13 *propè.*

} Are true, the reſt not ſo.

And if the Operation were repeated, as $3.60555129^2+13\div 3.60555129\times 2$, the Quote will give 16 places true in the Root; for

for every Approach affords double to that preceding. But the Reader may chuſe which of the 2 ways he likes as the beſt, this, or the 1ſt way, which I take to be ſomewhat eaſier and ſhorter in the Operations of Diviſion.

*Example* 2. *Of this Way of Converging; to extract the Square Root of* 29507710.4132345.

29507710.4132345 (5 = *r* the 1ſt.
25 ....

2 *r* = 10) 545 ..... (54 = *r* the 2d.
45

2950
+2916 = $54^2$

2 *r* = 108) 5866 ..... (5432 = *r* the 3d.
466
347
237

29507710 = of the Data.
+29506624 = $5432^2$

2*r* = 10864) = 59014334 ..... (5432.1000 = *r* the 4th.
46943
34873
22814
10864

29507710.41323450
+29507710.41000000

2*r* = 108642) 59015420.82323450 (5432.100000297721 = *r* the 5th Anſwer.
469442
348740
228148
108642
0323450
1061660
838820
783260
227660
103760

VI. *But*

VI. *But to perform the laſt Operation by the Method firſt taught making* $a =$ *each Approach, and* $t =$ *the Total of the Approaches; it will ſtand thus:*

$\frac{s-tt}{2t} = a$ 29507710.4132345 = $s$ (5 = $t$ and $t+a=t$ the 2d = 54
10) 45 . . . . . 04 = $a$

2950
2916 = $tt$ and $t$ the 2d + $a$ the 2d = $t$ the 3d = 5432

$2t = 108$) 347 . . . . . (0032. = $a$ the 2d.
237 $t$ 3d + $a$ the 3d = $t$ 4th
and $t$ the 4th = 5432.1000

29507710
29506624 = $tt$

$2t = 10864$) 10864 . . . . . (0000.1000 = $a$ 3d
0132

295077 10.41323450
295077 10.41000000 = $tt$

$2t = 10864.2000$) . . . . . = .00323450000 (0000.000000297721 = $a$
1061660 (the 4th.
838820 and $a$ the 4th + $t$ the 4th.
783260
227660 = 5432.100000297721 —
103760 = the Square Root ſought.

This Method is ſhorter than the laſt (as I ſaid before) becauſe the Figures in the Approaches are not repeated by dividing: or the Sum of the 1ſt ($t$) and the ($a$'s) is = the ſaid Root.

And as in the laſt Example the Operations in Diviſion are ſhorter than in that next before; ſo the Canon for the Approaches (next after the 2 firſt, when the Squaring of the Approaches grows operoſe) does ſhorten the Work of Multiplication; or that Prop. of *Euclid* (I think 1, 2.) which ſays, That of two Lines (or Numbers) if one be divided into any Number of Parts, the Sum of the Rectangles of one by the ſeveral Parts of the other, is = the Rectangle of one Whole by the other.

But it being my Design here, only to shew how the large Multiplications for producing the 4th, &c. Approaches may be contracted; I shall explain the said Canon, which is grounded on another Prop. of *Euclid*, *i. e.* Any Line (or Number) divided into two Parts; the Sum of the Squares of the Parts, more the double Rectangle of those Parts is = the Square of the Whole.

So in the last Example putting $t$ the 3d $= b$
$a$ the 3d $= c$
the Square Number given $= s$
$t$ the 4th $= b + c$

There arises agreeable to the Canon under the 2d Head of Converging toward the Square Root, this Equation.

$$\frac{s - bb - 2bc - cc}{2b + 2c} = a \text{ the 4th.}$$

Now whereas it would be a little tedious to square the whole of $t$ the 4th $= 5432.1000$ (especially if the Cyphers were significant Figures) I therefore always divide it into 2 parts just at the meeting of the Approaches, as $t$ the 3d, and $a$ the 3d, *viz.* into 5432, and .1000, then as the Canon directs:

$cc = .1$
$2bc = \begin{cases} 5432 \\ 5432 \end{cases}$
and $bb = 29506624 = tt$ in the last Approach before,
(ready done.)
$295077104.1$ = the Sum $= 5432.1^2 = tt$ in the last Example.

In this Operation (this way performed) you save the trouble of working and inserting 12 Figures in 29; and the Divisor $2b + 2c$ is = the $5432.1000 \times 2$ (as in the last Example) so that

$$\frac{s - tt}{2t} = \frac{s - bb - 2bc - cc}{2b + 2c} = a \text{ the 4th as before.}$$

But the latter Canon being used, after the 3d, &c. Approaches, it does much facilitate the Operations. And thus I think I have made this matter shorter, more certain in the Approaches, and plainer than has been done before.

VII. *Ano-*

VII. *Another Example of Approximation to the Cube Root* ſhall be to extract the Cube Root of 33148648517294436663656267 = *c*

$\frac{c-s^3}{3s^2} = a$ = the Theorem or Canon.

Root ſought = *r*
Each Approach = *a*
And the Sum of the Approaches = *s*

1 *Note*, You always bring down ſo much of *c*, as will admit of $s^3$ to be deducted.

2. If any thing had remained upon deducting the laſt $s^3$, you muſt have put Cyphers, and divided again by $3s^2$. See the Operation, and the Probation thereof.

33148648517294436663656267 (3 = *a* the 1ſt.

$3a^2 = 27$) 61 (..... 02 = *a* the 2d.
7
+*a* the 1ſt = 32 = *s*.

33148 = part of *c*.
32768 = $s^3$

$3s^2 = 3072$) 3806 (..... 00123 = *a* the 3d.
7344
12008
2792
+32 = 32123 = *s* the 2d.

33148648517294 = part of *c*.
33147310244867 = $s^3$ (the 2d.)

$3s^2 = 3095661387$) 13382724274 ..... 000004323 = *a* the 4th.
10000787263
7136031026
9447082526
+*s* the 2d = 32123
= 321234323
= *s* the 3d = *r*.

33148648517294436663656267 = *c*
33148648517294436663656267 = $s^3$ (the 3d.)

0 reſts, *ergo* 321234323 = Root required.

This might be proved by the common Way of Extracting, tho with very many more Figures: I shall chuse therefore rather to prove it by Involution of the Root, multiplying by 2 Digits at once, as taught *Compend. 6. Chap. 3.*

321234323=s=the Root.
× 321234323

7388389429
13813075889
7388389429
6745920783
963702969

103191490273268329=$s^2$
321234323=s

2373404276285171567
443723408175053814 7
2373404276285171567
216702129573863490 9
309574470819804987

331486485172944366636562 67=*a*=the Proof.

3 *Note*, That tho you put a Figure or 2 too many or too few, or too little or too great, next the right hand of your Quotients, it will be rectify'd in the following Operations, if you proceed as *per* the Theorem. But the safest way is not to proceed farther at one Division than in the Example.

VIII. To illustrate what is said in [*Note* 3.] I shall shew how the first Example of Approximation to the Cube-Root foregoing in this 13th Section, is done by different Approaches and Figures to what it is there: And how to manage when Negative Numbers fall in, as when the Cube of *a* or *s* exceed the Cube given from which they are to be subtracted.

*Thus*

*Thus to extract the Cube Root of* 13 $=c$ ; Approaches $= a$
Sum of the Approaches $= s$
The Theorem $\frac{c-a^3}{3a^2} = a$ the 2d, &c. Root $= r$

13.0 . . . . . (2.4 $=a$ the 1st
$3a^2=12$) 50

13.000 $=c$
13.824 $=a^3$

$3a^2=17.28$) —.824000 refts (—.0476 $=a$ the 2d + $a$ the 1st.
13280 $=2.3524=s$.
11840

13.
13.0176762 1824 $=s^3$

$3s^2=16.60135718$) —.0176762 1824 0000 (—.0010647 $=a$ the 3d.
10748610600 + 2.3524 $=s=$ 2.3513353
7877962920 $=s$ the 2d.
12374200480

13.
13.0000101554648092 37977 $=s^3$ (the 2d.)

$3s^2=16586333079078 27$)—.0000101554648092 379770000 (—.0000006 12278 $=a$
(the 4th.
203664961791 0150 + 2.3513353
3780163100023230 $=$2.3513346 87722
462896484207 5760 (= $s$.
131169822626 01060 = $r$ the Root of the
150454910724 62710 ($c$ given *propè*.

1. It may be obſerved from the laſt Operation, That there are different Digits in every Approach, from thoſe in the Approaches under the firſt Example of the Cube Root, in this *Sect.* foregoing. But 2*dly*, That tho the Digits toward the right hand in theſe Approaches be wrong, yet thoſe $a$'s ariſing from the ſubſequent Operations proceed in each nearer and nearer to the Truth.

2. Altho

2. Altho the Cube of $s$ does all along exceed $c$ the Cube Number given, and cannot therefore in the common way be ſubſtracted; yet by the Rules foregoing in Logarithms and Algebra, and under the word *Negative* (Arithmetic) in the Alphabetical Explanation at the beginning of Algebra, it may; (for example) 13.824 may be taken from 13: but then there will always remain ſo much leſs than nothing to be mark'd with the Sign (—) as the Subtrahend (or Cube of $s$) exceeds ($c$) *i. e.* in this Example of deducting $a^3$ (the firſt) from $c$ (or 13.824 from 13) there muſt remain —.824 to be divided by the laſt Approach ſquared and multiply'd by 3. And therefore that Quote muſt be negative, *i. e.* —.0476, which added to $2.4 = a$ the firſt (as *per* the Doctrine of Addition of Algebra) the Sum is $= 2.3524$. And ſo forward of the Management of the other Negative Numbers or Quantities; as is taught in the Arithmetic of Negatives under the word *Negative*, at the beginning of Algebra, *Chap* 10. and alſo *Sect.* 2, 3, 4, 5. of that Chapter; and alſo *Sect.* 3. of *Chap.* 7.

Theſe are the moſt Brief and Accurate Ways that I know of Converging Series, tho I am not ignorant of others which ſome Authors have been fond of exhibiting; but they are neither ſo ſhort nor accurate as the common Way of Extraction. I ſhall give the Inſtance which they have exemplify'd in finding the Square Root of 133225.

*Their Way by Converging Series.*

$$133225 = c$$

$r = 3)\ 66612.5 = \frac{c}{2}$

$$-45 = \frac{rr}{2}$$

*Diviſor* $r = 3)\ 216\ (6 = s$

$$18 = rs$$

$$+18 = \frac{ss}{2}$$

$= 198 =$ Sum.

$r = 36)\ 1812.5\ (5 = s$

$$180 = rs$$

$$12.5 = \frac{ss}{2}$$

1812.5 Sum.

0

*Root* $= 365$

*The Common Way of Extraction.*

133225 (365 = *Root.*

6) 432

72) 3625

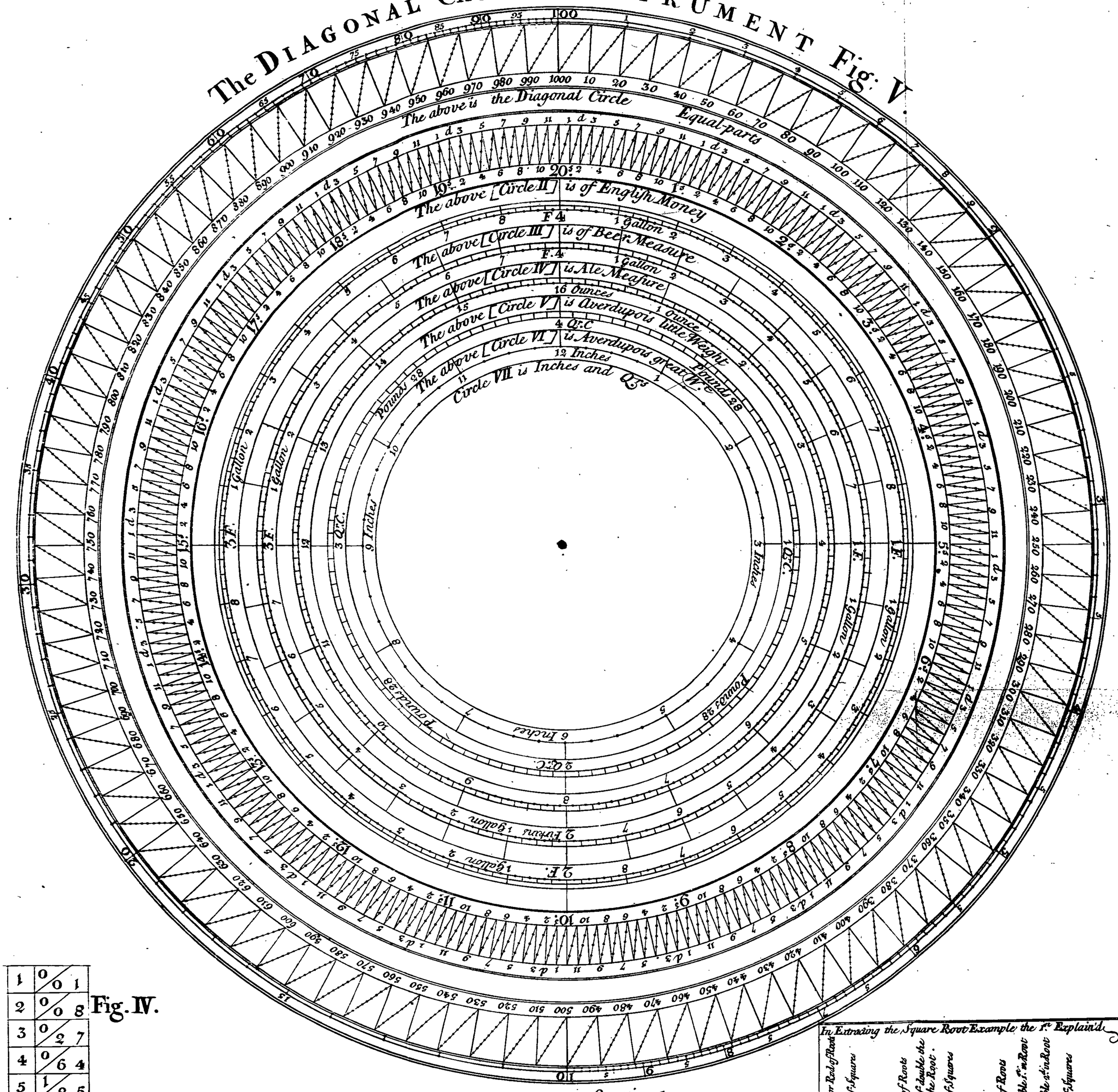
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Done by putting down 20 Figures: Whereas their Way by Converging takes 48, beſides the Signs and Symbals, and is therefore not worthy of Conſideration.

Their

*Plate B.*

# The DIAGONAL Circular INSTRUMENT Fig: V.

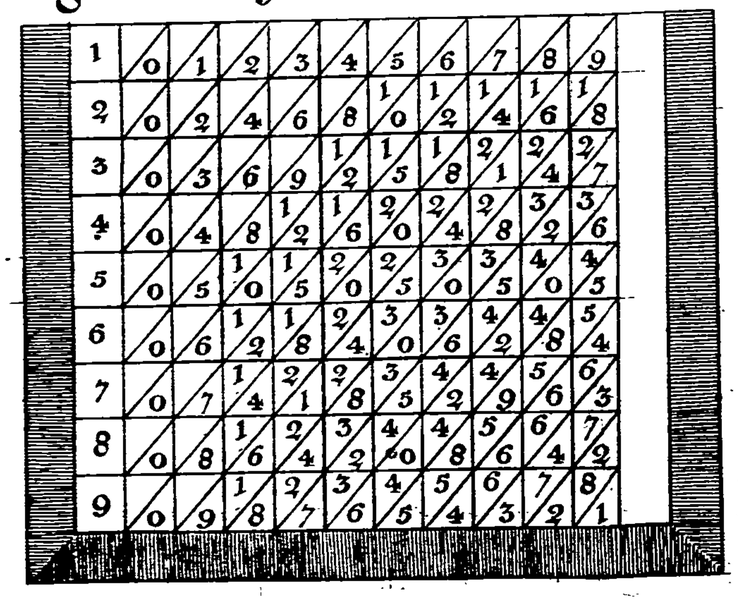



| | | |
|---|---|---|
| 1 | 0 | 0 1 |
| 2 | 0 | 0 8 |
| 3 | 0 | 2 7 |
| 4 | 0 | 6 4 |
| 5 | 1 | 2 5 |
| 6 | 2 | 1 6 |
| 7 | 3 | 4 3 |
| 8 | 5 | 1 2 |
| 9 | 7 | 2 9 |

*A Rod of Roots*
*A Rod of their Cubes*

Fig. IV.

Fig: I. *A Set of the Bones or Rods.*



*Question 1 Tabulated.*

*In Multiplication*

| | | | | | |
|---|---|---|---|---|---|
| 1 | 3 | 4 | 7 | 2 | 9 |
| 2 | 6 | 8 | 1 4 | 4 | 1 8 |
| 3 | 9 | 1 2 | 2 1 | 6 | 2 7 |
| 4 | 1 2 | 1 6 | 2 8 | 8 | 3 6 |
| 5 | 1 5 | 2 0 | 3 5 | 1 0 | 4 5 |
| 6 | 1 8 | 2 4 | 4 2 | 1 2 | 5 4 |
| 7 | 2 1 | 2 8 | 4 9 | 1 4 | 6 3 |
| 8 | 2 4 | 3 2 | 5 6 | 1 6 | 7 2 |
| 9 | 2 7 | 3 6 | 6 3 | 1 8 | 8 1 |

Fig: II.

*In Extracting the Square Root Example the 1st Explain'd*

| Index or Rod of Roots | Rod of Square | Rod of Roots | Rod of double the 1st in the Root. | Rod of Squares | Rod of Roots | Rod of double 1st in Root | Of double 2d in Root | Rod of Squares |
|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 4 | 1 | 1 | 4 | 6 | 1 |
| 2 | 4 | 2 | 8 | 4 | 2 | 8 | 1 2 | 4 |
| 3 | 9 | 3 | 1 2 | 9 | 3 | 1 2 | 1 8 | 9 |
| 4 | 1 6 | 4 | 1 6 | 1 6 | 4 | 1 6 | 2 4 | 1 6 |
| 5 | 2 5 | 5 | 2 0 | 2 5 | 5 | 2 0 | 3 0 | 2 5 |
| 6 | 3 6 | 6 | 2 4 | 3 6 | 6 | 2 4 | 3 6 | 3 6 |
| 7 | 4 9 | 7 | 2 8 | 4 9 | 7 | 2 8 | 4 2 | 4 9 |
| 8 | 6 4 | 8 | 3 2 | 6 4 | 8 | 3 2 | 4 8 | 6 4 |
| 9 | 8 1 | 9 | 3 6 | 8 1 | 9 | 3 6 | 5 4 | 8 1 |

200 — 30 — 4 = 234 *the Root*

Fig: III

Their other Inſtance of Converging, in the Example of 2 takes 25 Digits more in the Operation than the common Way of Extraction, and is not true in the 3d Decimal Place: Wonderful new Invention!

# *An* APPENDIX

Fig. 1.




Fig. 2.

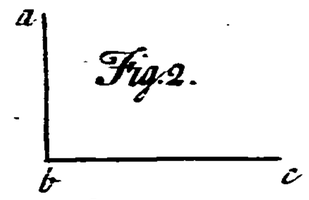



Fig. 3.

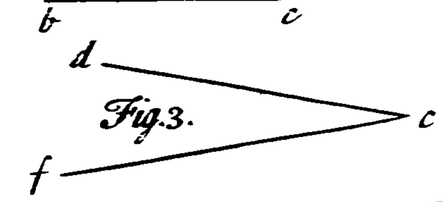



Fig. 5.

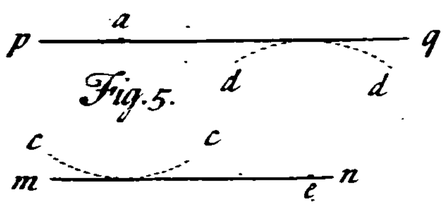



Fig. 7. This upper is the Line of Numbers or Logarithms

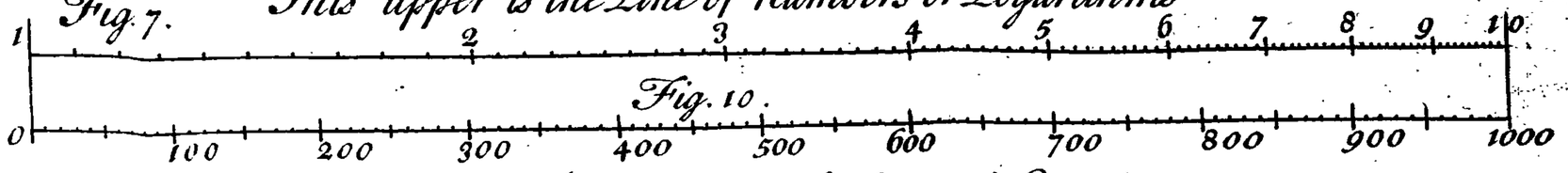



Fig. 10.




This Lower is ye Line of Equal parts

Fig. 12.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 6 | 1/2 | 1 | 2/4 | 3 | 3/6 | 4 | 4/8 | 5 | |
| | | 8 | | 0 | | 2 | | 4 | |

Fig. 13.

| | on ye Index Rod | there Stands on ye other 5 rods these figurs | Which Added by ye Rule above are these Numbers |
|---|---|---|---|
| | | 3 4 7 2 9 | |
| against | 5 | 1/5 2/0 3/5 1/0 4/5 | 173645 |
| against | 9 | 2/7 3/6 6/3 1/8 8/1 | 312561 |
| against | 7 | 2/1 2/8 4/9 1/4 6/3 | 243103 |
| against | 3 | /9 1/2 2/1 /6 2/7 | 104187 |
| against | 2 | /6 /8 1/4 4/4 1/8 | 69458 |
| Summ,or | | product required = 826376555 | |

Fig. 14.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1/2 | 0 | 1/6 | 0 | 3/6 | 3/6 | |

Fig. 15.

| |
|---|
| 3/6 |

Fig. 16.

| | |
|---|---|
| 2/7 | 2/7 |

the Index Rod — the Rod of Cubes

Fig. 17.

| |
|---|
| 2/7 |

Fig. 19.

| | | | |
|---|---|---|---|
| 1 | 1 | 8 | 1 |

Fig. 20.

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 9 | 6 | 1 |

Fig. 21.

| | | | | | |
|---|---|---|---|---|---|
| 1 | 1 | 9 | 7 | 4 | 1 |

Fig. 22.

| | |
|---|---|
| 1 | 0/0 1 |
| 2 | 0/0 8 |
| 3 | 0/2 7 |
| 4 | 0/6 4 |
| 5 | 1/2 5 |
| 6 | 2/1 6 |
| 7 | 3/4 3 |
| 8 | 5/1 2 |
| 9 | 7/2 9 |

Fig. 18. Dividend Quotient

2 5 3 2 4 = Divisor ) 2373469 1084 ( 93·7241.

| | | | | | |
|---|---|---|---|---|---|
| 9 | 1/8 | 4/5 | 2/7 | 1/8 | 3/6 |
| 3 | /6 | 1/5 | 9/6 | 1/2 | |
| 7 | 1/4 | 3/5 | 2/1 | 1/4 | 2/8 |
| 2 | /4 | 1/0 | 6/4 | /8 | |
| 4 | /8 | 2/0 | 1/2 | /8 | 1/6 |
| 1 | /2 | /5 | /3 | /2 | /4 |

against these Digits in the Tabulated Divisor are ye Products of the Divison by each Fig. in ye Quote as in the Rods.

Or ---- 227916 deduct
94309 ---------- 3
Or ------ 75972 deduct
183371 ---------- 7
Or ------ 177268 deduct
61030 ---------- 2
Or ------ 50648 deduct
103828 ---------- 4
Or ------ 101296 deduct
25324 ---------- 1
Or ------ 25324 deduct
(0) Rests

Fig. 23

| | | | | |
|---|---|---|---|---|
| 1 | 7 | 5 | 0 | 1 |
| 2 | 1/4 | 1/0 | 0/0 | 8 |
| 3 | 2/1 | 1/5 | 0/2 | 7 |
| 4 | 2/8 | 2/0 | 0/6 | 4 |
| 5 | 3/5 | 2/5 | 1/2 | 5 |
| 6 | 4/2 | | | |
| 7 | 4/9 | | | |
| 8 | 5/6 | 4/0 | 5/1 | 2 |
| 9 | 6/3 | 4/5 | 7/2 | 9 |

Place this plate A, — at the end of the Book, next to ye last leaf thereof.

Fig. 4.




Fig. 9.

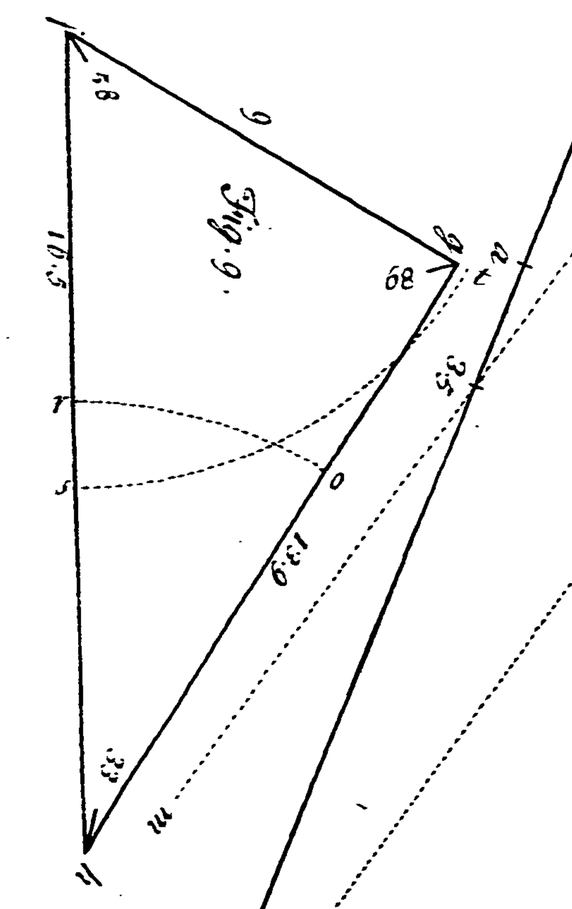



Fig. 6.




Fig. 10. See Fig. 10. also above




Fig. 8.




Fig. 24.

ye Divisor Tabulated — ye Rod of Cubes

| 1 | 9 | 4 | 0 | 8 | 0/0 | 1 |
|---|---|---|---|---|---|---|
| 2 | 1/8 | 8/ | 0 | 1/6 | 0/0 | 8 |
| 3 | 2/7 | 1/2 | 0 | 2/4 | 0/2 | 7 |
| 4 | 3/6 | 1/6 | 0 | 3/2 | 0/6 | 4 |
| 5 | 4/5 | 2/0 | 0 | 4/0 | 1/2 | 5 |
| 6 | 5/4 | 2/4 | 0 | 4/8 | 2/1 | 6 |
| 7 | 6/3 | 2/8 | 0 | 5/6 | 3/4 | 3 |
| 8 | 7/2 | 3/2 | 0 | 6/4 | 5/1 | 2 |
| 9 | 8/1 | 3/6 | 0 | 8/1 | 7/2 | 9 |

Fig. 25.

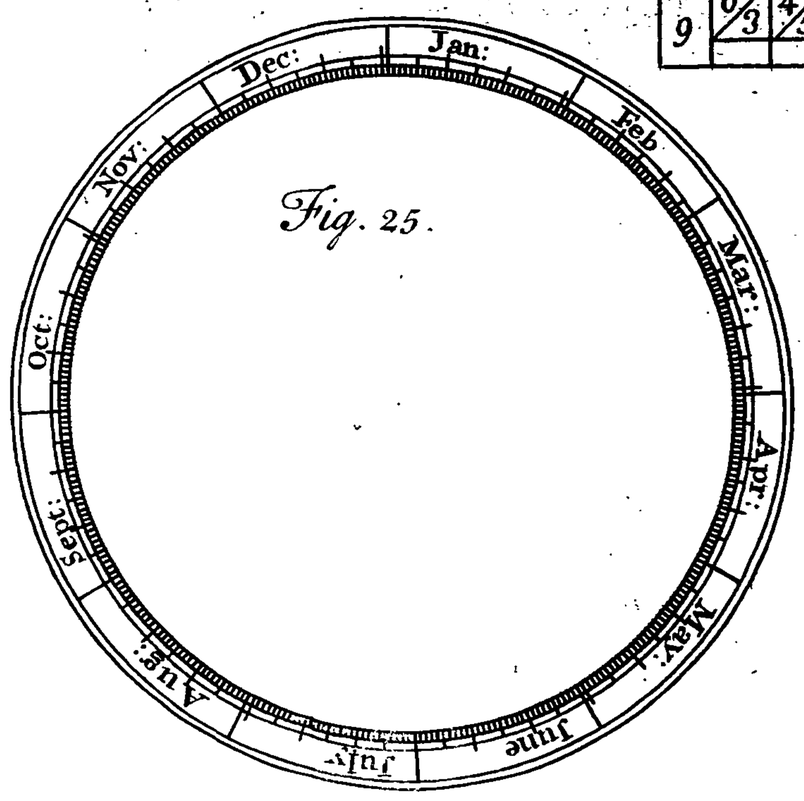



Fig. 27.

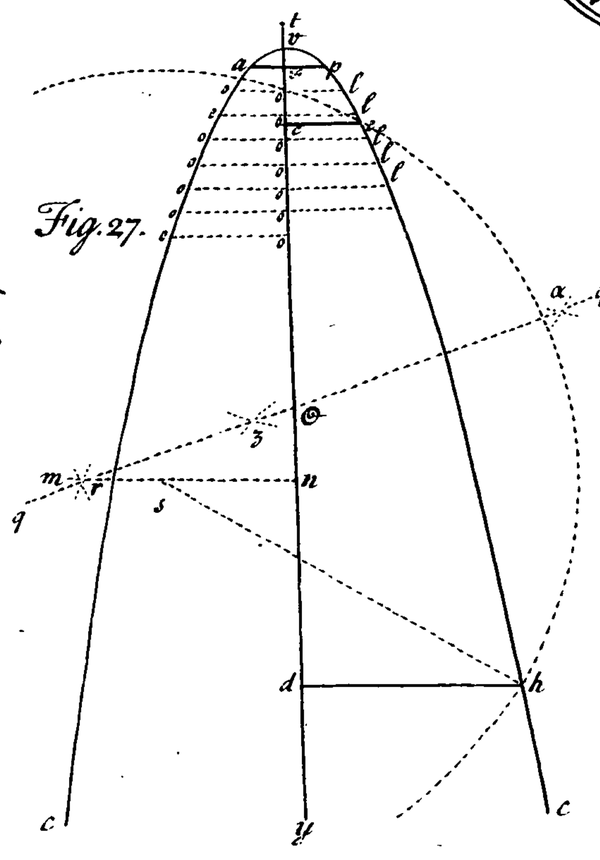



Fig. 26.

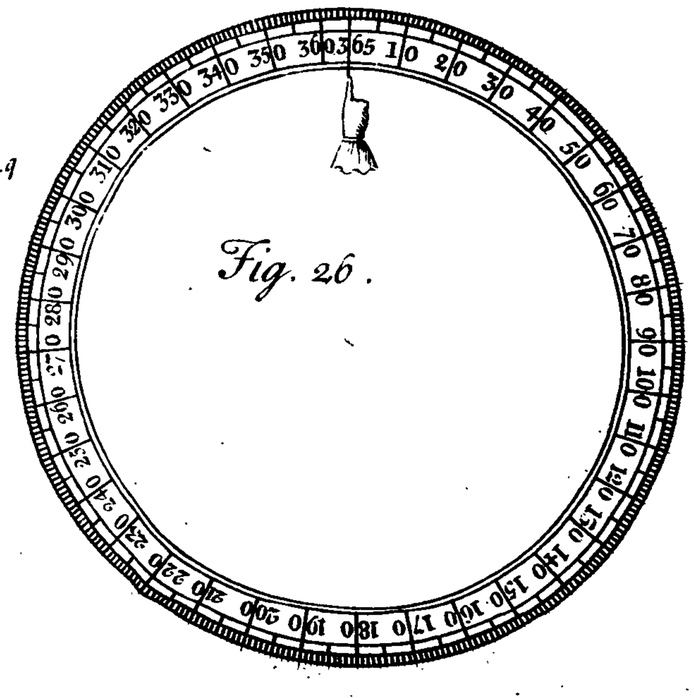


Plate C

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14.

Fig. 15

Fig. 16.

Fig. 17.

Fig. 18.

Fig. 19.

Plate C. place at ye end of the Book next B. viz.t A.B.C.

Fig. 20.

Fig. 21.

Fig. 22

Fig. 23.

Fig. 24

The 5 platonic bodys.

# *An* APPENDIX *to an Intire System of Arithmetic:*

*By* EDWARD HATTON *Gent.*

---

## *Of the Menſuration of Superficies and Solids.*

MY Intent is to be as Brief, and yet as Copious as poſſible; not doubting but the Variety of Figures here meaſured, more than any other Treatiſe of Meaſuring contains, that I have ſeen or believe to be publiſhed, will meet with the Approbation of the Studious in this Geometrical Science.

The Dimenſions I will ſuppoſe taken in Feet and Decimal Parts, as beſt anſwering that Accuracy and Brevity which will be expected from me: But at the Cloſe of this Appendix I ſhall give my Advice what Inſtruments are moſt proper for the ſeveral kinds of Workmen; and alſo by what Denomination the ſeveral kinds of Work is valued.

## CHAP. I.

### *To meaſure any Superficies.*

*Superficies* is properly *defined* to conſiſt only of Length and Breadth or Girt, without taking any Notice of Thickneſs.

## SECT. I. *The Geometrical Square and Parallelogram, Rombus and Romboide.*

*Propoſition* 1. *To meaſure the Geometrical Square.*

*Definition.* This is a Figure conſiſting of 4 equal Sides, and as many right Angles: as *Fig.* 1. *Plate* C.

*Rule.* Take the Dimenſion of one Side, as *a b*, and multiply it by itſelf, gives the Content.

*Example.* The Side of any Square being 11.28, the Content is found 127.238, as *per* Marg.

| | |
|---|---|
| | 11.28 |
| | 11.28 |
| | 31584 |
| | 12408 |
| *Anſw.* = | 127.2384 |

*Prop.* 2. *To find the Area of a Parallelogram.*

*Defin.* This is a Figure conſiſting of or bounded by 2 longer equal parallel Sides, and 2 ſhorter equal and parallel Sides, having 4 right Angles: as *Fig.* 2. *Plate* C.

*Rule.* Multiply the Length by the Breadth, and the Product is the Area.

*Example.* The Length *a b* = 23.53, Breadth *b c* = 2.15. See the Work.

| | |
|---|---|
| | 23.53 |
| | 2.15 |
| *Anſw.* = | 50.5895 |

*Prop.* 3. *To find the Area of a Rombus.*

*Defin.* This Figure is bounded by 4 equal Sides, and hath 4 Angles, the 2 oppoſite ones being equal, and the Sides parallel: as *Fig* 3. *Plate* C.

*Rule.* The Diſtance between 2 oppoſite Angles, as *b d*, multiplied by the Line *e c*, gives the Anſwer.

*Example.* The Diagonal (*b d*) = 1.34; *e c* = .85; the Anſwer is 1.139.

| | |
|---|---|
| | 1.34 |
| | .85 |
| | 1.1390 |

*Prop.* 4. *To find the Area of the Romboides.*

*Defin.* This Figure is bounded with 4 right Lines, the oppoſite Sides equal and parallel, and the oppoſite Angles equal: as *Fig.* 4. *Plate* C.

*Rule.* Multiply the Diagonal *a b* by the Perpendicular *c n* or *m r*, and the Rectangle anſwers.

*Example.* The Diagonal *a b* = 12.47, the Perpendicular *c n* = 4.127, and the Anſwer is therefore, as *per* Margin, = 51.46369.

| | |
|---|---|
| | 12.47 |
| | 4.127 |
| | 33669 |
| | 51127 |
| *Anſw.* = | 51.46369 |

SECT.

## SECT. II. *To meaſure the Conic Sections.*

PROP. 1. *To meaſure a Triangle.*

*Defin.* Right-lined Triangles are of 3 kinds (beſides the equilateral, whoſe Sides are equal) *viz.* Right-angled, as (*a r n*) or (*x r n*) right-angled at *r* in *Fig.* 5. 2*dly*, Scalenous Triangles, as (*o r o*) the Sides of which are all unequal. And, 3*dly*, Iſoceles Triangles, as (*a n x*) appears; 2 of whoſe Sides are equal. This laſt is one of the Sections of a Cone, made by cutting the Cone (*e w a g x n a*) down the Axis, as (*o p*) *Fig.* 5. *Plate* C.

*Rule.* Multiply the Baſe (as *x a*) by half the Perpendicular (as *n r*) or half the Baſe by the Perpendicular, or the Perpendicular by the Baſe; and taking half the Product, will give the Area of any Triangle as above mentioned: for all Triangles are half a Square of the ſame Baſe and Perpendicular.

*Example.* The Baſe (*a x*) being = 9.35, the Perpendicular *n r* = 12.42, the Area is 58.0635, as *per* Margin.

| |
|---|
| 9.35 |
| 6.21 |
| 19635 |
| 5610 |
| *Anſw.* = 58.0635 |

*Prop.* 2. *To meaſure the Circle.*

*Defin.* This is formed by cutting the Cone (*x w a g x n a*) parallel to the Baſe (*x w a g x*) as (*m c*) *Fig.* 5. *Plate* C.

*Rule.* There are ſeveral ways of finding the Area of a Circle. As,

1*ſt*, As 7 to 22, ſo the Diameter (as *r s*, *Fig.* 6.) to the Circumference.

2*dly*, As 1 to 3.1416, ſo the Diameter to the Circumference.

Then multiply the Circumference by a 4th of the Diameter, or a 4th of the Circumference by the Diameter gives the Anſwer.

Or the Area of a Circle is found without the Circumference thus:

As 1 is to .7854 :: ſo is the Square of any Diameter to the Area.

*Example.* Admit the Diameter (*r s*, *Fig.* 6.) be = 14.32, what is the Area?

By the laſt Method

1. .7854 :: 205.0624. 161.056 = Anſw.

| |
|---|
| 14.32 |
| 14.32 |
| 45824 |
| 20048 |
| □ = 205.0624 |

*Prop.* 3. *To meaſure the Ellipſis, or Oval.*

*Defin.* This is a Section of a Cone, made by cutting it through both its Sides, but not parallel to the Baſe; as the Section (*l e*) *Fig.* 5. *Plate* C. which produceth *Fig.* 7. *Plate* C.

*Rule.* Multiply the *Latus tranſverſum* (or tranſverſe Ax) *l t*, by the Diameter Conjugate (*c d*) and the Product is = the Square of the Diameter of a Circle equal to the Oval: then work as in the laſt.

*Example.* Admit *l t* = 15.34, and *c d* = 12.28, the Anſwer is found 147.95 *fere*: for

1. .7854 :: 188.3752. 147.9499

*l t* = 15.34
*c d* = 12.28

42952
18408

∠ = 188.3752

*Prop.* 4. *To find the Area of the Parabola.*

*Defin.* This Section is formed by cutting the Cone parallel to the oppoſite Side, as in the Line *t s*, *Fig.* 5. which is parallel to the Side (*a n*) and produceth a Parabola; as *Fig.* 8. *Plate* C.

*Rule.* This Figure being 2 Thirds of a Parallelogram made of the whole Ordinat (*p a*) and the Abſciſſa (*s s*) therefore multiplying thoſe 2 Lines together, 2 Thirds of the Rectangle is the Area.

3.84
2.54

20736
768

*Example.* Admit *a p* = 3.84, and *s s* = 2.54; 2 Thirds of the Rectangle is = the Anſwer = 6.5024.

*Rectangle* = 9.7536
$\frac{1}{3}$ = 3.2512
$\frac{2}{3}$ *or Anſwer* = 6.5024

*Prop.* 5. *To find the Area of the Hyperbola (or Hyperbolic Space.)*

*Defin.* This Section of a Cone is formed by cutting it (not parallel to the Side, as in the Parabola, but) ſo that if the Axis of the Section were continued upward, it would interſect the oppoſite Side of the Cone produced: the Section's Axis is repreſented by the Line *y z*, *Fig.* 5. and produceth an Hyperbola, as *Fig.* 9. *Plate* C.

*To meaſure this Figure*, there is no general Rule that can be given for Practice: But the Inveſtigation is performed by the help of the Aſymptotes, an Infinite Series, and partly by the Method of Fluxions; which 'tis not proper here to inſert.

## SECT. III. *To meaſure any of the Regular Polygons.*

*Defin.* Theſe are Figures conſiſting of above 4 equal Sides, and as many equal Angles. As,

The Pentagon having 5 equal Sides, and as many Angles.
Hexagon . . . . . 6
Heptagon . . . . . 7
Octogon . . . . . 8

The

The Nonogon having 9 equal Sides, and as many Angles.
Decagon . . . . . 10
Undecagon . . . . . 11
Dodecagon . . . . 12

*Rule.* To measure any of these, draw 2 Lines from any 2 Angles that include a Side to the Center: then find the Content of that Triangle, as taught *Prop.* 1. § 2. above. Lastly, Multiply the Area of that Triangle by the Number of Sides, and the Product is the Content of the whole Polygon. Or multiply half the Sum of the Sides by the nearest Distance from the Center to one of the Sides, gives the Answer.

*Example.* The Side of the Pentagon (*s d, &c.*) *Fig.* 10. is = 15.34, the Perpendicular (or Radius of a Circle inscribed) is *rc* = 10.5: So that 15.34 multiplied by 5, the Number of Sides, gives = 76.7 = Sum of the Sides; half of that is = 38.35, which multiplied by *r c* = 10.5, gives the Area = 402.675.

38.35
10.5
*Answer* = 402.675

## Sect. IV. *To measure the Trapezium, the Parallelogramic, the Polygram, (or Multangle, or irregular Polygon) the Area or Superficies of the Cylinder, Cone, and Sphere.*

*Prop.* 1. *To measure the Trapezium.*

*Defin.* This is a Figure of 4 Sides and 4 Angles, uncertain whether parallel or equal, but is commonly neither; as *Fig.* 11. *Plate* C.

*Rule.* Multiply the Diagonal (*a n*) by half the Sum of the Perpendiculars *m p* and *c p*; or the Sum of the Perpendiculars by half the Diagonal, and either of the Products is the Answer.

*Example.* The Diagonal *a n* = 17.42; the Perpendicular *m p* = 4, and *c p* = 2.6 *r*, half the Diagonal is = 8.71, and *m p* more *c p* = 6.6 *r*, the Product of which 2 last is 58 06666, *&c.*

8.71
6.6*r*
5226
*Answ.* = 58.06*r*1

Or half the Product of the Diagonal in the Perpendiculars gives the Answer.

*Prop.* 2. *To find the Area of the Parallelogramic, or Parallelopleuron.*

*Defin.* This Figure hath 4 Sides, 2 of which opposite are parallel, and of 4 Angles the 2 at each end being equal: as *Fig.* 12. *Plate* C.

*Rule.*

*Rule.* Multiply half the Sum of the 2 Sides ($v\,r$) and ($s\,t$) by the Length ($t\,w$) gives the Answer. Or a Diagonal ($r\,t$) multiplied by half the Sum of 2 Perpendiculars let fall on that Diagonal from $s$ and $v$.

*Example.* The Ends $r\,u = 0.79$; $s\,t = 0.7$; Length $w\,t = .8888$, *&c.* half the Sum of .79 and .7 multiplied in .8 $r$ = .662 $r$ 1.

*Prop.* 3. *To find the Area of the Polygram, (sometimes called an irregular Polygon, or Multangle.)*

*Defin.* This is any Geometrical Figure having above 4 Sides, which are uncertain, and generally unequal; as are also its Angles: as *Fig.* 13. *Plate* C.

*Rule.* First divide your Figure into Trapeziums and Triangles, and then measure the same as under *Prop.* 1. §. 2. and §. 4.

*Example.* Figure 13 is divided into,

1. The Trapeziums ($u\,t\,m\,q\,v$)
2. . . . . and $q\,m\,p\,r\,q$
3. And the Triangle $q\,r\,a\,q$.

Sum $r\,u$ more $s\,t$ = 1.49

$\frac{1}{2}$ = .745

.8 $r$

5960

.662 $r$ 1

In the 1st, $q\,t$ = 18.26, the $\frac{1}{2}$ = 9.13

$u\,x$ = 5.17, more $h\,m$ = 3.5 = 8.67

61171

7304

*Prod.* = 79.1571

In 2d Trapez. $q\,p$ = 15.73, the half = 7.865
$m\,n$ = 4.5, more $r\,s$ = 2.31 = 6.81 — *Prod.* = 53.56065

In 3d or the $\triangle$, $q\,a$ = 10.72; the half = 5.36
$r\,o$ = the Perpend. = 3.27 — *Prod.* = 17.5272

The Sum = the Area of the whole *Fig.* 13. = 150.24495

*Prop.* 4. *To find the Area of a Cylinder.*

*Defin.* This Figure is defined under *Prop.* 1. *Chap.* 2.

*Rule.* Multiply the Ambit or Circumference of the Cylinder by its Length, and to that Rectangle add the double Area of the Base or End, and the Sum is the whole Area.

*Area of the curved part of the Cylinder is therefore* = 3.1416

*Area of the Base* = .7854

*Ditto* = .7854

*Whole Area (or Sum)* = 4.7124

*Example.* Ambit of the Cylinder = 3.1416; Altitude as also the Diameter of the Base = 1.

But there are 2 other ways of doing this, where the Diameter and Altitude are equal. As

As, 2*dly*, Multiply the Area of the Sphere having the ſame Diameter by 3, and divide by 2.

Or, 3*dly*, 6 times the Area of the Baſe or End is the Anſwer. Thus 6 times .7854 is = 4.7124, as above.

*Prop.* 5. *To find the Convex Superficial Content of a Sphere.*

*Defin.* This Body is defined where its Solidity is meaſured, *Prop.* 4. *Chap.* 2. and 'tis repreſented *Fig.* 6. *Plate* C.

*Rule.* The Area is 4 times that of a great Circle; therefore having by the Diameter found the Area of a great Circle of any Sphere, multiply it by 4, and you have the Anſwer.

*Example.* The Diameter of a Sphere being 12, the Area of the ſame is 452.39, as *per* Margin.

1. .7854 :: 144.
144

*Prod.* = 113.0976 *Area Circle.*
4

452.3904 = *Area of the Sphere.*

*Prop.* 6. *To find the Superficies (or Area) of a Pyramid.*

*Defin.* This is a ſolid Body, having (generally) a Square or ſome regular Polygon for its Baſe, being tapering like the Spire of a Steeple, *&c.*

*Rule.* Multiply the Circumference of the Baſe (or Sum of the Sides) by half the Altitude, and it gives the Superficial Content.

*Example.* The Side of the Baſe of a Pyramid (being an Octogon) is = 5.65, that multiplied by 8 = the Number of Sides, and that Product by half the Altitude = 30.7, the laſt Product is the Area ſought.

5.65
8

*The Total Sides* = 45.20
*Half the Altitude* = 30.7

*Area* = 1387.64

*Prop.* 7. *To find the Area of a Cone.*

*Defin.* The Cone is a Solid which hath a Circle for its Baſe, from whence it is gradually tapering upward till it terminate in a Point directly over the Center of its Baſe; as *Fig.* 5. *Plate* C.

*Rule.* As the Radius of the Baſe: Is to the Side of the Cone :: So is the Area of the Baſe to the Area of the Cone.

*Example.* Diameter of the Baſe = 6; Side of the Cone = 15: So the' Area of the Baſe of the Cone is found = 28.2744; and the Area of the Cone, as by the 2d Analogy = 141.37.

1. .7854 :: 36
36

28.2744 = *Area of the Baſe.*

*Rad. Baſ. Side. Ar. Baſe Ar. Cone*
3. 15 :: 28.2744. 141.37

Or

Or *Note* farther, That the Area of a Cone is = that of a Triangle, whose Perpendicular is the Altitude of the Cone, and the Base = the Circumference of the Cone's Base: For in Example, 1. 3.1416 :: 6. 18.8496 = the Base or Circumference; half of which multiplied by the 15, gives the Answer as before = 141.37.

SECT. V. *To find the Area of the Parts of a Circle; as the Semicircle, Quadrant, Sector, Segment, Lune or Crescent, and the Trochoid or Cycloidal Space.*

PROP. 1. *To measure the Semicircle, Quadrant, and Sectors of a Circle, also the Segment.*

*Defin.* The Semi or Half-Circle is the Space between (*d m b d*) *Fig.* 14. *Plate* C; the Quadrant or Quarter of the Circle is (*dmcd*) the Sector is *cbrec* or the like, less or more than a Quadrant.

*Rule.* Multiply 1 fourth of the Ark, or Curve Line, by the Diameter of the Circle, and the Product is the Area required.

*Example.* The Circumference is = 3.1416, the Diameter of that Circle = 1.

The Ark of the Half-Circle *dcbmd* as *bmd* = 1.5708
of the Quadrant *cmdc* as *mmd* = .7854
of the Sector *cbrec* as *erb* = 1.05

So that 1 fourth of 3.1416 = .7854 × by 1, gives .7854 the Area of the Circle.

1 fourth of 1.5708 = .3927 × by 1, gives .3927 the Area of the Half-Circle.

1 fourth of .7854 = .1963 × by 1, gives .19635 the Area of the Quadrant.

1 fourth of 1.05 = .26211 × by 1, gives .26211 the Area of the Sector.

And the Area of the Sector (*c b r e c*) less the Area of the Triangle (*c b n e c*) gives the Area of the Segment (*e n b r e*) *Fig.* 14. *Plate* C.

*Prop.* 2. *To find the Area of the Lune.*

*Defin.* This is a Figure like a Crescent or increasing Moon; as (*a r n o a*) *Fig.* 15.

*Rule.* Find first the Area of the Semicircle (*n c a r n*) and from that deduct that of the Segment (*n c a o n*) and the Remainer is the Area of the Lune.

*Example.* The Diameter *n a* = 1, and consequently the Ark *arn* = 1.5708, and Area = .3927.

The Area of the Sector *e a o n e* is = .256, from which deducting the Area of the Triangle *n e a n* = .135,

.3927
.121
.2717
the

the reſt is the Area of the Segment = *n o a c n* = .121; which taken from the Area of the Semicircle, there reſteth the Anſwer = .2717.

*Prop.* 3. *To find the Area of the Trochoid, or Cycloidal Space.*

*Defin.* This Figure (as 16, *Plate* C.) is generated by the Circle (*l b o r*) which turneth round on the Line *a r c*, the Point at *b* beginning at *a*, and when it touches the Line again at *c*, the Cycloidal Curve *a b c* is finiſhed, and the Space from that to the Line *a r c* contains 3 times the ſaid generating Circle.

*Rule.* Therefore having found the Area of a Circle, whoſe Diameter is (*r b*) multiply that by 3, and it produceth the Area of the Cycloidal Space.

*Example.* The Line *r b* = 1; Area of the Circle = .7854, and of the Cycloidal Space = 2.356.

# CHAP. II.

## *The Menſuration of Solids.*

### SECT. I. *To find the ſolid Content of a Cylinder, Parallelopiped and Priſm.*

PROP. 1. *To meaſure the Cylinder.*

*Defin.* A Cylinder is a ſolid Body having a Circle for its Baſe, and is of equal Circumference from one end to the other, like a Rolling-Stone for Walks, &c. as the *Figure* 17. *Plate* C.

*Rule.* Multiply the Area of the Baſe (found as under *Prop.* 2. of *Sect.* 2. *Chap.* 1.) by the Length, and the Product is the Anſwer.

*Example.* The Diameter of the Baſe, as (*a b*) or (*c d*) = 14.32, the Area of that Circle will be found 161.056; which multiplied by the Length (*b c*) = 14.32, or (*a d*) the ſolid Content, is = 2306.32192, as *per* Margin.

```
161.056
 14.32
-------
 5153792
 2254784
-------
2306.32192
```

*Prop.* 2. *To find the ſolid Content of the Parallelopipedon, and of the Priſm.*

*Defin.* This Solid is bounded by 4 Parallelograms, equal and parallel, for the Sides; and 2 Squares or Parallelograms, equal and parallel, for the Ends: as *Fig.* 18. *Plate* C.

*Rule.* Multiply the Area of the Baſe or End by the Length, and the Rectangle is the Anſwer.

*Example.* The End is a Square whoſe Side is 3.72, and the Length = 31.44. See the Operation, where (according to the Method of working *Caſe* 5. of *Sect.* 5. of *Chap.* 3.) the Solidity is found 435.0793.

*Area at the end* 138384
*Length inverted* 44.13

415152.
138384
55354
5535

435.0793 = *Solid Content.*

3.72
3.72

26784
1116

13.8384 = *Area*

*Note*, That a *Priſm* is meaſured alſo by multiplying the Area of the Baſe (which is either a Triangle or a Polygon) by the Length of the Priſm. This or the laſt Figure are three times a Pyramid of the ſame Baſe and Altitude.

*Prop.* 3. *To find the ſolid Content of the Cone: And, 2dly, of a Pyramid.*

*Defin.* This Figure hath a Circle for its Baſe, and is defined *Prop.* 7. *Sect.* 4. of *Chap.* 1. of this Appendix.

*Rule.* Multiply the Area of the Baſe by one third of the perpendicular Altitude of the Cone, and that Rectangle is the Anſwer.

*But Note*, That to find the Height (it being a ſolid Body) muſt be done by this Rule, as *Euclid* 47. 1.

From the Square of the ſlant Height (*m l*, *Fig.* 19.) deduct the Square of the Semidiameter of the Baſe, as (*m a*) and the Square Root of the Remainer is the perpendicular Altitude (*x l*).

*Example.* The Diameter *m d* is = 6, the ſlant Height *m l* = 5; and conſequently the true Altitude (*a l*) = 4: for 5 ſquared leſs $\frac{6}{2}$ = *m a* ſquared, = 16 = the Square of *a l*; that is, 5 ſquared leſs 3 ſquared, is = 16; whoſe Root is 4 = *a l*. So that as 1. .7854 :: 36. 28.274 = the Area of the Baſe: Which multiplied by 1 third of 4 = 1.3*r*, the Product is = 37.6986*r*1.

28.274
1.3*r*1

84822

94246
28274

37.6986*r*1 = *Anſw.*

*To meaſure the Pyramid*, you muſt alſo multiply the Area of the Baſe by a Third of the perpendicular Altitude: for as a Cone is 1 Third of a Cylinder of the ſame Baſe and Altitude, ſo is a Pyramid 1 Third of a Priſm or Parallelopiped. 'Tis defined *Prop.* 6. *Sect.* 4. *Chap.* 1. of this Appendix.

***Prop.* 4. *To find the ſolid Content of a Sphere.***

*Defin.*

*Defin.* This is a solid Body, every way perfectly round, so that all Lines drawn from the Convex Area to the Center are equal. It is 2 Thirds of a Cylinder of the same Diameter of the Base and Altitude with the Diameter of the Sphere, or of a Cylinder circumscribed.

*Rule.* Find the Area of a great Circle, as *Prop.* 5. of *Sect.* 4. *Chap.* 1. which multiply by the Diameter; and 2 Thirds of the Rectangle is the solid Content required.

*Example.* The Diameter of a Sphere being 12, the Area of a great Circle (as *Fig.*6. *Plate* C.) is = 113.0976; which multiplied by the Diameter (or Length of a Cylinder) 12, the Rectangle is = 1357.171, the Solidity of the Cylinder; 2 Thirds of which is the solid Content of the Sphere = 904.7808.

## Sect. II. *To find the Solidity of the five Platonic, or Regular Bodys*, viz. *the Tetrahedron, Hexahedron, Octahedron, Dodecahedron, and Icosihedron.*

PROP. 1. *To measure the Tetrahedron.*

*Defin.* This is a solid Body bounded with 4 equal equilateral Triangles; and being measured as a Pyramid, it needs no other Rule than as under *Prop.* 3. of *Sect.* 1. *Chap.* 2. See *Fig.* 20. *Plate* C.

*Prop.* 2. *To find the Solidity of the Hexahedron or Cube.*

*Defin.* This Solid is bounded with six equal Geometrical Squares.

*Rule.* Multiply a Side in itself, and that Rectangle by the Side gives the Answer. See *Fig.* 21. *Plate* C.

*Prop.* 3. *To find the Solid Content of the Octahedron.*

*Defin.* This Figure is bounded with 8 equal equilateral Triangles, which are the Bases of as many Pyramids, which meet in the Center of a Sphere, (out of which this Body is cut) so that it being 8 Pyramids. (See *Fig.* 22. *Plate* C.)

*Rule.* First find the Area of one of the Triangles, then take with Callipers, *&c.* the Distance between the Center of that Triangle and that of the opposite; by one sixth part of which multiply the Area of any one Triangle; and that Product multiply'd by 8, this last Rectangle gives the Solidity required.

*Prop.* 4. *To find the Solid Content of the Dodecahedron.*

*Defin.* This is a Solid bounded by 12 equal Pentagons, which are the Bases of so many Pyramids that meet in the Center of this Solid, or of a Sphere out of which this Body is cut. See *Fig.*23. *Plate* C.

*Rule.* Therefore take with a Pair of Callipers the Distance from the Center of one Pentagon to that of its opposite, and by 1 sixth of

that multiplying the Area of any of the Pentagons; 12 times that Rectangle gives the Solidity. *Or twice the Diſtance of the oppoſite Pentagons multiplied by the Area of* 1 *Pentagon*, gives the Anſwer.

*Prop.* 5. *To find the Solidity of the Icoſihedron.*

*Defin.* This is the laſt of the 5 Platonic or Regular Bodies: it is bounded with 20 equal equilateral Triangles, which are Baſes to ſo many Pyramids, whoſe ſmall Ends terminate in a Point in the Center of a Sphere which circumſcribeth this Solid, and out of which it is cut. (See *Fig.* 24. *Plate* C.)

*Rule.* Multiply the Area of one Triangle by 1 ſixth of the Diſtance of the Centers of 2 oppoſite Triangles (taken with Callipers) and that Rectangle multiplied by 20 (the Number of Pyramids in the Body) gives the Solidity ſought.

I have not thought it neceſſary to give Examples to theſe 5 Rules, becauſe there is not much more therein beſides the repeated Menſuration of a Pyramid, which is taught above. But I have chiefly inſerted them for the ſake of the Definitions, which I do not remember to be in any Tract of Meaſuring, no more than many other things contained in other Propoſitions of this Appendix.

## SECT. III. *To find the ſolid Content of the Parabolic Conoid, the Parabolic Spindle, the Cylindroid, the Sphereoid, and the Hyperbolic Conoid.*

PROP. 1. *To find the ſolid Content of the Parabolic Conoid.*

*Defin.* This Solid is generated by the Rotation of a Semi-Parabola, as (*p s s*, *Fig.* 8. *Plate* C.) round its Axis *s s*; and is = half of the circumſcribing Cylinder *p q s x a s p*, (or 1 and half of the Cone inſcribed.) Therefore

*Rule.* Multiply the Area of the Baſe by the Axis or greateſt Abſciſs, and half the Rectangle is the Anſwer.

*Example.* The Diameters of the Figure (as *a p*) = 14.32, the Area of the Baſe will be found 161.056; which multiplied by the Abſciſs (*s s*) = 1432, the Rectangle is = 2306.32192; the half of which is = 1153.16096 the Anſwer.

*Prop.* 2. *To find the Solidity of the Parabolic Spindle.*

*Defin.* This Figure is generated by the Rotation of the Semi-parabola round its Ordinat, and is $\frac{8}{15}$ of the circumſcribing Cylinder. Therefore

*Rule.* Multiply the Content found as a Cylinder by 8, and divide by 15, the Quote is the Anſwer, which needs no Example.

*Prop.* 3. *To find the ſolid Content of the Cylindroid.*

*Defin.*

*Defin.* This Figure only differs from that under *Prop.* 1. *Sect.* 1. of *Chap.* 2. inaſmuch as that hath a Circle, this hath an Ellipſis for its Baſe: ſo that to meaſure it,

*Rule.* Multiply the Baſe found as *per Prop.* 3. *Sect.* 2. *Chap.* 1. by the Length, and that Product is the Anſwer.

*Example.* Admit the *Latus tranſverſum* (as *l t*, *Fig.* 7. *Plate* C.) by *c d* = the Diameter Conjugate, (*i. e.* ſuppoſe *l t* = 15.34, and *c d* = 12.28) the Area is = 147.95, which multiply'd by the Length = 1.881, the Anſwer is = 279.45.

*Prop.* 4. *To find the ſolid Content of the Sphereoid.*

*Defin.* This Figure is generated by the Rotation of the Semi-Ellipſis round its Axis, or *Latus tranſverſum*; and 'tis by ſome called the Prolate Sphereoid, to diſtinguiſh it from that which is generated by the Rotation of a Semi-Ellipſis, which turns round its Diameter Conjugate, and is called an Oblate Sphereoid, (of which Figure, 'tis aſſerted by the moſt Learned, our Earth is, the Diameter at the Equator being greater than between the Poles.) This Prolate Sphereoid is 2 Thirds of a Cylinder, whoſe Baſe's Diameter is = the greateſt Diameter of the Sphereoid, and its Altitude = the *Latus tranſverſum* of the Ellipſis.

*Rule.* Multiply the Area of a Circle, whoſe Baſe is the Conjugate Diameter, (or here the greateſt Diameter of the Sphereoid) by the Length, and 2 Thirds of the Product is the Content of the Sphereoid.

*Example.* The Diameter in the middle = 14.32, as *c d*, *Fig.* 7. the Area of that Circle 161.056, which multiply'd in the Length *l t* = 14, gives 200.48; 2 Thirds of which = 133.65, the Anſwer.

*Prop.* 5. *To find the Solidity of the Hyperbolic Conoid.*

*Defin.* This Figure is generated by the Rotation of the Semi-Hyperbola (*z h r i*) *Fig.* 9. round the Abſciſſa *z i*; which formeth the Hyperbolic Conoid *h r i n z h*.

*Rule.* The *Latus tranſverſum* = *i o*, multiply'd by 6; more the Axis or the Abſciſſa *i z* multiply'd by 6 is = the Diviſor.

2*dly*, Multiply (*h a i s n z h*) = the Content of the Cylinder (whoſe Diameter is equal to the whole Ordinat *h n*, and its Length = the Abſciſſa *z i*) in 3 times the *Latus tranſverſum*, more 2 times the Abſciſſa, and that Rectangle is = a Dividend; which divided, there ariſeth the Quotient, which is the Solidity required.

*Example.* The whole Ordinat (or Diameter) *h n* = 3.51, the Abſciſs or Axis *i z* = *h a* = 2.43, and the *Latus tranſverſum i o* = 2.16.

The

The Content of the Cylinder *basnzh*, is therefore = 23.513
The *Latus tranſverſum* mult. in 3 = 6.48
The Axis or Abſciſſa mult. in 2 = 4.82
Sum = 11.3 } Mult.

The *Latus tranſverſum* mult. in 6 = 12.96
The Axis multiplied in 6 is ... = 14.46
= Sum or Diviſor = 27.42

265.6969 = *Product*
18916
24649
9.69 = *the Anſw.*

## SECT. IV. *To find the Solidity of the Fruſtums of a Cone, Sphere, and Sphereoid.*

PROP. 1. *To find the ſolid Content of the Fruſtum of a Cone or Pyramid.*

*Defin.* This may be done ſeveral ways: as to find the Content of that part of the Cone (*drrmad*) *Fig.* 19. *Plate* C.

*Rule.* 1. Find the Content of the whole Cone (*dlmd*) then deducting the Content of the upper part (*rlr*) the reſt is the Content of the Fruſtum (*drrmd.*)

All the difficulty (more than as *Prop.* 3. *Sect.* 1. of this *Chap.*) is to find the Height (*al*) which is done by this Rule, That the homogeneal Sides of ſimilar Triangles are in proportion: ſo that as *mn. mr :: ma. ml*; whence a Perpendicular let fall to *a*, is the true Altitude of the Cone.

*Rule* 2. *But more general for a Cone, Pyramid,* &c. without the upper part: Multiply the Areas of the greater and leſſer Ends together, extract the Square Root of the Product; then multiply the Sum of the ſaid Root, and 2 Areas of the Ends by one third of the Altitude of the Fruſtum, and the Product is the Content.

*Example.* In *Fig.* 19. *Plate* C. the Area of the greater Baſe (*modxm*) is = 36.72; of the leſſer End (as *rsrsr*) = 15.5; the Product of which is = 569.16, whoſe Square Root is 23.857; which added to the 2 Areas, gives = 76.077; which multiply'd by 1 third of the Altitude *nr* = 6.6*r*, the Product is the ſolid Content of the Fruſtum = 507.17971.

*Prop.* 2. *To find the ſolid Content of the Fruſtum of a Sphere or Globe.*

*Defin.* By a Segment is meant a part of the Sphere cut off parallel to the Diameter by a right Line: as *bnerb*, *Fig.* 14. *Plate* C.

*Rule.* Find the Area of the Segment's Baſe, which multiply by the Altitude, reſerving the Product. Then

To half the Altitude of the greater Fruſtum add 1 ſixth of the Altitude of the leſſer, and multiply the Sum by the reſerv'd Rectangle, and divide this laſt by the greater Fruſtum's Altitude, and the Quote is the leſſer Fruſtum's Solidity required.

*Example.*

*Example.* Diameter of the leſſer Fruſtum = 6 = *b e*, Area = 28.2744
Its Altitude = *n s* . . . . . = 5
Altit. of the greater Fruſtum = *n o* = 7 | Reſerv'd Rectang. = 14.3720
Half that Altitude is . . . . . = 3.5
1 Sixth of the Altit. of the leſſer = .83 r 1 } Sum multipl. = 4.3r1
Product = 612.6119r1
1 Seventh of which laſt Product is = 87.516 = the Solidity of the Fruſtum required.

*Note, That if you divide the Square of the Diameter of the Fruſtum's Baſe by the Altitude of either Fruſtum, the Quote is the other Fruſtum's Altitude.*

*Prop.* 3. *To find the ſolid Content of the middle Fruſtum of a Sphereoid.*

*Defin.* This is repreſented by *r a i i c r*, *Fig.*7. like a Cask.

*Rule.* The eaſieſt Rule, and what is near the Truth, is this: Multiply the Difference of the Bung and Head Diameters by .7, and add the Product to the Head Diameter; to that Sum find the Area of a Circle, and multiply it by the Length of the Cask, and that Product is the Anſwer.

*Example.* The Bung Diameter 20 = *c d*, Head 15 = *r a* or *i i*, Length *o o* = 12. The Difference between the Bung and Head = 5, which multiply'd by .7 is 3.5; which added to the Head 15, gives 18.5 for a mean Diameter, whoſe Area is = 268.803; which multiply'd by the Length = 12, the Solidity is = 3225.638 = the Content of the ſaid Fruſtum (*r a d i o i r.*)

*Note*, That if the Sphereoid be cut in the middle of (*t m*) by a Plain (*r a*) that part (*r s t a o r*) is 5 Ninths of a Cylinder, whoſe Baſe is (*r o a*) and the Altitude (*o t.*)

I believe I have, in the Pages preceding of this Appendix, ſhewn the beſt Ways of Meaſuring a greater Variety of Superficies and Solids than any Book wrote purely on that Subject contains: and to cloſe this Chapter of Solid Meaſure, I may farther add, There is no Body, tho never ſo irregular, but what may have its Content diſcover'd by the help of Water. For example, I would know the ſolid Inches in a Faggot.

Firſt, I put Water into a Veſſel (ſuppoſe 36 Inch by 20, and is a Parallelopipedon; and taking the Depth, I find it 30 Inches deep) into which I immerſe the Faggot, ſo as to cover it with Water; and taking the Depth again, I find it 33 Inches deep, or 3 more than before the Faggot was in: ſo the ſaid 36 by 20 = 720 multiply'd in this 3, produceth 2160, the Inches ſolid contained in the Faggot.

*Brief-*

## *Brief and Useful Observations on the most Proper* INSTRUMENTS *to be used in the Mensuration of several things, &c.*

B*Ricklayers Work* is best measured by the Foot and 100 Parts: for the Divisor being 272 and a quarter, (which is the Feet in one Rod, or the Square of 16 Foot and a half) to reduce Feet into Rods, by which Denomination they value their Work; it is necessary that the Superficies of the Walls being the Dividends, should be Feet and Decimal Parts, as the Divisor is ——— = 272.25 Feet.

Or without the Rod, the shortest Way is to multiply any Number of Feet by the Value of 1 Foot at the Rate agreed on *per* Rod, as at *l.*5 : 10 : —, the Foot is in Value *l.* .020202, &c.

| | |
|---|---|
| | .02 |
| | 5.4450 |
| | 5.49991 |

Here the Multiplier being .020202, *&c.* or .02r2, I only multiply the Feet on the flat Wall by 2, and add every other Figure of the Product till there are but 2 : which are the Integers; and the Fractions to the right hand.

*Example* 2. What is the Value of 1361.25 Foot, Brick and half thick, at *l.* .0202020202, *&c.* or .02r2 *l. per* Foot, or *l.*5 : 10 *per* Rod. See the Operation.

Foot 1361.25
Multiply .... = .02r

As *per* Rule, add every other of = 27.2250
Sum or Answer = 27.4999, &c. or *l.* 27.5

*Carpenters Work.* The common two Foot serves as well as any other Instrument to measure Flooring, Partitioning, and Roofing; they being each valued by the Square of 100 Foot : so that Feet multiply'd by Feet, and 2 Figures cut off towards the right hand, gives the Squares towards the left hand.

*Painters and Plaisterers* giving in and valuing their Work by the Yard, I think the best Instrument for them to take their Dimensions with, to be a Yard divided into 100 equal Parts.

*Glaziers Work* and *Glass* being valued by the Foot, and Crown-Glass being to be done accurately, ought to be measured by the Foot, and Decimal, or 100 Parts of a Foot.

*Joiners Work* (as Wainscot, especially Oak) being valued by the Yard, ought to be measured by the Foot divided into 100 Parts, (as being of considerable Value) and the Sum of the Feet divided by 9, (the Square Feet in a Yard.)

*Land.* The best Instrument to measure it with, is a 4 Pole Chain, divided into 100 Parts or Links : for 40 single Pole long and 4 broad making 160 square Perches or 1 Acre; if the Chain be 4 Pole long, then 10 long and 1 broad make an Acre. So that whatever the Dimensions be in square Chains and Links (or hundred Parts) instead of dividing them by 160, if they were measured by a 1 Pole Chain, here you have nothing to do but to cut off 5 places from any Product to give the Acres. As suppose a Field be 40 Chains and 30 Links long, and the Breadth 30.40 Chains; the Content, by dividing the Product by 10, (or cutting off 1 place besides the 4 which are Decimals) is 122.512 Acres.

| *Chains.* |
|---|
| 40.30 |
| 30.40 |
| 122.51200 *Acres.* |

*F I N I S.*

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